Maximal Regularity for the Instationary Stokes System in an Aperture Domain

Andreas Fröhlich

Darmstadt University of Technology, Schlossgartenstr. 7,
D-64289 Darmstadt, Germany, email: froehlich@mathematik.tu-darmstadt.de

Abstract

We prove estimates in $L^s(0,T;L^q_\omega(\Omega))$ for the solution of the instationary Stokes system in an aperture domain, where $1 < s, q < \infty$ and the weight function ω is in the Muckenhoupt class A_q.

The result is achieved by combining a characterisation of maximal regularity by $\mathcal{R}$-bounded operator families with the fact that $\mathcal{R}$-boundedness follows from weighted estimates for Muckenhoupt weights.

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1 Introduction

Let $d \geq 0$, $n \geq 2$ and $\mathbb{R}^n_\pm = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n \ : \ \pm x_n > d/2\}$. Then an open connected set $\Omega \subset \mathbb{R}^n$ is called an aperture domain if there is a ball $B = B_R$ of radius $R > 0$ such that $\Omega \cup B = \mathbb{R}^n_+ \cup \mathbb{R}^n_- \cup B$.

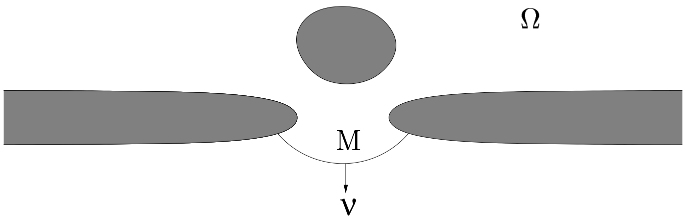

Figure 1: An aperture domain

In an aperture domain $\Omega \subset \mathbb{R}^n$ we consider the instationary Stokes system

$$u_t - \Delta u + \nabla p = f \tag{1}$$

$$\operatorname{div} u = 0 \tag{2}$$

$$u\big|_{\partial\Omega} = 0 \tag{3}$$

$$u(0) = u_0. \tag{4}$$

Heywood [16] pointed out that in order to single out a unique solution in the L^2-theory an additional auxiliary condition must be imposed. For example the flux $\Phi(u) = \int_M u \cdot \nu \, d\sigma$ through aperture can be prescribed, where M is a smooth bounded $(n-1)$-dimensional