Experimental Validation of an Enhanced System Synthesis Approach

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Abstract Optimizing the design and operating of a technical system is a common task for an engineer. Typically, the workflow is divided into two consecutive stages: First, a set-up is found by experience or by heuristic methods. Secondly, optimization techniques are used to compute an optimal usage strategy. This usually results in an optimal operating of a suboptimal system topology. In contrast, we apply Operations Research (OR) methods to find a cost-optimal solution for both stages simultaneously via mixed integer programming (MILP). Technical Operations Research (TOR) allows one to find a provable global optimal solution within the model formulation. However, the modeling error due to the abstraction of physical reality remains unknown. We address this ubiquitous problem of OR methods by comparing our computational results with measurements in a test rig. For a practical test case we compute a topology and control strategy via MILP and verify that the objectives are met up to a deviation of 8.7%.

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1 Introduction

Mixed-integer linear programming (MILP) [3] is the outstanding modeling technique for computer-aided optimization of real-world problems, e.g. logistics, flight or production planning. Regarding the successful application in other fields, it is desirable to transfer Operations Research (OR) methods to the optimization of technical systems. The design process of a technical system is typically divided into two consecutive stages: First, a set-up is found by an experienced engineer or by heuristic methods. Secondly, optimization techniques are used to compute an optimal usage strategy. This usually results in an optimal operating of a suboptimal system topology. Therefore, we strive to establish Technical Operations Research (TOR) in engineering sciences, and provide engineers with a methodical procedure. The TOR approach allows to find an optimal solution for both the topology decision and the usage strategy simultaneously via MILP [2].

While this formulation allows to prove global optimality and to assess feasible solutions using the global optimality gap, the modeling error often cannot be quantified. In this paper, we address this problem. We examine a practical test case and compare the computed results with measurements in a test rig.

2 Problem Description

We replicate MILP predictions for the topology and operating of a technical system in a test rig and compare the computed optimal solution to experimental results. A manageable test case is a water- conveying system, in which a certain amount of water per time has to be pumped from the source to the sink. Such a time-dependent volume flow demand can for example be observed when people shower in a multistory building. To fulfill this time-varying load, a system designer may choose one single speed-controlled pump dimensioned to meet the peak demand.

Another option is a booster station. It consists of an optional accumulator and a set of pumps which are able to satisfy the peak load in combined operation. Compared to the single pump, this set-up allows for a more flexible operating that may lead to lower energy consumption. The speed of each active pump can be adjusted according to the demand, so that they may operate near their optimal working point and thus with higher efficiency. The designer's challenging task is to trade off investment costs and energy efficiency while considering all possible topology and operating options.

3 Mixed Integer Linear Program

Our model consists of two stages: First, find a low-priced investment decision in an adequate set of pumps, pipes, accumulators and valves. Secondly, find energy-optimal operating settings for the selected components. The goal is to compare all possible systems that fulfill the load and to minimize the sum of investment and energy costs over a given deprication period.

All possible systems can be modelled by a graph $G = (V,E)$ with edges E corresponding to possible components, and vertices V representing connection points between these components. A binary variable $p_{i,j}$ for each optional component $(i, j) \in V$ indicates the purchase decision. Since accumulators can store volume, we generate a time-expansion $\mathcal{G} = (\mathcal{V},\mathcal{E})$ of the system graph G by copying it once for every time step [1]. Each edge $(i,t_i, j,t_j) \in \mathcal{E}$ connects vertices $(i,t_i) \in \mathcal{V}$ at time t_i and $(j,t_j) \in \mathcal{V}$ at time t_j. An accumulator is represented by edges in time, connecting one point in time with the next, while the other components are edges in space, representing quasi-static behavior. Binary variables a_{i,t_i,j,t_j} for each edge of the expanded graph allow to deactivate purchased components during operation. The conservation of the volume flow Q_{i,t_i,v,t_v} in space and time is given by

$$\forall\, v \in \mathcal{V}: \qquad \sum_{(i,t_i,v,t_v)\in\mathcal{E}} Q_{i,t_i,v,t_v} \cdot \Delta t = \sum_{(v,t_v,j,t_j)\in\mathcal{E}} Q_{v,t_v,j,t_j} \cdot \Delta t \tag{1}$$

with time step Δt. An additional condition with an adequate upper limit $Q_{\max}$ makes sure that only active components contribute to the volume flow conservation:

$$\forall\, e \in \mathcal{E}: \qquad Q_{i,t_i,j,t_j} \leq Q_{\max} \cdot a_{i,t_i,j,t_j} \tag{2}$$

Another physical constraint is the pressure propagation

$$\forall\, (i,t_i, j,t_i) \in \mathcal{E}: \qquad p_{j,t_i} \leq p_{i,t_i} + \Delta p + M \cdot a_{i,t_i,j,t_i} \tag{3}$$

$$p_{j,t_i} \geq p_{i,t_i} + \Delta p - M \cdot a_{i,t_i,j,t_i} \tag{4}$$

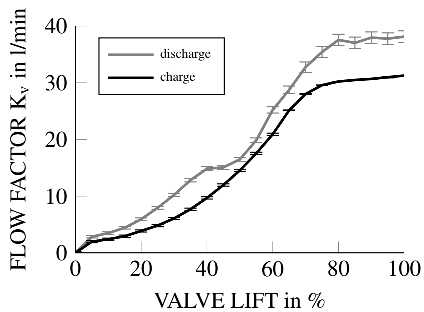
which has to be fulfilled along each edge in each time step, if the component is active. Regarding pumps, the resulting increase of pressure depends on the rotational speed of the pump and on the volume flow that is conveyed, cf. Fig. 1(b). For pipes and valves, pressure loss increasing with the volume flow is observed, cf. Figs. 1(a), 1(c) and 1(d). All of the measured characteristic curves were linearly approximated and included in the model by a convex combination formulation. [4]

4 Experimental Validation

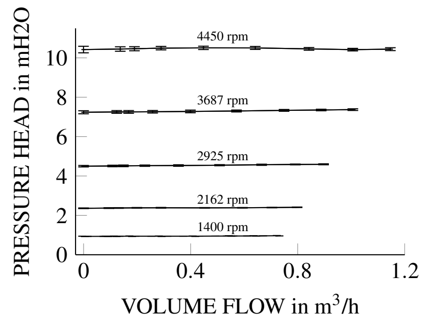
To validate our mathematical model, we look at three test cases with different time-dependent demand profiles. To assess the modeling error, the computed optimal combination of the available components is replicated in an experimental set-up, and the settings of the system (e.g. the speed of the used pumps or the valve lift) are adjusted according to the computed optimal variable assignment. Subsequently, we verify if the demand profiles are met in each time step. Moreover, the energy consumption of the set-up is measured and the resulting energy costs are calculated and compared to the objective value of the mathematical model.

4.1 The Test Rig

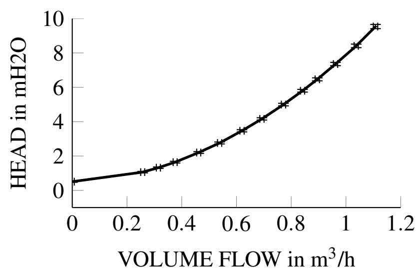
Fig. 2 shows the modular test rig used for validation measurements. It consists of a combination of up to three speed-controlled centrifugal pumps in a row and an optional acrylic barrel which serves as volume and pressure accumulator. The three pumps differ in their maximum rotating speed (S: 2800 rpm, M: 3400 rpm,



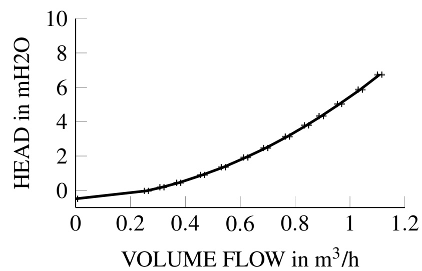
(a) Characteristic curves of the valve. Discharging the accumulator causes more pressure loss than charging it.



(b) Characteristic curve of the most powerful of the available three pumps with a speed range of 1400 rpm - 4500 rpm.



(c) Characteristic curve of the system for the section from the source to the junction, cf. Fig. 2. A geodetic offset of around 0.5 m has to be overcome from the source to the junction.



(d) Characteristic curve from junction to sink. Since the geodetic height of the sink is around 0.5 m lower than that of the junction, this curve starts with negative pressure values.

Fig. 1 Input data for the model are the measured characteristic curves of the components of the fluid system. Each data point is the mean value of 10,000 samples. The error bars depict the corresponding standard deviation.

L: 4450 rpm) and power consumption. Fig. 1(b) depicts the characteristic curves of pump L. The accumulator has a maximum volume of 50 l and a maximum storable pressure of $\approx 0.2\,\text{bar}$. The barrel can be charged and discharged via a controllable valve, cf. Fig. 1(a). Closing the ball valve allows to charge the accumulator without conveying water to the sink. The volume flow is measured by a magnetic flow meter with a tolerance of $\pm 0.1\,\text{l/min} = \pm 0.006\,\text{m}^3/\text{h}$. Pressure measurements are performed by manometers with a tolerance range of $\pm 0.01\,\text{bar} \approx \pm 0.1\,\text{mH2O}$. All data points represent the mean value of 10,000 samples, collected within 10 s.

4.2 Comparison of Optimization Results and Measurements

Three different load profiles are given as an input to the optimization program. We built every calculated first-stage solution on our test rig, set up the control strategy and measured the volume flows at the sink and the power consumption of the pumps. The measurement results are given in Figs. 3 - 5.

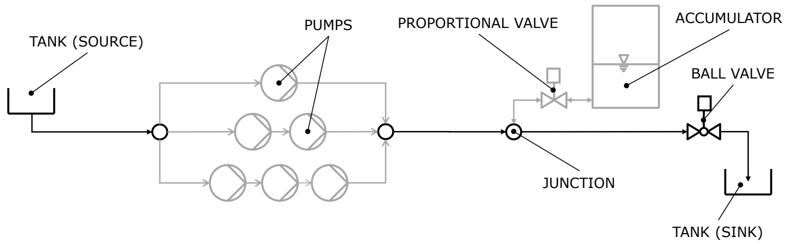


Fig. 2 The test rig consists of a combination of up to three out of three different speed-controlled centrifugal pumps. An optional accumulator can be used to fullfill the demand at the sink. It can be charged and discharged via a controllable valve.

The time-varying flow demand of the first test case is between $0.25\,\mathrm{m^3/h}$ and $0.6\,\mathrm{m^3/h}$. It can be fulfilled by pump M, but not by pump S. As pump M is at a lower price than pump L, the optimal result via MILP is to buy pump M. The measured flow is in good agreement with the demand profile, cf. Fig. 3, if the pump is driven with the predicted control settings. The computed total energy consumption for a recovery period of 10 years is 1.9126×10^3 kWh, corresponding to energy costs of €478.14, and total costs of €923.14. The measured energy consumption for one repetition of the load cycle is $(663 \pm 112) \times 10^3$ kWh, which sums up to $(1.9369 \pm 0.1117) \times 10^3$ kWh and €(484.23 ± 9.57) within 10 years.

The second test case contains higher flow demands than the first one: $0.4\,\mathrm{m^3/h}$ to $0.9\,\mathrm{m^3/h}$. The optimization result is to use pumps L and M to cover the load. The demanded and measured volume flow rates match, cf. Fig. 4. During a recovery period of 20 years the pumps consume 1.0565×10^4 kWh according to the optimization result, compared to $(1.0765 \pm 0.0239) \times 10^4$ kWh derived from the measurements. This leads to total optimal costs of €3436.27, compared to €(3486.32 ± 59.85). Pump L could have also been used, but its energy consumption is higher for flow demands around $0.7\,\mathrm{m^3/h}$.

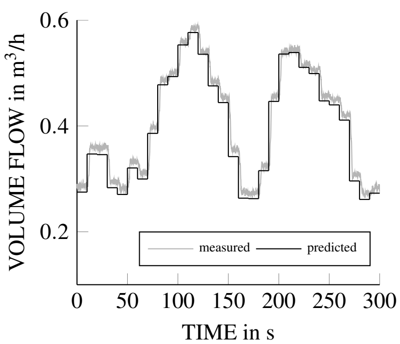


Fig. 3 Test case 1. One pump fulfills the load.

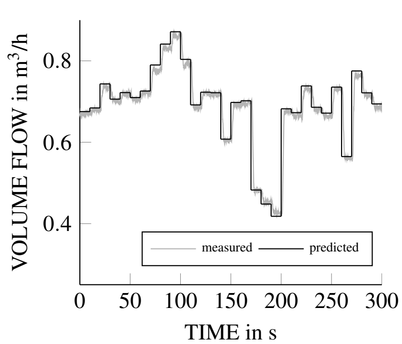


Fig. 4 Test case 2. Two pumps fulfill the load.

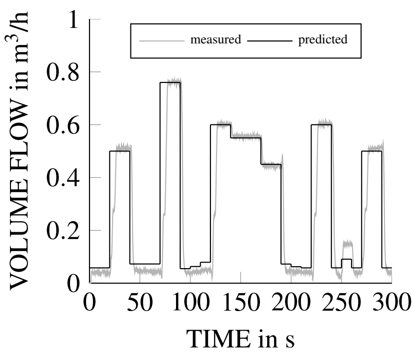


Fig. 5 Test case 3. One pump in combinaton with accumulator fulfils the load.

The flow demands in the third test case range from 0.1 m^3/h to 0.8 m^3/h. The optimal topology consists of pump L, of the accumulator and the valve. In the test rig configuration this pump cannot convey volume flows as low as 0.05 m^3/h. The optimization model correctly predicts the usage of the accumulator during time steps with these small demands. Though the accumulator is already loaded in the first time step with a filling level of 13.34 cm. It has to be recharged till the last time step to forbid an energy gain for free. In Fig. 5 the measured data is in satisfactory agreement with the time-varying demand. The optimal energy costs are € 537.57. Compared to € (584.27 ± 27.91) derived from the measurements this corresponds to the highest observed deviation of 8.7 %. For all test cases a delayed step response of around 5 s - 10 s to the changed rotational speed settings can be observed.

5 Conclusion

In this paper, we presented a MILP model for a system synthesis problem. We were able to find the best combination out of a set of pumps, valves and accumulators to satisfy a given time-dependent flowrate demand with minimal weighted purchase and energy costs. The predicted topology and operating decisions were validated in an experimental setup for three different load demands. In each case, the measured volume flows and the power consumption of the pumps resembled the predicted values with satisfying accuracy, even though our model is based on a quasi-static formulation. One reason for these deviations could be the delayed response of the pumps when changing their speed settings.We plan to investigate the influence of the time steps size on the modeling error in a future research project. This will allow us to determine to which degree the components' start-up characteristics and deferred adaptation should be included into our model formulation.

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