Prof. Stefan Ulbrich (TU Darmstadt)

Reversible solutions of transport equations in the context of optimal control of hyperbolic conservaton laws

We consider optimal control problems for hyperbolic systems of conservation or balance laws, where the control acts in the initial data, right hand side and possibly also in the boundary data. Since the solution can develop discontinuities (shocks, contact discontinuities) which move depending on the control, the rigorous development of a variational calculus, of optimality conditions and of differentiability results for objective functionals is involved.

It turns out that in quite general settings a large class of objective functions is nevertheless differentiable with respect to the control and that its derivative can be represented by an adjoint PDE, which is a transport equation with discontinuous coefficient. This transport equation has a whole family of solutions and the concept of reversible solutions is required to select uniquely the correct solution. We will discuss reversible solutions of the adjoint PDE for the optimal control of scalar balance laws with strictly convex flux as well as for the optimal control of the generalized Riemann problem for strictly hyperbolic systems of balance laws in one space dimension.

We will also briefly discuss how the correct reversible solution of the adjoint PDE can be computed by suitable numerical schemes.