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Summer 2025

Algebraic Geometry I

Exercise Sheet 7

Exercise 1:

Let X be a topological space and define $\mathcal{F}$ by setting $\mathcal{F}(X) = 0, \mathcal{F}(\emptyset) = 0$ and $\mathcal{F}(U) = \mathbb{Z}$ for every other open subset. The restriction maps $\mathcal{F}(U) \to \mathcal{F}(V)$ equal to the identity if U and V are different from $\emptyset$ and X and the zero map otherwise. Show that $\mathcal{F}$ is a presheaf. Is $\mathcal{F}$ a sheaf?

Exercise 2:

Let X be a topological space and $x \in X$. Denote $i : \{x\} \hookrightarrow X$ the inclusion. Let $\mathcal{F} = \underline{\mathbb{Z}}_{\{x\}}$ be the constant sheaf on $\{x\}$. What is $i_*\mathcal{F}$?
Now let $p : X \to pt$ be the obvious map and let $\mathcal{F}$ be the constant sheaf $\underline{\mathbb{Z}}_{pt}$ on pt. What is $p^*\mathcal{F}$ and $p^{-1}\mathcal{F}$?

Exercise 3:

Let $X = \{x, y, z\}$ and define the following topology on X: $\tau = \{\emptyset, \{x\}, \{x, y\}, \{x, z\}, X\}$. Further define a sheaf of rings $\mathcal{O}_X$ on X by $\mathcal{O}_X(\{x\}) = \mathbb{Q}(t)$ and $\mathcal{O}(\{x, y\}) = \mathcal{O}(\{x, z\}) = \mathcal{O}_X(X) = \mathbb{Q}[t]_{(t)}$. Show that $(X, \mathcal{O}_X)$ is a sheaf which is not isomorphic to an affine scheme.

Exercise 4:

Let X be a scheme of characteristic p. Show that there exists a unique morphism of schemes $(F, F^\flat) : X \to X$ such that on topological spaces, $F = \mathrm{id}_X$, and for open subsets $U \subseteq X$, $F^\flat$ is the Frobenius endomorphism of $\Gamma(U, \mathcal{O}_X)$, $s \mapsto s^p$. This morphism is called the *absolute Frobenius morphism of X*.