# Analysis Day on Fluid Dynamics

on the occasion of the retirement of

# Professor Reinhard Farwig

## Virtual Workshop, TU Darmstadt, via Zoom
## June 18th, 2021

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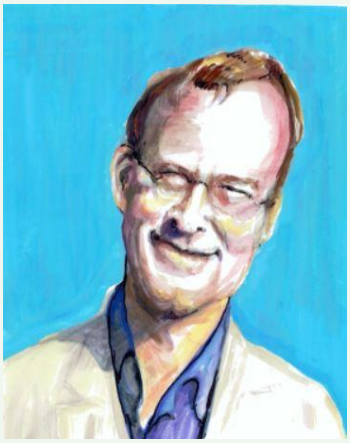
| 09h30 | Opening |
| 09h35-10h05 | **Hideo Kozono, Tokyo**<br>Topics on the Navier-Stokes equations – my joint work with Prof. Reinhard Farwig |
| 10h20-10h50 | **Jonas Sauer, Delft**<br>A Priori Bounds for Quasi-Linear SPDEs in the Full Sub-Critical Regime |
| 11h05-11h20 | Coffee Break |
| 11h20-11h50 | **Luigi Berselli, Pisa**<br>Natural second-order regularity for parabolic system |
| 12h05-13h30 | **Toast** & Lunch |
| 13h30-14h00 | **Gregory Seregin, Oxford/St. Petersburg**<br>On Liouville-Type Theorems for Steady-State Navier-Stokes Equations |
| 14h15-14h45 | **Jiří Neustupa, Prague**<br>On stability or instability of a steady flow of a viscous liquid past a rotating Obstacle |
| 15h00-15h15 | Coffee Break |
| 15h15-15h45 | **Raphaël Danchin, Paris**<br>On the use of Lorentz spaces with respect to time in global well-posedness issues for viscous fluids |
| 16h00 | **Toast** & Closing |



Illustrations by Prof. K.H. Hofmann

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# Abstracts

Luigi Berselli, Pisa
**Natural second-order regularity for parabolic system**

We consider parabolic problems with stress tensor depending only on the deformation tensor. By developing a new approximation method we provide an approach to the regularity which is valid for general stress tensors with (p,delta) structure, for all p>1 and for all delta>0. This allows to prove the ``natural'' second order regularity -- up to the boundary-- in the case of homogeneous Dirichlet boundary conditions. (Joint work with M. Ruzicka)

Raphaël Danchin, Paris
**On the use of Lorentz spaces with respect to time in global well-posedness issues for viscous fluids**

We are concerned with global existence and uniqueness issues for models of nonhomogeneous viscous gases in the case where the initial has no regularity (it is only bounded). Here we look at models of pressureless gases that are used for describing phenomena in astrophysics or collective behavior (joint work with P.B. Mucha and P. Tolksdorf), and at inhomogeneous incompressible viscous fluids (collaboration with S. Wang). In both cases, we obtain global existence results for small variations of density and small velocities in critical Besov space. A maximal regularity estimate involving Lorentz spaces for the time variable plays a key role.

Hideo Kozono, Tokyo
**Topics on the Navier-Stokes equations - my joint work with Prof. Reinhard Farwig**

We consider the following problems on the Navier-Stokes equations.
1. strong energy inequality of weak solutions in general unbounded domains
2. regularity criterion of weak solutions based on the continuity in time of the energy norm
3. Leray's inequality associated with the flux condition in multi-connected domains
4. stability of solutions in exterior domains with moving boundary
These problems have been discussed for longer time in my joint work with Prof. Reinhard Farwig.
In the talk, I will give a short survey.

Jiří Neustupa, Prague
**On stability or instability of a steady flow of a viscous liquid past a rotating obstacle**

We discuss the dependence of stability or instability of a steady flow of an incompressible viscous fluid past a rotating obstacle on spectral properties of a certain associated linear operator L. As a result of independent interest, we show that the uniform growth bound of the C_0-semigroup $e^{tL}$ equals the spectral bound of the operator L.

Jonas Sauer, Delft
**A Priori Bounds for Quasi-Linear SPDEs in the Full Sub-Critical Regime**

In my talk I will be concerned with quasi-linear parabolic partial differential equations of the form
(1) u_t -a(u) u_xx = f, where the coefficient field a(u) is sufficiently smooth and uniformly elliptic. In line with the pathwise approach to stochastic analysis of Lyons, the external forcing is deterministic and viewed as a realization of a singular noise which a.s. belongs to the (negative) parabolic Hölder space. For sufficiently irregular regimes the PDE is not expected to be well-posed in the traditional sense and a re-centering will be needed for the non-linear term, which amounts to adjusting the equation (1) with certain counter-terms, known as a renormalization. I will present a framework that applies to all sub-critical regularities in the sense of Hairer and all space dimensions. The input for our theory is a structural assumption on the forcing, which amounts to assuming that various multi-linear functionals of the 'noise' f have already been renormalized in an "off-line" probabilistic step. The output is a local Hölder a priori estimate on solutions to the renormalized version of (1). We emphasize that the renormalization terms are local in the solution and can be constructed explicitly in terms of derivatives of the non-linearity a and partial information on f.
The talk is based on ongoing work with Felix Otto, Scott Smith and Hendrik Weber.

Gregory Seregin, Oxford/St. Petersburg
**On Liouville-Type Theorems for Steady-State Navier-Stokes Equations**

In the expository talk, we shall discuss the state of the art in the theory of the so-called D-solutions to the steady-state Navier-Stokes equations in the whole space.