Hilbert Modular Forms

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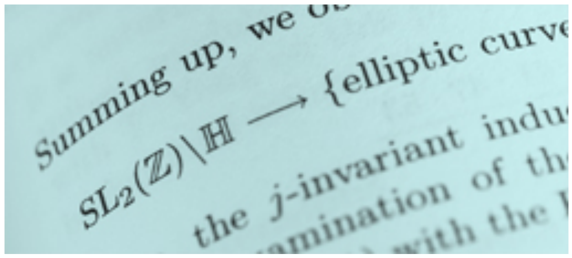
Termine: Fr 11:40-13:20 (Starting on April 16)

Location: Zoom. For detail, please check moodle or contact the lecturer.



Elliptic modular forms are functions on the upper-half plane satisfying certain symmetries. These functions are important in number theory, as demonstrated their roles in the classical theory of complex multiplication, the modularity theorem (proved in the late 1990's) and the BSD conjecture (one of open millennium problems).

The quotient of the upper-half complex plane by the symmetry group is the usual modular curve, and can be viewed as the Shimura variety associated to the algebraic group GL_2 over the rationals $\mathbb{Q}$. When this is replaced by its restriction of scalar from a totally real number field F to $\mathbb{Q}$, the corresponding Shimura variety is the Hilbert modular variety, and the analogous forms are Hilbert modular forms. Many features of elliptic modular forms still exist for Hilbert modular forms. Yet, interesting phenomena appear when F is no longer $\mathbb{Q}$ and the geometry of the Hilbert modular variety is more intricate. Furthermore, studying Hilbert modular forms can lead to applications in number theory, such as rationality of special value of Dedekind zeta function.

The goal of this course is to describe these phenomena and applications. During the first half of the course, we will cover the basic notions of Hilbert modular forms and Hilbert modular varieties, from both the classical and adelic perspectives. In the second half, we will focus on the case when F is a real quadratic fields, when the Hilbert modular surface also happens to be an orthogonal Shimura variety. If time permits at the end, we will see how Hilbert modular forms can be applied to prove new results about singular moduli.

Basic understanding of complex analysis and algebraic number theory will be assumed. Knowledge of elliptic modular forms would be useful, but is not required.

Literatur:

J. Bruinier, G. van der Geer, D. Zagier, *The 1-2-3 of Modular Forms*, Springer, (2008).
E. Freitag, *Hilbert Modular Forms*, Springer, (1990).
P. Garrett, *Holomorphic Hilbert Modular Forms*, Wadsworth & Brooks/Cole, (1990).
G. van der Geer, *Hilbert Modular Surfaces*, Springer, (1988).