An mTSP-CTW Model for Optimising School Starting Times and Public Bus Services

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Abstract. In rural areas the public bus service is typically demand-oriented: By far the biggest group of customers are pupils who are transported to their schools within certain strict time limits. Usually, all schools start around the same time, which causes a morning peak in the number of deployed buses. However, schools are allowed to change their starting times within some intervall. The question is, how to simultanenously rectify the starting times for all schools and bus trips in a certain county so that the number of scheduled buses is minimized. We present a mixed-integer programming formulation for this optimization problem and address its solution for a real-world instance.

1 Introduction

In rural areas pupils on their ways to school and back home are usually the biggest group of customers for public means of transportation. In Germany it has become custom that schools start in a small time interval around 8:00. On the transportation side, this leads to a high number of deployed vehicles (i.e., buses) that have to transport the pupils in time, with a peak from 7:00 to 8:00. After having served the morning rush hour, most of the buses are sent back to the depot, for there is nearly no further demand. The afternoon peak at the end of school is much lower, for schools do not release all pupils at the same time. Hence the main focus for optimization, i.e., the reduction of the total number of deployed buses, is the morning peak. The optimization problem can be manifested by the following questions:

- When should schools start, subject to the legal bounds?
- When should the bus trips start so that customers (pupils and others) can use the current changes between trips, and pupils don't have to wait unacceptably long at school?
- What is a bus doing after serving a trip: driving back to the depot or serving another trip?

Suppose for a moment we have some kind of tool that could answer all these questions. Then, despite that, there is one important point, which has nothing to do with optimization, but nevertheless is as important as the previous ones:

- Who is implementing the solution in real-world, negotiating with the county administration, the transportation companies, the schools and teachers, the pupils and their parents, for short, all groups of interest that are affected by the changes in public transit?

To give an example, if some school doesn't start at, say, 7:50, but at 8:30, then all respective pupils leave home more than half an hour later, which might cause troubles for working parents. In the afternoon, the child returns later, and therefore might no longer be able to attend a sport's club. Here the consulting company *BPI-Consult*, a subsidiary of the Finish *Jaakko Pöyry*, enters the stage. Appointed from the county administration, they develop new trip time tables, bus schedules, and school starting times. Then, BPI accompanies the whole process of embedding their solution into real-life, which includes negotiations with all participants and potentially re-optimization, when new constraints appear that make previous solutions infeasible. Up to now, BPI successfully consulted four counties and one city, where the number of buses was reduced by 15 – 20%, which yields a yearly cost saving of 10 – 15%, see [11] for details. In each of these cases, the solutions were generated manually.

Note that BPI-Consult is not interested in changing the routes of bus trips, in the creation of new lines and trips, or the deletion of some existing trips, even if the number of deployed buses could be lowered. This is mainly a political decision, because changing only the starting times of trips and schools already creates enough opposition. In addition it is required that after the optimization all pupils are using the same bus trips to come to school as they do now. The mathematical model we present in this article reflects these restrictions. It was designed to become part of a software tool that supports the planning part of BPI's work.

A wide range of transportation problems involving public bus transit, pupils and/or schools were already studied before, see [2], [4], [3], [5], [7], or [8], to name just a few. However, none of the presented models completely fits to our problem, mainly for some or all of the following reasons. With exception of [8], the time windows of school starting times are fixed and cannot be changed to save buses. In all modelling approaches, pupils are always transported directly to school, and changing the bus is not allowed. Locating bus stops, designing routes (trips) and assigning pupils to routes is sometimes part of the optimization, but for us these are input figures. Finally, scheduling drivers is not an issue for us: Since our time horizon is small (from 6:00 till 8:30), no planning of breaks is needed.

In Section 2 we come up with a new model. This model is applied to real-world data collected by BPI, the results are discussed in Section 3. Finally, Section 4 gives an outlook on future directions of our research.

2 A Mixed-Integer Programming Model

In general, our problem belongs to the class of vehicle routing problems, or VRP, for short, where a fleet of vehicles starting at a depot is sent to customers, picks up some commodities up to a maximum load and drives back to the depot. Dropping the capacity restrictions in VRP results in a multiple traveling salesman problem, or mTSP. The model we present now is based on a model for the classical single-depot multiple traveling salesman problem with (static) time windows, or mTSP-TW for short, see [6], for instance. We start with a detailed description of all sets and parameters needed as input figures.

2.1 Sets and Parameters

Let $\mathcal{S}$ be the set of all schools in the given county. For every school $s \in \mathcal{S}$, a time window $\underline{\tau}_s, \overline{\tau}_s \in \mathbb{Z}_+, \underline{\tau}_s \leq \overline{\tau}_s$, for the school starting time is given.

A *trip* t is a list of bus stops, where a departure time is assigned to every bus stop of this list. Let $\mathcal{V}$ be the set of all bus trips in the county under consideration. The starting time of trip $t \in \mathcal{V}$, i.e., the time of departure at the first bus stop, is allowed to be shifted within the time window $\underline{\alpha}_t, \overline{\alpha}_t \in \mathbb{Z}_+, \underline{\alpha}_t \leq \overline{\alpha}_t$. The time a bus needs to serve trip t, i.e., the time difference between first and last bus stop, is denoted by $\delta_t^{\text{trip}} \in \mathbb{Z}_+$. We distinguish four different types of bus trips, called school trips, feeder and collector trips, and free trips.

A trip $t \in \mathcal{V}$ is called *school trip* for school $s \in \mathcal{S}$, and we write $(s,t) \in \mathcal{P}$, if trip t transports pupils to a bus stop of school s. The driving time from the first bus stop of the trip till the school's bus stop is settled by parameter $\delta_{st}^{\text{school}} \in \mathbb{Z}_+$. For the pupils there is given a minimal and maximal waiting time $\underline{\omega}_{st}^{\text{school}}, \overline{\omega}_{st}^{\text{school}} \in \mathbb{Z}_+, \underline{\omega}_{st}^{\text{school}} \leq \overline{\omega}_{st}^{\text{school}}$ relative to the starting time of the school in which they must arrive at the school bus stop. The lower bound on this time interval $\underline{\omega}_{st}^{\text{school}}$ actually reflects the walking time from the school bus stop to the classroom in school, whereas the maximum waiting time $\overline{\omega}_{st}^{\text{school}}$ is specified by law.

Not all customers (pupils and others) arrive at their final destination using only one trip. If they start their journey with some trip $t_1 \in \mathcal{V}$ and transfer to some other trip $t_2 \in \mathcal{V}$ at a so-called *changing bus stop*, then we call trip t_1 a *feeder trip* for *collector trip* t_2, and we write $(t_1, t_2) \in \mathcal{C}$. The driving time for feeder trip t_1 from the first bus stop of the trip till the changing bus stop is settled by parameter $\delta_{t_1 t_2}^{\text{feeder}} \in \mathbb{Z}_+$, for collector trip t_2 this is $\delta_{t_1 t_2}^{\text{collector}} \in \mathbb{Z}_+$. The minimum and maximum waiting time at this bus stop is given by the time window $\underline{\omega}_{t_1 t_2}^{\text{change}}, \overline{\omega}_{t_1 t_2}^{\text{change}} \in \mathbb{Z}_+, \underline{\omega}_{t_1 t_2}^{\text{change}} \leq \overline{\omega}_{t_1 t_2}^{\text{change}}$. Note that a trip can have more than one type, p.e., it can be a school and feeder trip.

All other trips that are not school, feeder or collector trips are called *free trips*. These trips don't play a role for the transport of pupils. Nevertheless, they also have to be served.

When a bus finishes a trip, it either starts serving another trip or it is sent back to the depot. The connection of several trips that are served by the same bus is called *schedule* or *block*. The set $\mathcal{A} \subset \mathcal{V} \times \mathcal{V}$ consists of all pairs of trips that might be connected in some schedule. The intermediate trip from the last bus stop of trip t_1 to the first bus stop of trip t_2, where no passangers are transported, is called a *shift* or a *deadhead trip*. The driving time for a shift is denoted by $\delta^{\text{shift}}_{t_1 t_2} \in \mathbb{Z}_+$ for all $(t_1, t_2) \in \mathcal{A}$. If we formulate this problem in a graph, where the nodes correspond to the trips and the arcs to the shifts, then the blocks are paths in the directed graph $(\mathcal{V}, \mathcal{A})$.

In principle, every trip may be served by a new bus from the depot. The assignment of a new bus from the depot to a trip t contributes with cost $C_t \in \mathbb{Z}_+$ to the objective value.

2.2 Variables and Bounds

For every bus trip $t \in \mathcal{V}$ we introduce a decision variable $v_t \in \{0,1\}$ with $v_t = 1$ if and only if trip t is the first trip in some block. In the same manner, $w_t \in \{0,1\}$ is a decision variable with $w_t = 1$ if and only if trip t is the last trip in a block.

For the connection of two trips $(t_1, t_2) \in \mathcal{A}$ in a block we make use of a decision variable $x_{t_1 t_2} \in \{0,1\}$ with $x_{t_1 t_2} = 1$ if and only if some bus serves trip t_2 directly after finishing trip t_1. The starting time of trip $t \in \mathcal{V}$ (i.e., departure of a bus at the first bus stop of trip t) is settled by the real-valued variable $\alpha_t \in \mathbb{R}_+$ which is bounded by the respective time window, $\underline{\alpha}_t \leq \alpha_t \leq \overline{\alpha}_t$.

For every school $s \in \mathcal{S}$, the starting time is modeled by a variable $\tau_s \in \mathbb{R}_+$ with $\underline{\tau}_s \leq \tau_s \leq \overline{\tau}_s$. It is required that the new starting time is a multiple of 5 (minutes), thus we introduce an integer variable $\tau^{\text{I}}_s \in \mathbb{Z}_+$ with $\tau^{\text{I}}_s \leq \frac{\overline{\tau}_s - \underline{\tau}_s}{5}$ and set $\tau_s = \underline{\tau}_s + 5 \cdot \tau^{\text{I}}_s$.

2.3 mTSP-TW

In a feasible solution, every trip must be served by exactly one bus. The trip can either be the first one in a block or it has a unique predecessor:

$$\sum_{(t_1,t_2)\in\mathcal{A}} x_{t_1 t_2} + v_{t_2} = 1, \quad \forall\, t_2 \in \mathcal{V}. \tag{1}$$

Furthermore, it can either be the last one in a block or it has a unique successor:

$$\sum_{(t_1,t_2)\in\mathcal{A}} x_{t_1 t_2} + w_{t_1} = 1, \quad \forall\, t_1 \in \mathcal{V}. \tag{2}$$

If trips t_1 and t_2 are connected, then trip t_2 can only start after the bus has finished trip t_1 and shifted from the end of t_1 to the start of t_2. Waiting is permitted, if the bus arrives there before the start of t_2. Using a sufficient big value for M, these constraints can be formulated in terms of linear equations:

$$\alpha_{t_1} + \delta^{\text{trip}}_{t_1} + \delta^{\text{shift}}_{t_1 t_2} - M \cdot (1 - x_{t_1 t_2}) \leq \alpha_{t_2}, \quad \forall\, (t_1, t_2) \in \mathcal{A}. \tag{3}$$

Note that these inequalities are already sufficient to avoid the appearence of sub-cycles within any feasible solution.

The main goal is the saving of buses, a secondary goal is to use the deployed buses in an efficient way and avoid long shifts where no customers are transported. Thus for sufficient big values of C_t, an objective function reflecting this goal can be stated as

$$z = \text{minimize} \sum_{t \in \mathcal{V}} C_t \cdot v_t + \sum_{(t_1,t_2) \in \mathcal{A}} \delta^{\text{shift}}_{t_1 t_2} \cdot x_{t_1 t_2}. \tag{4}$$

So far, the presented formulation is a model for the the classical single-depot multiple traveling salesman problem with (static) time windows, or mTSP-TW for short. Next, we extend this model with respect to restrictions due to school starting times and restrictions for pupils who have to change the bus on their way to school.

2.4 mTSP-CTW

In contrast to the classical mTSP-TW, the time windows for the trips are not fixed from the beginning, but depend on other time windows. For this reason, we call our problem *mTSP-CTW*, where the character "C" represents the coupling aspect of the time windows.

The problem under consideration gives rise for two different kind of coupled time windows. The first one we call *internal coupling* of time windows. In our application, the time windows for feeder and collector trips are coupled internally. For $(t_1, t_2) \in \mathcal{C}$, bus trip t_2 must arrive at the changing bus stop after trip t_1 within a small time window specified by $\underline{\omega}^{\text{change}}_{t_1 t_2}$ and $\overline{\omega}^{\text{change}}_{t_1 t_2}$. This synchronization is done by the following inequalities:

$$\alpha_{t_1} + \delta^{\text{feeder}}_{t_1 t_2} + \underline{\omega}^{\text{change}}_{t_1 t_2} \leq \alpha_{t_2} + \delta^{\text{collector}}_{t_1 t_2}, \tag{5}$$

$$\alpha_{t_1} + \delta^{\text{feeder}}_{t_1 t_2} + \overline{\omega}^{\text{change}}_{t_1 t_2} \geq \alpha_{t_2} + \delta^{\text{collector}}_{t_1 t_2}, \qquad \forall\ (t_1, t_2) \in \mathcal{C}. \tag{6}$$

The second one we call *external coupling* of time windows. In our application, the external source are the schools, where coupling of time windows arises from the minimal and maximal waiting time restriction for pupils at their respective schools. Thus, we have to add the following inequalities to the model in order to synchronise the start of bus trips and schools:

$$\alpha_t + \delta^{\text{school}}_{st} + \underline{\omega}^{\text{school}}_{st} \leq \tau_s, \tag{7}$$

$$\alpha_t + \delta^{\text{school}}_{st} + \overline{\omega}^{\text{school}}_{st} \geq \tau_s, \qquad \forall\ (s, t) \in \mathcal{P}. \tag{8}$$

Summing up, our MIP model consists of optimizing the linear objective function (4) subject to the set of linear constraints (1) – (3) and (5) – (8). Note that an instance of mTSP-CTW, where all trips are free trips (i.e., $\mathcal{P} = \mathcal{C} = \emptyset$), is in fact an instance of mTSP-TW.

From a theoretical point of view mTSP-CTW is NP-hard. Checking feasibility for mTSP-CTW with a fixed number of vehicles is NP-complete, for it contains mTSP-TW with static time windows as a special case, which is known to be NP-complete, see Savelsbergh [10]. Thus we cannot expect a polynomial algorithm for its solution, unless $P = NP$.

3 Computational Results

In order to quickly solve the model, we developed a primal solution method, i.e., a greedy-type heuristic algorithm[1]. For an evaluation of potential savings for the simultaneous optimization of school starting times and public bus services, we tested our heuristic on real-world data from a county in the north-west of Germany. This county is rural, for the population density is rather low. The biggest city has no more than 30,000 inhabitants, at most 150,000 people live in an area of about 1,000 square kilometers. More than half of all pupils are coming to school by public transit, that is, about 15,000 pupils take the bus to 102 different schools. The average way to school has a length of around 10 km, a few pupils travel even more than 30 km twice a day. Today, most of the schools start between 7:50 and 8:00. In the morning from 6:00 till 8:30, the bus company deploys 227 buses to serve 490 trips. There are 406 internally coupled time windows to ensure that all existing transfers between two trips will also be possible after the optimization. Moreover, there are 574 couplings of school and trip time windows. Our solution found by the heuristic (after only a few seconds of computations) reduces the number of deployed buses by 17% (from 227 down to 188). Figure 1 shows the current (dashed line) and the planned (solid line) school starting time distribution. Figure 2 shows the current (dashed line) and the planned (solid line) number of simultaneously deployed buses. Note that the maxima of these curves are lower than the total number of deployed buses, for not all buses are in use at the same time.

However, there is no guarantee on the quality of this solution, perhaps even more savings might be possible. Standard state-of-the-art mixed integer solvers (`CPlex`, see [1]) are able to give lower bounds on the number of vehicles. From this we know that no feasible solution can use less than 163 buses. However, `Cplex` was not able to close the gap between lower bound and best known feasible solution, even after some days of computations.

4 Conclusions and Future Research

In this article we presented mTSP-CTW as a generalization of the mTSP-TW problem, and gave a mixed-integer programming model. We applied this model to the problem of the simultaneous optimization of school starting

[1] The details of this algorithm will be described in a forthcoming article.

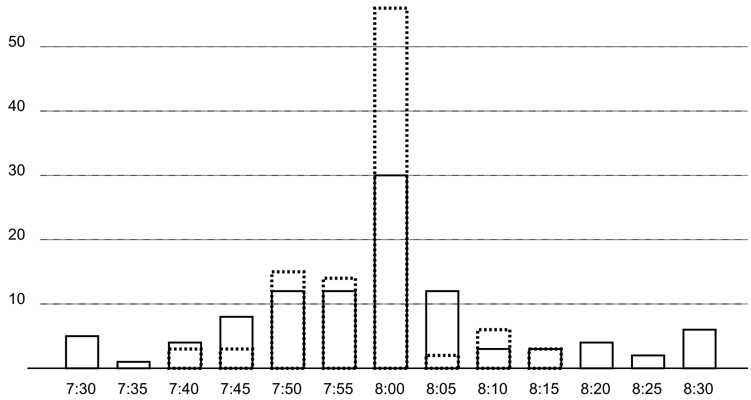

Fig. 1. Current and planned school starting time distribution.

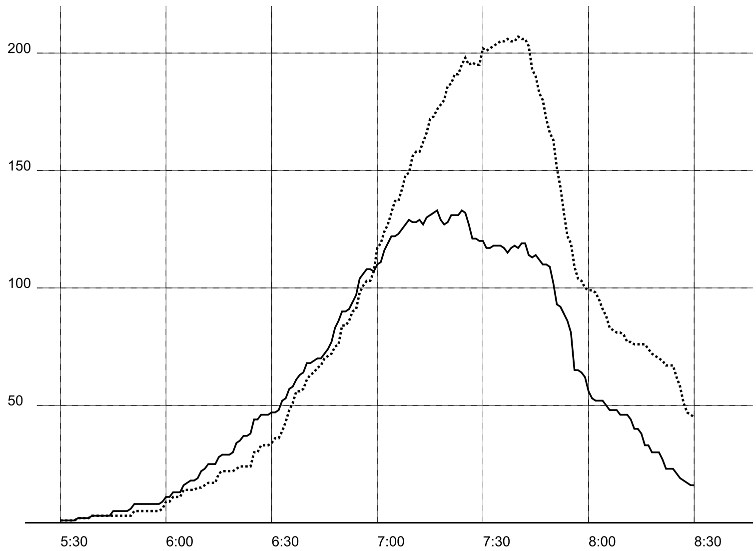

Fig. 2. Current and planned number of simultaneously deployed buses.

times and public bus services. The computational results presented show that our heuristic is able to obtain savings. In the future, we will refine the primal heuristic approach and improve the dual bound by strengthening the LP-relaxation to obtain optimal solutions.

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