Exact and numerical invertibility of pseudodifferential operators and applications to signal processing

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Abstract

A large class of time-varying filters can be described via pseudodifferential operators belonging to the Hörmander class $OPS^0_{0,0}$. The questions whether and how an input signal can be reconstructed from a known output lead to the problems of invertibility of pseudodifferential operators in that class and of (at least, numerical) solution of pseudodifferential equations. We are going to derive effective conditions for the invertibility for pseudodifferential operators with globally slowly varying symbols as well as for causal pseudodifferential operators, and we study the stability of the finite sections method with respect to time and frequency for these operators.

1 Introduction

We start with recalling some basic definitions and facts from signal processing theory. Standard references to this field are [8, 9, 4, 22], for instance.

An *analog complex signal* (ACS for short) is a function $u : \mathbb{R} \to \mathbb{C}$. We will only consider ACS with finite energy, that is, we suppose that u belongs to the Hilbert space $L^2(\mathbb{R})$ with norm

$$\|u\|_2 := \left(\int_{\mathbb{R}} |u(t)|^2 \, dt \right)^{1/2} < \infty.$$

*The authors are grateful for the support by the German Research Foundation (DFG), grant 444 MEX-112/2/05.