Weil Conjectures

Prof. Dr. Timo Richarz

Contact

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Time and Place

Wednesdays, 09:50-11:30 in S1|02-144
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Contents

The course is an introduction to the Weil Conjectures. We start with Congruences and Zeta functions, formulate the Weil Conjectures, make the connection to étale cohomology and discuss elements of the proof starting with the case of curves.

Prerequisites are basic algebraic geometry such as Hartshorne, §§2-3 and étale cohomology, e.g., as covered in the last winter term link. Constructions in étale cohomology relevant to the course will be stated during the lecture with details to be discussed in a block seminar, see exercise session below.

Literature

- A. Weil: Numbers of Solutions of Equations in finite fields.
- P. Deligne: La conjecture de Weil I & II, Publ. math. de l'I.H.É.S.
- N. Katz: An overview of Deligne's proof of the Riemann hypothesis for varieties over finite fields.
- E. Freitag, R. Kiehl: Etale Cohomology and the Weil Conjecture, Springer.

Exercise session

M.Sc. Patrick Bieker

This will be in form of a block seminar accompanying the lectures. We cover topics in étale cohomology such as the Lefschetz trace formula which are used, but not proven throughout the lectures and complement the topics from last semester. See the plan below.

Weil Conjectures - Seminar on Étale Cohomology

Patrick Bieker, Timo Richarz

The goal of this seminar is to review three topics in étale cohomology which are used, but not proven throughout the lectures. In each talk one of the following theorems should be stated and the main ideas of the proof should be sketched. For all three talks it is suggested to follow [2] or [3]. Another good reference are the seminar notes that can be found at [1].

1 Smooth Base Change

Define locally acyclic morphisms and give some basic properties. Show that smooth morphisms are locally acyclic. In order to do so, reduce the assertion to the case of an affine space of dimension 1, which can be treated by explicit calculations.

Conclude by proving the base change theorem. Reduce the theorem to the case of a locally acyclic base change map and an open immersion as structure map. Use the machinery introduced before to finish the proof.

2 Poincaré Duality

Use the cohomology of curves to define the trace map for (smooth) curves. Give a proof of the duality theorem in this case. Explain how to obtain a trace map in the general case. Finally, indicate how to reduce the duality theorem in the general case to the assertion for curves discussed before.

3 Lefschetz Trace Formula

Define constructible $\mathbb{Q}_\ell$-sheaves and their cohomology. Recall the Frobenius action on sheaves in order to state the trace formula. Explain how to reformulate the trace formula for torsion sheaves and indicate how to prove it in this case.

References

[1] `math.stanford.edu/~conrad/Weil2seminar/`.

[2] P. Deligne, *Cohomologie Étale*, SGA 4 $\frac{1}{2}$, Springer.

[3] E. Freitag and R. Kiehl, *Etale Cohomology and the Weil Conjecture*, Springer.