A Simple Model for Climate Change

Lange Nacht der Mathematik 2025

Climate Change

Climate change refers to long-term shifts in temperatures and weather patterns, primarily caused by human activities, especially the burning of fossil fuels.

The increase in atmospheric carbon dioxide (CO_2) is a significant driver of climate change. CO_2 is a greenhouse gas that traps heat in the atmosphere, leading to global warming. Since the Industrial Revolution, human activities such as burning fossil fuels, deforestation, and certain industrial processes have increased atmospheric CO_2 concentrations by about 50%.

This rise in CO_2 enhances the greenhouse effect, resulting in higher global temperatures and associated climatic changes.

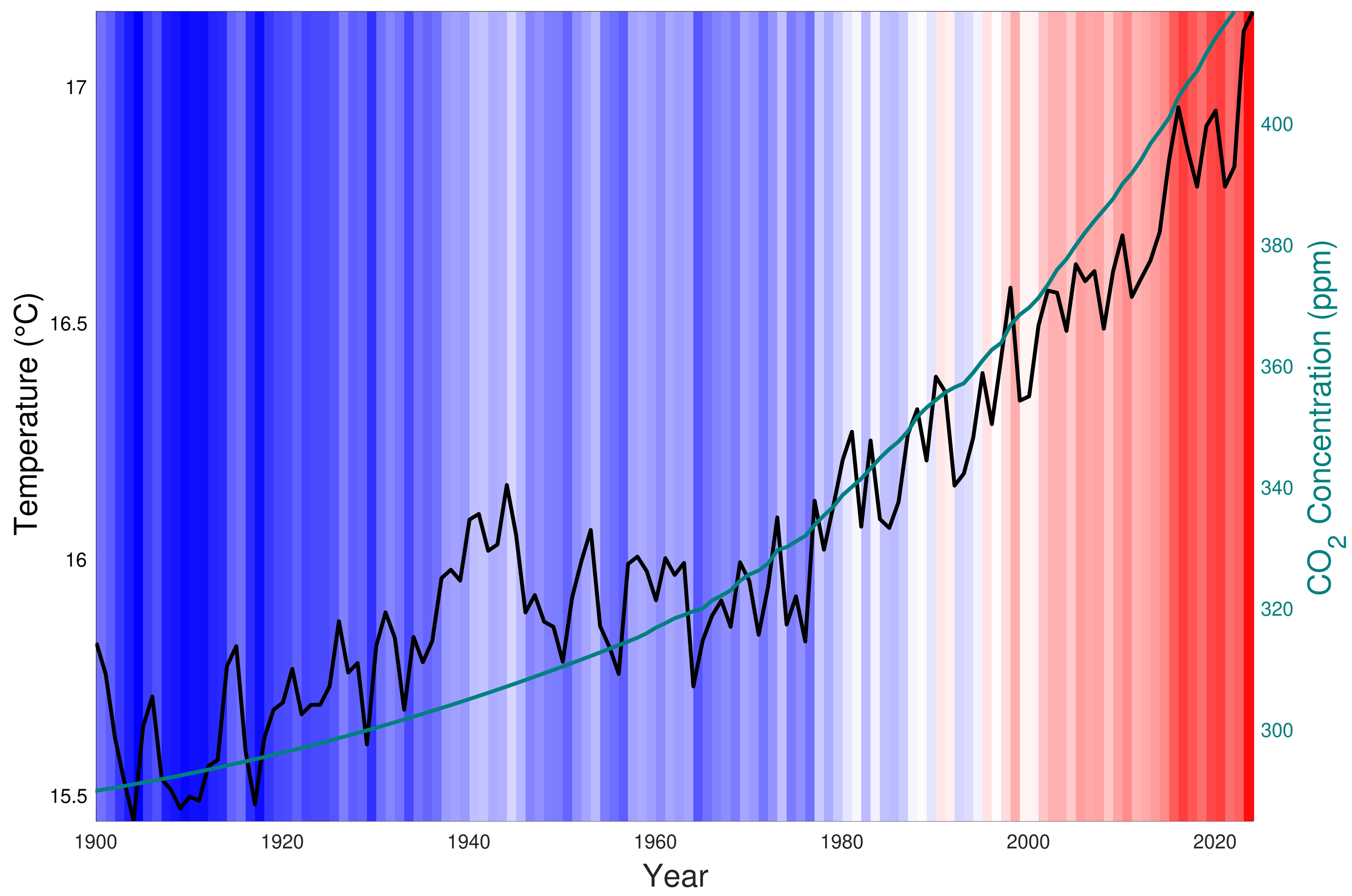

Fig. 1: Global temperature time series (black) and CO_2 concentration in the atmosphere (blue-green).

A simple climate model

We consider an energy balance model for the global mean temperature (GMT) $T(t)$, which depends on time t. The model assumes that GMT evolves in time according to the balance between the radiation absorbed and the radiation emitted by the planet:

$$\frac{dT}{dt} = Q\beta + q - (A + BT), \quad T(0) = T_0,$$

where:

- $Q = 340\,\mathrm{Wm}^{-2}$ is the solar radiation,
- $\beta = 0.7$ is the globally averaged co-albedo,
- q represents the effect of CO_2 concentration,
- $A + BT$ represents the outgoing long-wave radiation, with $A, B > 0$ determined empirically (see Fig. 2).

Equilibrium temperature and CO_2 increase

The equilibrium temperature T_{eq} of the model, i.e., the average of the Earth's temperature we experience every day, is given by the solution of

$$0 = Q\beta + q - (A + BT_{eq}),$$

which can be expressed as

$$T_{eq} = \frac{Q\beta + q - A}{B}.$$

The larger q is, the larger T_{eq} becomes.

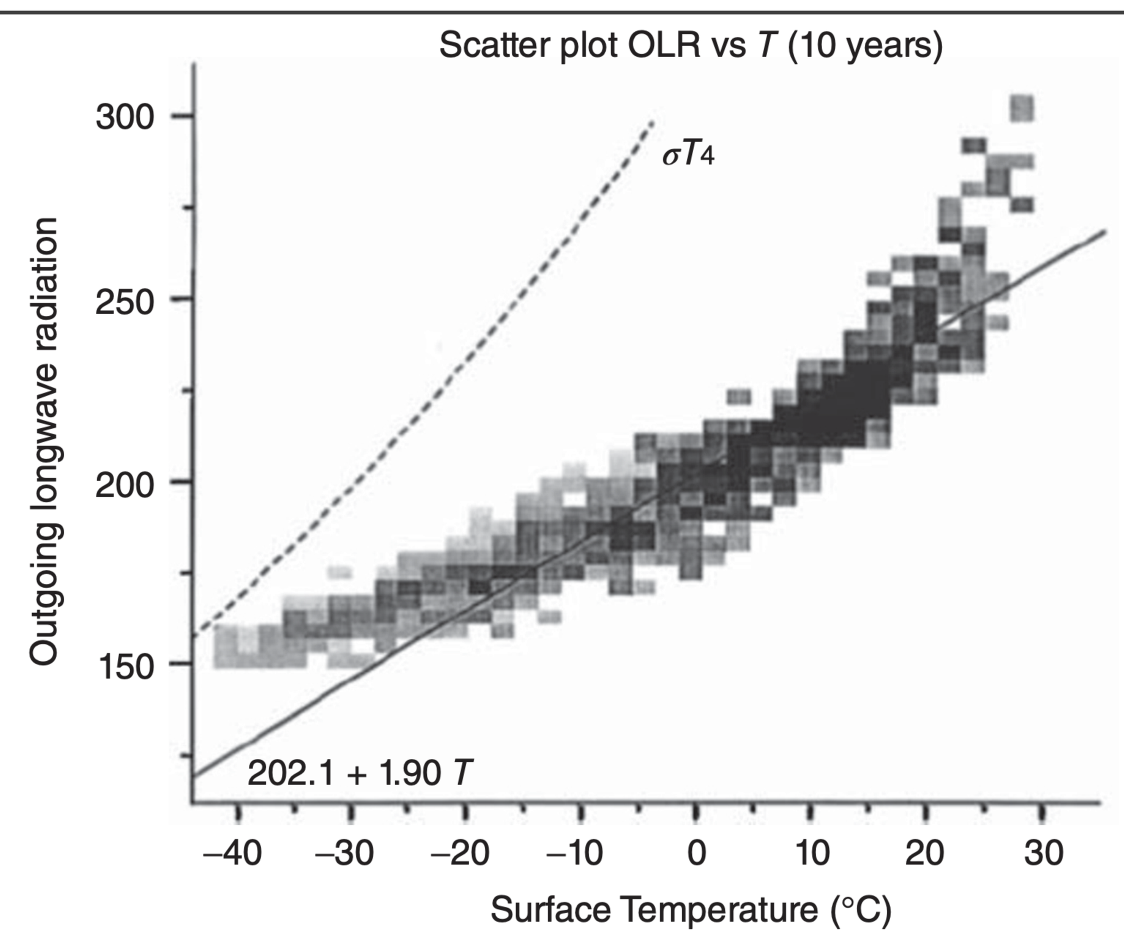

Fig. 2: Density plot of outgoing radiation versus surface temperature. Darker shading indicates greater frequency of occurrence (source: North, Gerald R., and Kwang-Yul Kim. Energy balance climate models. John Wiley & Sons, 2017).

Conclusions

Although elementary, the previous model is valuable because it explains the fundamental mechanisms of climate dynamics.

We can prove that, as long as the CO_2 concentration will continue to increase, the same will be true for the global mean temperature of our planet.

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