



**International Conference  
Nonlinear PDEs in Fluid Dynamics  
CIRM, Luminy  
May, 9th-13th, 2022**

**Within the framework of the  
JEAN-MORLET CHAIR  
Nonlinear PDEs in Fluid Dynamics  
Matthias Hieber  
Sylvie Monniaux**

**Speakers:**

H. Abels  
T. Alazard  
P. Auscher  
L. Brandolese  
D. Cordoba  
K. Disser  
E. Feireisl  
G.P. Galdi  
S. Ibrahim  
M. Ifrim  
P. Kaplicky

H. Kozono  
I. Kukavica  
P. Kunstmann  
M. Lopes Filho  
N. Masmoudi  
S. Modena  
C. Nobili  
H. Nussenzveig Lopes  
S. Shimizu  
G. Simonett  
A. Swierczewska-Gwiazda

D. Tataru  
E. Titi  
P. Tolksdorf  
M. Tucsnak  
K. Widmayer  
E. Wiedemann  
Z. Xin  
H-M. Yin  
P. Zhang





---

## Contents

---

<b>1 Program</b>	<b>3</b>
<b>2 Abstracts</b>	<b>9</b>
<b>3 Participants</b>	<b>25</b>
<b>4 Notes</b>	<b>29</b>



# 1 Program

Conference Nonlinear PDEs in Fluid Dynamics  
Luminy, May 9th-13th, 2022

Time	Monday, May 9th	Tuesday, May 10th	Wednesday, May 11th	Thursday, May 12th	Friday, May 13th
08:45h	Opening				
09:00h-09:35h	Daniel Tataru	Diego Cordoba	Nader Masmoudi	Zhouping Xin	Pascal Auscher
09:45h-10:20h	Senjo Shimizu	Eduard Feireisl	Hideo Kozono	Helena Nussenzveig Lopes	Klaus Widmayer
10:30h-11:00h	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00h-11:55h	Gieri Simonett	Ping Zhang	Mihaela Irim	Edris Titi	Emil Wiedemann
11:45h-12:20h	Igor Kukavica	Stefano Modena	Thomas Alazard	Lorenzo Brandolese	Marius Tucsnak
12:20h					Closing
12:30h-14:00h	Lunch	Lunch	Lunch	Lunch	Lunch
14:00h-15:30h	free discussion	Free discussion	Excursion calanques (14:00h-19:00h)	Free discussion	
15:30h-16:05h	Paolo Galdi	Milton Lopes Filho		Heimut Abels	
16:15h-16:50h	Patrick Tolksdorf	Slim Ibrahim		Karoline Disser	
16:50h-17:20h	Coffee Break	Coffee Break		Coffee Break	
17:20h-17:55h	Short Presentations (17:20h-19:00h)	Camilla Nobili		Agnieszka Swierczewska-Gwiazda	
18:05h-18:40h		Petr Kaplicky		Hong-Ming Yin	
18:50h-19:25h	Kr de bienvenue (19:00h-19:30h)	Short Presentations		Peer Christian Kunstmann	
19:30h-20:30h	Dinner	Dinner	Dinner	Bouillabaisse	
20:30h-21:30h			Soirée musicale		

---

---

**Monday, 9th May 2022**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
09:00-09:35	Daniel Tataru	<i>Global solutions in one dimensional dispersive flows</i>
09:45-10:20	Senjo Shimizu	<i>Free boundary problems for the incompressible Navier-Stokes equations in critical spaces</i>
11:00-11:35	Gieri Simonett	<i>On the Navier-Stokes equations on surfaces</i>
11:45-12:20	Igor Kukavica	<i>The global existence for a fluid-structure interaction system</i>
15:30-16:05	Paolo Galdi	<i>Navier-Stokes Flow past a Rigid Body that Moves by Time-Periodic Motion</i>
16:15-16:50	Patrick Tolksdorf	<i>On off-diagonal behavior of the generalized Stokes operator</i>
17:20-19:00	Short presentations	

---

---

**Tuesday, 10th May 2022**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
09:00-09:35	Diego Cordoba	<i>Instant blow-up for the generalized SQG equations</i>
09:45-10:20	Eduard Feireisl	<i>Statistical solutions to the compressible Navier-Stokes system: Analysis and numerics</i>
11:00-11:35	Ping Zhang	<i>Global hydrostatic approximation of hyperbolic Navier-Stokes system with small Gevrey class 2 data</i>
11:45-12:20	Stefano Modena	<i>Non-newtonian fluids and convex integration</i>
15:30-16:05	Milton Lopes Filho	<i>Small obstacle limit for the inviscid Euler-alpha system</i>

---

<b>16:15-16:50</b>	Slim Ibrahim	<i>Revisit singularity formation for the inviscid primitive equations</i>
<b>17:20-17:55</b>	Camilla Nobili	<i>Bounds on mixing norms for advection diffusion equations</i>
<b>18:05-18:40</b>	Petr Kaplicky	<i>On solutions for a generalized Navier-Stokes-Fourier system fulfilling the entropy equality</i>
<b>18:50-19:25</b>	Short presentations	

---



---

**Wednesday, 11th May 2022**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
<b>09:00-09:35</b>	Nader Masmoudi	<i>tba</i>
<b>09:45-10:20</b>	Hideo Kozono	<i>Analyticity in space-time of solutions to the Navier-Stokes equations via parameter trick based on maximal regularity</i>
<b>11:00-11:35</b>	Mihaela Ifrim	<i>The time-like minimal surface equation in Minkowski space: low regularity solutions</i>
<b>11:45-12:20</b>	Thomas Alazard	<i>The Cauchy problem for the Muskat equation</i>

---

---

---

**Thursday, 12th May 2022**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
<b>09:00-09:35</b>	Zhouping Xin	<i>On The Existence of Multi-dimensional Compressible MHD Contact Discontinuities</i>
<b>09:45-10:20</b>	H. Nussenzveig Lopes	<i>Vanishing viscosity and conserved quantities for 2D incompressible flow</i>
<b>11:00-11:35</b>	Edriss Titi	<i>A New Blow-up Criterion for the 3D Euler Equations: A Computational Study</i>
<b>11:45-12:20</b>	Lorenzo Brandolese	<i>Large global solutions of the parabolic-parabolic Keller–Segel system in higher dimensions</i>
<b>15:30-16:05</b>	Helmut Abels	<i>On a fluid-structure interaction problem for plaque growth</i>
<b>16:15-16:50</b>	Karoline Disser	<i>Global solutions for fluid-elastic interaction with small data</i>
<b>17:20-17:55</b>	A. Swierczewska-Gwiazda	<i>Euler-Poisson equation - weak-strong uniqueness principle for dissipative measure-valued solutions</i>
<b>18:05-18:40</b>	Hong-Ming Yin	<i>On a Reaction-Diffusion System and Application to a Reactive-Flow Model</i>
<b>18:50-19:25</b>	Peer Kunstmann	<i>Functional calculi for Stokes operators with first order boundary conditions on unbounded domains</i>



---

---

**Friday, 13th May 2022**

---

<b>Time</b>	<b>Speaker</b>	<b><i>Title of Talk</i></b>
<b>09:00-09:35</b>	Pascal Auscher	<i>Tent spaces techniques for evolution PDE's</i>
<b>09:45-10:20</b>	Klaus Widmayer	<i>Global axisymmetric Euler flows with rotation</i>
<b>11:00-11:35</b>	Emil Wiedemann	<i>Statistical Solutions of the 2D Euler Equations</i>
<b>11:45-12:20</b>	Marius Tucsnak	<i>Large time behavior for solids driven by a viscous flow</i>

---

# On a fluid-structure interaction problem for plaque growth

Helmut Abels

Universität Regensburg,  
Regensburg, Germany

helmut.abels@mathematik.uni-regensburg.de

We study a free-boundary fluid-structure interaction problem with growth, which arises from the plaque formation in blood vessels. The fluid is described by the incompressible Navier-Stokes equation, while the structure is considered as a viscoelastic incompressible neo-Hookean material. Moreover, the growth due to the biochemical process is taken into account. Applying the maximal regularity theory to a linearization of the equations, along with a deformation mapping, we prove the well-posedness of the full nonlinear problem via the contraction mapping principle. The system is considered in the case of a bounded and smooth domain and a suitable cylindrical domain with suitable boundary conditions. This is a joint work with Yadong Liu.

# The Cauchy problem for the Muskat equation

Thomas Alazard

ENS Paris-Saclay,  
Paris, France

thomas.alazard@ens-paris-saclay.fr

This talk is about a series of papers with Omar Lazar and Quoc-Hung Nguyen, devoted to the study of the Cauchy problem for the Muskat equation. I will explain how to decompose the nonlinearity in order to commute Fourier multipliers with the equation. This allows to study solutions with critical regularity. I will also mention one application to the analysis of a nonlinear and nonlocal parabolic equation introduced by Stefan Steinerberger to study the roots of polynomials under differentiation.

---

# Tent spaces techniques for evolution PDE's

Pascal Auscher

Université Paris-Saclay,  
Université Paris-Saclay, CNRS, Laboratoire de Mathématiques d'Orsay,  
91405 Orsay, France  
pascal.auscher@universite-paris-saclay.fr

We wish to give a panorama of actual possibilities brought by tent space theory to solve some evolution PDE's.

## Large global solutions of the parabolic-parabolic Keller–Segel system in higher dimensions

Lorenzo Brandolese

Université de Lyon 1,  
Lyon, France  
brandolese@math.univ-lyon1.fr

We study the global existence of the parabolic-parabolic Keller–Segel system in  $\mathbb{R}^d$ . We prove that initial data of arbitrary size give rise to global solutions provided the diffusion parameter  $\tau$  is large enough in the equation for the chemoattractant. This fact was observed before in the two-dimensional case by Biler, Guerra and Karch (2015) and Corrias, Escobedo and Matos (2014). Our analysis improves earlier results and extends them to any dimension  $d \geq 3$ . Our size conditions on the initial data for the global existence of solutions seem to be optimal, up to a logarithmic factor in  $\tau$ , when  $\tau \gg 1$ : we illustrate this fact by introducing two toy models, both consisting of systems of two parabolic equations, obtained after a slight modification of the nonlinearity of the usual Keller–Segel system. For these toy models, we establish in a companion paper finite time blowup for a class of large solutions.

---

# Instant blow-up for the generalized SQG equations.

Diego Cordoba

Instituto de Ciencias Matematicas-CSIC,  
Madrid, Spain  
d cg@icmat.es

In this talk we present recent results on the existence of solutions of the generalized Surface Quasi-geostrophic equations (SQG) that initially are in  $C^k$ ,  $C^{k,\gamma}$  or in super-critical Sobolev spaces, but lose that prescribe regularity for  $t > 0$ . This is a joint work with Luis Martinez-Zorua.

# Global solutions for fluid-elastic interaction with small data

Karoline Dissler

Universität Kassel  
Institut für Mathematik, Heinrich-Plett-Strasse 40, 34132 Kassel, Germany  
karoline.dissler@mathematik.uni-kassel.de

In this talk, we show global existence of strong solutions for a non-linear system modelling the dynamics of a linearly elastic body immersed in an incompressible viscous fluid without damping. We also identify and discuss long-time asymptotic dynamics of the system. This is a joint work with Michelle Luckas (Universität Kassel).

# Statistical solutions to the compressible Navier-Stokes system: Analysis and numerics

Eduard Feireisl

Academy of Sciences of the Czech Republic,  
Prague, Czech Republic  
feireisl@math.cas.cz

We discuss several concepts of statistical solutions to the compressible fluid models. In particular, we identify a suitable class of solutions based on the semiflow

---

selection. Then we consider two numerical approximations based on the stochastic collocation method and the Monte Carlo simulation, respectively. We show convergence towards the exact solution under the condition that the numerical solutions remain bounded.

## Navier-Stokes Flow past a Rigid Body that Moves by Time-Periodic Motion

Giovanni Paolo Galdi

Department of Mechanical Engineering and Materials Science,  
University of Pittsburgh, USA  
galdi@pitt.edu

We study existence, uniqueness and asymptotic spatial behavior of time-periodic strong solutions to the Navier-Stokes equations in the exterior of a rigid body,  $\mathcal{B}$ , moving by time-periodic motion of given period  $T$ , when the data are sufficiently regular and small. Our contribution improves all previous ones in several directions. For example, we allow both translational,  $\xi$ , and angular,  $\omega$ , velocities of  $\mathcal{B}$  to depend on time, and do not impose any restriction on the period  $T$  nor on the averaged velocity,  $\bar{\xi}$ , of  $\mathcal{B}$ . If  $\xi \neq \mathbf{0}$  we assume that  $\xi$  and  $\omega$  are both parallel to a constant direction, while no further assumption is needed if  $\xi \equiv \mathbf{0}$ . We also furnish the spatial asymptotic behavior of the velocity field,  $\mathbf{u}$ , associated to such solutions. In particular, if  $\mathcal{B}$  has a net motion characterized by  $\bar{\xi} \neq \mathbf{0}$ , we then show that, at large distances from  $\mathcal{B}$ ,  $\mathbf{u}$  manifests a wake-like behavior in the direction  $-\bar{\xi}$ , entirely similar to that of the velocity field of the steady-state flow occurring when  $\mathcal{B}$  moves with velocity  $\bar{\xi}$ .

---

# Revisit singularity formation for the inviscid primitive equations

Slim Ibrahim

University of Victoria,  
Victoria, Canada  
ibrahims@uvic.ca

In this talk, I will review the methods to show blowup of inviscid solutions of the Primitive Equation (also known as hydrostatic Euler), then I will share more recent progress on the qualitative properties of the finite-time singularity formation. Most notably, I will provide a full description of two blowup mechanisms, for a reduced PDE that is satisfied by a class of particular solutions to the PEs. In the first one a shock forms, and pressure effects are sub-leading, but in a critical way: they localize the singularity closer and closer to the boundary near the blow-up time (with a logarithmic in time law). This first mechanism involves a smooth blow-up profile and is stable among smooth enough solutions. In the second one, the pressure effects are fully negligible; this dynamics involves a two-parameters family of non-smooth profiles, and is stable only by smoother perturbations.

This is a joint work with C. Collot and Q. Lin.

# The time-like minimal surface equation in Minkowski space: low regularity solutions

Mihaela Ifrim

UW Madison,  
Madison, Wisconsin, USA  
ifrim@wisc.edu

It has long been conjectured that for nonlinear wave equations which satisfy a nonlinear form of the null condition, the low regularity well-posedness theory can be significantly improved compared to the sharp results of Smith-Tataru for the generic case. The aim of this article is to prove the first result in this direction, namely for the time like minimal surface equation in the Minkowski space-time. Further, our improvement is substantial, namely by  $3/8$  derivatives in two space dimensions and by  $1/4$  derivatives in higher dimensions. This work is joint with Albert Ai and Daniel Tataru.

---

# On solutions for a generalized Navier-Stokes-Fourier system fulfilling the entropy equality

Petr Kaplický

Charles University of Prague,  
Prague, Czech Republic  
kaplicky@karlin.mff.cuni.cz

We consider a flow of a non-Newtonian heat conducting incompressible fluid in a bounded domain subjected to the homogeneous Dirichlet boundary condition for the velocity field and the Dirichlet boundary condition for the temperature. In three dimensions, for the power-law index greater or equal to  $11/5$ , we show the existence of a solution fulfilling the entropy equality. The entropy equality can be formally deduced from the energy equality by renormalization. However, such a procedure can be justified by the DiPerna–Lions theory only for  $p > 5/2$ . The main novelty is that we do not renormalize the temperature equation, but we rather directly construct a solution, which fulfills the entropy equality. This is joint work with Anna Abbatiello, Miroslav Bulíček.

## Analyticity in space-time of solutions to the Navier-Stokes equations via parameter trick based on maximal regularity

Hideo Kozono

Waseda University & Tohoku University  
Tokyo 169–8555, Sendai 980-8578, Japan  
kozono@waseda.jp, hideokozono@tohoku.ac.jp

We consider analyticity in space-time variables of solutions to the Navier-Stokes equations by using the method of “parameter trick”. Based on maximal Lorentz regularity of the Stokes equations, we prove that the solution of the Navier-Stokes equations in the Serrin class is real analytic in the time variable. Our method is also applicable to the proof of analyticity in both space and time variables of solutions in the the whole space problem. This is the joint work with Prof. Peer C. Kunstmann(Karlsruhe) and Senjo Shimizu(Kyoto).

---

# The global existence for a fluid-structure interaction system

Igor Kukavica

University of Southern California,  
Los Angeles, California, USA  
ikukavica@gmail.com

We address a system of partial differential equations modeling a motion of an elastic body inside an incompressible fluid. The fluid is modeled by the incompressible Navier-Stokes equations while the structure is represented by the wave equation. We will review the local for large and global existence theorems and present the most most recent global existence result, which is joint with A. Tuffaha and W. Ozanski.

## Functional calculi for Stokes operators with first order boundary conditions on unbounded domains

Peer Christian Kunstmann

Karlsruher Institut für Technologie,  
Institut für Analysis, Englerstrasse 2, 76131 Karlsruhe, Germany  
peer.kunstmann@kit.edu

We study functional calculi in  $L^q$  for Stokes operators with Hodge, Navier, and Robin type boundary conditions on uniform  $C^{2,1}$ -domains  $\Omega \subseteq \mathbb{R}^d$ . Our research complements recent results on the  $L^q$ -theory of such operators and also sheds new light on the cases  $q = 1$  and  $q = \infty$ .



---

# Small obstacle limit for the inviscid Euler-alpha system

Milton Lopes Filho

Federal University of Rio de Janeiro,  
UFRJ - Instituto de Matemática (IM), Rio de Janeiro, Brasil  
mlopes@im.ufrj.br

We consider a family of solutions of the 2D Euler-alpha equations with no-slip boundary conditions, in the region  $\{\epsilon < |x|\}$ . We prove that this family converges to a solution of a modified Euler system in the full plane when  $\epsilon$  approaches zero.

# Non-newtonian fluids and convex integration

Stefano Modena

Technische Universität Darmstadt  
Fachbereich Mathematik, Schlossgartenstrasse 7, 64285 Darmstadt, Germany  
modena@mathematik.tu-darmstadt.de

The viscosity of a fluid is usually a constant, independent of the stress. There are however in nature several examples of fluids (ice, molten lava, blood, certain polymers, some salt solutions) where viscosity changes under applied forces. Such fluids are called non-Newtonian. I will focus on a simple model for such fluids, the “power law” model (Ladyzhenskaya, 1966): it is known that such model is well-posed in the “subcritical” regime and it has energy solutions above the “compactness threshold”. In a recent joint work with J. Burczak and L. Székelyhidi, we show that a picture dual to the above one holds: the power-law model is ill posed below the “compactness threshold” and it has many (very) weak solutions in the “supercritical regime”.

---

# Bounds on mixing norms for advection diffusion equations

Camilla Nobili

University of Surrey  
Guildford, Surrey, UK  
c.nobili@surrey.ac.uk

I will start this talk introducing a conjecture of Charles Doering and Christopher Miles (stated in “Diffusion-limited mixing by incompressible flows”, Nonlinearity 2018) on long-time “convergence to the Batchelor scale” of a suitable norm for mixing in passive scalars. Motivated by this conjecture, I will present mixing estimates on  $\mathbb{R}^n$  obtained in collaboration with Steffen Pottel. These estimates, obtained by Fourier splitting methods, were recently (substantially) improved by relaxing the assumptions on initial conditions.

# Vanishing viscosity and conserved quantities for 2D incompressible flow

Helena Nussenzveig Lopes

Universidade Federal do Rio de Janeiro,  
Rio de Janeiro, Brasil,  
hlopes@im.ufrj.br

Weak solutions of the incompressible Euler equations which are weak limits of vanishing viscosity Navier-Stokes solutions inherit, in two dimensions, conservation properties which are not available for general weak solutions. Research has focused on the behavior of energy, enstrophy and, more generally, the distribution function of vorticity, always in fluid domains with no boundary, with and without forcing. In this talk I will report on recent work in this direction.

---

# Free boundary problems for the incompressible Navier-Stokes equations in critical spaces

Senjo Shimizu

Kyoto University  
Kyoto, Japan

shimizu.senjo.5s@kyoto-u.ac.jp

Time-dependent free surface problem for the incompressible Navier-Stokes equations which describes the motion of viscous incompressible fluid whose initial boundary is given by the graph of a function are considered. We obtain global well-posedness of the problem for small initial data in scale invariant critical Besov spaces. Our proof is based on maximal  $L^1$ -regularity of the corresponding Stokes problem in the half-space and special structures of the quasi-linear term appearing from the Lagrangian transform of the coordinate. This is a joint work with Takayoshi Ogawa (Tohoku University).

## On the Navier-Stokes equations on surfaces

Gieri Simonett

Vanderbilt University,  
Nashville, USA

gieri.simonett@vanderbilt.edu

I will consider the motion of an incompressible viscous fluid on compact surfaces without boundary. Local in time well-posedness is established in the framework of  $L_p$ - $L_q$  maximal regularity for initial values in critical spaces. It will be shown that the set of equilibria consists exactly of the Killing vector fields. Each equilibrium is stable and any solution starting close to an equilibrium converges at an exponential rate to a (possibly different) equilibrium. In case the surface is two-dimensional, it will be shown that any solution with divergence free initial value in  $L_2$  exists globally and converges to an equilibrium.

---

# Euler-Poisson equation - weak-strong uniqueness principle for dissipative measure-valued solutions

Agnieszka Świerczewska-Gwiazda

University of Warsaw,  
Banacha 2, 02-097 Warsaw, Poland,  
aswiercz@mimuw.edu.pl

We will start with the statement of weak-strong uniqueness principle for general hyperbolic conservation laws and show that Euler-Poisson fails to fit into this framework. We consider several pressureless variants of the compressible Euler equation driven by nonlocal repulsion-attraction and alignment forces with Poisson interaction. Under an energy admissibility criterion, we prove existence of global *measure-valued solutions*, i.e., very weak solutions described by a classical Young measure together with appropriate concentration defects. We then investigate the evolution of a relative energy functional to compare a measure-valued solution to a regular solution emanating from the same initial datum. This leads to a (partial) weak-strong uniqueness principle.

## Global solutions in one dimensional dispersive flows

Daniel Tataru

University of California, Berkeley,  
Berkeley, CA, 94720, USA,  
tataru@math.berkeley.edu

The aim of this talk is to present some recent work concerning global existence and scattering results for small data problems of Schrödinger type in one space dimension. This is joint work with Mihaela Ifrim.

---

# A New Blow-up Criterion for the 3D Euler Equations: A Computational Study

Edriss S. Titi

Univeristy of Cambridge  
Texas A&M University  
and

The Weizmann Institute of Science  
est42@cam.ac.uk

In this talk we will report the results of a computational investigation of a new blow-up criterion for the 3D incompressible Euler equations, which does not rely on the seminal Beale-Kato-Majda blow-up criterion. This criterion is based on an inviscid regularization of the Euler equations known as the 3D Euler-Voigt equations, which are known to be globally well-posed. Moreover, simulations of the 3D Euler-Voigt equations also require less resolution than simulations of the 3D Euler equations for fixed values of the regularization parameter  $\alpha > 0$ . Therefore, the new blow-up criteria allow one to gain information about possible singularity formation in the 3D Euler equations indirectly, namely by simulating the better-behaved 3D Euler-Voigt equations. The new criterion is only known to be sufficient criterion for blow-up. Therefore, to test the robustness of the inviscid-regularization approach, we also investigate analogous criteria for blow-up of the 1D Burgers equation, where blow-up is well known to occur.

Notably, the Voigt inviscid regularization approach applies equally to other hydrodynamical models, and it can be shown that its solutions converge, as the regularization parameter  $\alpha \rightarrow 0$ , to the corresponding solutions of the underlying hydrodynamical model for as long as the latter exist.

## On off-diagonal behavior of the generalized Stokes operator

Patrick Tolksdorf

Johannes Gutenberg-Universität Mainz  
Institut für Mathematik, Staudingerweg 9, 55128 Mainz, Germany  
tolksdorf@uni-mainz.de

Let  $L = -\nabla \cdot \mu \nabla$  denote a second-order elliptic operator in divergence form and let  $(e^{-tL})_{t \geq 0}$  denote the corresponding strongly continuous heat semigroup on

---

$L^2(\mathbb{R}^d)$ . If  $E, F \subset \mathbb{R}^d$  denote measurable sets with  $\text{dist}(E, F) > 0$  and if  $f \in L^2(\mathbb{R}^d)$  is supported in  $E$ , then by the strong continuity of the semigroup, one finds that

$$\|e^{-tL}f\|_{L^2(F)} \rightarrow \|f\|_{L^2(F)} = 0 \quad \text{as } t \rightarrow 0.$$

An estimate that quantifies the convergence rate is often viewed as an off-diagonal estimate and it is well-known that heat semigroups satisfy the following type of off-diagonal decay

$$\|e^{-tL}f\|_{L^2(F)} \lesssim e^{-\frac{c\text{dist}(E,F)^2}{t}} \|f\|_{L^2(E)}.$$

In this talk, we study off-diagonal behaviour of the generalized Stokes semigroup  $(e^{-tA})_{t \geq 0}$  that is generated on  $L^2_\sigma(\mathbb{R}^d)$  by the generalized Stokes operator with bounded measurable coefficients  $\mu$ , formally given by

$$Au := -\text{div}(\mu \nabla u) + \nabla \Phi, \quad \text{div}(u) = 0 \quad \text{in } \mathbb{R}^d. \quad (1)$$

In contrast to the elliptic operator  $L$ , the operator  $A$  exhibits a non-local behaviour due to the presence of the pressure function  $\Phi$ . This non-locality affects the non-local behaviour of the generalized Stokes semigroup  $e^{-tA}$  and it is not clear how fast the support of a divergence free vector field  $f$  that is supported in a set  $E$  is smeared out. In this talk, first results in this direction are presented. We further discuss how possible optimal estimates could look like and try to pinpoint what has to be improved in the existing proof.

## Large time behavior for solids driven by a viscous flow

Marius Tucsnak

Université de Bordeaux,

Bordeaux, France

marius.tucsnak@u-bordeaux.fr

We present some recent advances on the PDE system describing the motion of a rigid body in a viscous incompressible flow. The fluid-solid system is supposed to fill the whole three dimensional space. The main results concern the global wellposedness for small initial data and the large time behavior of trajectories. Some of the presented results can be generalized for the case of several solids of arbitrary shape.

---

# Global axisymmetric Euler flows with rotation

Klaus Widmayer

University of Zürich,  
Winterthurerstrasse 190, 8057 Zürich,  
klaus.widmayer@math.uzh.ch

We discuss the construction of a class of global, dynamical solutions to the 3d Euler equations near the stationary state given by uniform “rigid body” rotation. These solutions are axisymmetric, of Sobolev regularity and have non-vanishing swirl. At the heart of this result is a dispersive effect due to rotation, which is captured in our “method of partial symmetries”. This approach is adapted to maximally exploit the symmetries of this anisotropic problem, both for the linear and nonlinear analysis, and allows to globally propagate sharp decay estimates.

This is joint work with Y. Guo and B. Pausader (Brown University).

# Statistical Solutions of the 2D Euler Equations

Emil Wiedemann

Universität Ulm,  
Institut für Angewandte Analysis, Helmholtzstrasse 18, 89081 Ulm, Germany  
emil.wiedemann@uni-ulm.de

It has been well-accepted for a long time that turbulence requires a probabilistic description. Accordingly, concepts of statistical solution for the Navier-Stokes equations were introduced by Foias and Vishik-Fursikov in the 1970s. In contrast, similar notions for the Euler equations have received comparatively little attention. We show how the deterministic existence theory for the 2D Euler equations with unbounded vorticity (even in the Delort class) can be established in the statistical context, and discuss the relation with the measure-valued solutions of DiPerna-Majda and the Young measure-based statistical solution concept of Fjordholm-Lanthaler-Mishra. This is joint work with Raphael Wagner.

---

# On The Existence of Multi-dimensional Compressible MHD Contact Discontinuities

Zhouping Xin

The Chinese University of Hong Kong,  
Hong Kong  
zpxin@ims.cuhk.edu.hk

Contact discontinuities for the ideal compressible magnetohydrodynamics (MHD) are most typical interfacial waves for astrophysical plasmas and prototypical fundamental waves for systems of hyperbolic conservations. Such waves are characteristic discontinuities for which there is no flow across the discontinuity surface while the magnetic field crosses transversally, which lead to a two-phase free boundary problem where the pressure, velocity and magnetic field are continuous across the interface whereas the entropy and density may have discontinuities. Some of the major difficulties for the existence of the Multi-dimensional ideal MHD contact discontinuities are the possible nonlinear Rayleigh-Taylor instability and loss of derivatives due to the non-ellipticity of the associated linearized problem. In this talk, I will present the recent work where we have proved the local existence and uniqueness of MHD contact discontinuities in both 2D and 3D in Sobolev spaces without any additional constraints such as Rayleigh-Taylor sign condition or with surface tensions. The key ingredients of our analysis are on the Cauchy formula for MHD, the transversality of the magnetic field, and an elaborate viscous approximation. This talk is based on a joint work with Professor Yanjin Wang of Xiamen University.

# On a Reaction-Diffusion System and Application to a Reactive-Flow Model

Hong-Ming Yin

Department of Mathematics and Statistics,  
Washington State University, Pullman, WA 99164, USA,  
hyin@wsu.edu

In this presentation I will discuss the recent progress about the global solvability for nonlinear reaction-diffusion systems. The focus will be on the system with balanced mass with nonlinear coupling in reaction terms. I will show the idea on how to derive a priori  $L^8$ -estimate for the solution of the nonlinear system. One of



---

the key ideas for the proof is based on some estimates in Morrey-John-Nirenberg-Campanato space as well as the dual technique. I will also give some examples to illustrate how our general results can be applied to some mathematical models arising from biological and health sciences as well as a model about the reactive flow in fluid mechanics.

Some results are based on a joint work with William Fitzgibbon, Jeffray Morgan at University of Houston and Bao Q. Tang at University of Graz.

## **Global hydrostatic approximation of hyperbolic Navier-Stokes system with small Gevrey class 2 data**

Ping Zhang

Academy of Mathematics & Systems Science,

Academy of Mathematics & Systems Science and Hua Loo-Keng Key Laboratory of Mathematics, The Chinese Academy of Sciences, Beijing 100190, China,,  
zp@amss.ac.cn

We investigate the hydrostatic approximation of a hyperbolic version of Navier-Stokes equations, which is obtained by using Cattaneo type law instead of Fourier law, evolving in a thin strip  $\mathbb{R} \times (0, \varepsilon)$ . The formal limit of these equations is a hyperbolic Prandtl type equation. We first prove the global existence of solutions to these equations under a uniform smallness assumption on the data in Gevrey 2 class. Then we justify the limit globally-in-time from the anisotropic hyperbolic Navier-Stokes system to the hyperbolic Prandtl system with such Gevrey 2 class data. Compared with our previous paper for the hydrostatic approximation of 2-D classical Navier-Stokes system with analytic data, here the initial data belong to the Gevrey 2 class, which is very sophisticated even for the well-posedness of the classical Prandtl system, furthermore, the estimate of the pressure term in the hyperbolic Prandtl system arises additional difficulties. (This is joint work with M. Paicu)

---

### 3 Participants

---

Aarach Nacer, Université de Bordeaux  
Abels Helmut, University of Regensburg  
Agresti Antonio, Institute of Science and Technology Austria  
Alazard Thomas, CNRS ENS Paris-Saclay  
Amrouche Chérif, Université de Pau  
Auscher Pascal, Université Paris-Saclay  
Balogh Andras, University of Texas Rio Grande Valley  
Berkemeier Stefanie Elisabeth, Bielefeld University  
Binz Tim, Technical University of Darmstadt  
Bleitner Fabian, University of Hamburg  
Boutros Daniel, University of Cambridge  
Brandolese Lorenzo, Université de Lyon 1  
Brandt Felix, Technical University of Darmstadt  
Bravin Marco, Université Lyon 1  
Bresch Didier, CNRS Université Savoie Mont-Blanc  
Brzezniak Zdzislaw, University of York  
Caggio Matteo, Institute of Mathematics of the Czech Academy of Sciences  
Carigi Giulia, University of Reading  
Ciani Simone, Technical University of Darmstadt  
Cordoba Diego, CSIC Spain  
Crin-Barat Timothée, Université Paris-Est Créteil  
Danchin Raphaël, Université Paris-Est Créteil Val-de-Marne  
Denis Clément, Aix-Marseille Université  
Denk Robert, University of Konstanz  
Dhifaoui Anis, University of Monastir-Tunisia  
Disser Karoline, University of Kassel

---

Eiter Thomas, Weierstrass Institute for Applied Analysis and Stochastics  
Farwig Reinhard, Technical University of Darmstadt  
Feireisl Eduard, Academy of Sciences of the Czech Republic  
Fernandez Pedro, Cergy Paris Université  
Flandoli Franco, SNS Pisa  
Furukawa Ken, Institute of Physical and Chemical Research Tokyo  
Galdi Giovanni, University of Pittsburgh  
Galeati Lucio, University of Bonn  
Gallenmueller Dennis, Ulm University  
Gaudin Anatole, Aix-Marseille Université  
Gérard-Varet David, Université de Paris  
Giga Yoshikazu, University of Tokyo  
Golding William, University of Texas at Austin  
Hieber Matthias, Technical University of Darmstadt  
Hu Yiran, University of Texas at Austin  
Hussein Amru, Technical University of Kaiserslautern  
Ibrahim Slim, University of Victoria  
Ifrim Mihaela, UW Madison  
Ionescu-Kruse Delia, IMAR Academie Roumanie  
Jo Min Jun, The University of British Columbia  
Kaplicky Petr, Charles University of Prague  
Khoshnasib-Zeinabad Fariba, University of Texas at Dallas  
Klein Rupert, Freie University of Berlin  
Koch Herbert, University of Bonn  
Koley Ujjwal, Tata Institute of Fundamental Research  
Korn Peter, Max Planck Institute for Meteorology  
Kozono Hideo, Waseda University/Tohoku University  
Kukavica Igor, University of Southern California  
Kunstmann Peer Christian, Karlsruhe Institute of Technology

---

Laheurte Vincent, ENS Paris-Saclay  
Lai Chen-Chih, Columbia University  
Lange Theresa, Bielefeld University  
Leblond Antoine, Sorbonne Université  
Li Jinkai, South China Normal University  
Liu Yadong, University of Regensburg  
Llerena Montenegro David, Université Paris-Saclay  
Lopes Filho Milton, Universidade Federal do Rio de Janeiro  
Mahdi Jawdat Tufaha Amjad, American University of Sharjah  
Maremonti Paolo, Vanvitelli University  
Maringová Erika, Institute of Science and Technology Austria  
Martínez Zoroa Luis, Instituto de Ciencias Matematicas  
Marveggio Alice, Institute of Science and Technology Austria  
Masmoudi Nader, NYU Courant  
Miller Evan, University of British Columbia  
Modena Stefano, Technical University of Darmstadt  
Monniaux Sylvie, Aix-Marseille Université  
Mucha Piotr, University of Warszawa  
Necasova Sarka, Czech Academy of Sciences  
Nguyen Thieu Huy, Hanoi University of Science and Technology  
Nobili Camilla, University of Surrey  
Nussenzeig Lopes Helena, Universidade Federal do Rio de Janeiro  
Palasek Stan, University of California, Los Angeles  
Pauron Matthieu, Université de Bordeaux  
Perrin Charlotte, CNRS Aix-Marseille Université  
Roy Arnab, Basque Centre for Applied Mathematics, Bilbao  
Saal Jürgen, University of Düsseldorf  
Schenke Andre, Bielefeld University  
Shimizu Senjo, Kyoto University

---

Simonett Gieri, Vanderbilt University  
Sun Changzhen, Université Toulouse III-Paul Sabatier  
Swierczewska-Gwiazda Agnieszka, University of Warsaw  
Szekelyhidi Laszlo, University of Leipzig  
Tan Jin, Cergy Paris Université  
Tataru Daniel, UC Berkeley  
Tendani Soler Adrien, Université de Bordeaux  
Titi Edriss, University of Cambridge  
Tolksdorf Patrick, University of Mainz  
Tucsnak Marius, Université de Bordeaux  
Voso Riccardo, University of Vienna  
Welter Roland, University of Hamburg  
Widmayer Klaus, University of Zurich  
Wiedemann Emil, Ulm University  
Wilke Mathias, University Halle-Wittenberg  
Wróblewska Kamińska Aneta, Institute of Mathematics, Polish Academy  
Xin Zhouping, The Chinese University of Hong Kong  
Yin Hong-Ming, Washington State University  
Zhang Ping, Chinese Academy of Sciences  
Zi Ruizhao, Central China Normal University



---

## 4 Notes

---



Conference  
**Nonlinear PDEs in Fluid Dynamics**  
 Luminy, May 9th-13th, 2022

Time	Monday, May 9th	Tuesday, May 10th	Wednesday, May 11th	Thursday, May 12th	Friday, May 13th
08:45h	Opening				
09:00h-09:35h	<b>Daniel Tataru</b>	<b>Diego Cordoba</b>	<b>Nader Masmoudi</b>	<b>Zhouping Xin</b>	<b>Pascal Auscher</b>
09:45h-10:20h	<b>Senjo Shimizu</b>	<b>Eduard Feireisl</b>	<b>Hideo Kozono</b>	<b>Helena Nussenzveig Lopes</b>	<b>Klaus Widmayer</b>
10:30h-11:00h	<i>Coffee Break</i>	<i>Coffee Break</i>	<i>Coffee break</i>	<i>Coffee Break</i>	<i>Coffee Break</i>
11:00h-11:35h	<b>Gieri Simonett</b>	<b>Ping Zhang</b>	<b>Mihaela Ifrim</b>	<b>Edriss Titi</b>	<b>Emil Wiedemann</b>
11:45h-12:20h	<b>Igor Kukavica</b>	<b>Stefano Modena</b>	<b>Thomas Alazard</b>	<b>Lorenzo Brandolese</b>	<b>Marius Tucsnaк</b>
12:20h					Closing
12:30h-14:00h	<i>Lunch</i>	<i>Lunch</i>	<i>Lunch</i>	<i>Lunch</i>	<i>Lunch</i>
14:00h-15:30h	free discussion	Free discussion	<i>Excursion catanques (14:00h-19:00h)</i>	Free discussion	
15:30h-16:05h	<b>Paolo Galdi</b>	<b>Milton Lopes Filho</b>		<b>Helmut Abels</b>	
16:15h-16:50h	<b>Patrick Tolksdorf</b>	<b>Slim Ibrahim</b>		<b>Karoline Disser</b>	
16:50h-17:20h	<i>Coffee Break</i>	<i>Coffee Break</i>		<i>Coffee Break</i>	
17:20h-17:55h	<b>Short Presentations</b> <i>(17:20h-19:00h)</i>	<b>Camilla Nobili</b>		<b>Agnieszka Swierczewska-Gwiazda</b>	
18:05h-18:40h		<b>Petr Kaplicky</b>		<b>Hong-Ming Yin</b>	
18:50h-19:25h	<i>Kir de bienvenue (19:00h-19:30h)</i>	<b>Short Presentations</b>		<b>Peer Christian Kunstmann</b>	
19:30h-20:30h	<i>Dinner</i>	<i>Dinner</i>	<i>Dinner</i>	<i>Bouillabaisse</i>	
20:30h-21:30h			<i>Soirée musicale</i>		

