Prof. Timo Richarz
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Algebraic Geometry II

4. Exercise sheet

Exercise 1:
Let $X = \mathbb{P}^2_k - \{(0 : 0 : 1)\}$ with k a field. Determine the cohomology groups $H^p(X, \mathcal{O}_X)$ for $p = 0, 1$.
Hint: Use the open affine covering $\mathcal{U} = \{D_+(x_0), D_+(x_1)\}$.

Exercise 2:
Let $f : X \to Y$ be an affine morphism of Noetherian, separated schemes. Show that there is a canonical isomorphism

$$\mathrm{H}^p(X, \mathcal{F}) = \mathrm{H}^p(Y, f_*\mathcal{F})$$

for all quasi-coherent sheaves $\mathcal{F}$ on X and all $p \in \mathbb{Z}_{\geq 0}$.

Exercise 3:
Let $f \in k[x_0, x_1, x_2]$ be a homogeneous polynomial of degree $d \in \mathbb{Z}_{\geq 1}$ over a field k. Let $X = V_+(f)$ be the plane curve in $\mathbb{P}^2_k$ defined by $f = 0$. Determine the cohomology groups $H^p(X, \mathcal{O}_X)$ for all $p \in \mathbb{Z}_{\geq 0}$.
Hint: If $i : X \to \mathbb{P}^2_k$ is the inclusion, then there is an exact sequence $0 \to \mathcal{I}_X \to \mathcal{O}_{\mathbb{P}^2_k} \to i_*\mathcal{O}_X \to 0$ and an isomorphism $\mathcal{I}_X \cong \mathcal{O}(-d)$.

Exercise 4:
Let X and Y be separated schemes of finite type over a field k. Let $\mathcal{F}$ and $\mathcal{G}$ be two finite type quasi-coherent sheaves on X and Y respectively. Show that there is a canonical isomorphism

$$H^n(X \times_k Y, p_1^*\mathcal{F} \otimes p_2^*\mathcal{G}) \;=\; \bigoplus_{p+q=n} H^p(X, \mathcal{F}) \otimes_k H^q(Y, \mathcal{G}).$$

Hint: Use a Čech complex using products of affine opens.