

SEMINAR ON CATALAN'S CONJECTURE

WINTERSEMESTER 2018
ORGANIZER: YINGKUN LI

In 1844, Eugène Charles Catalan proposed the following conjecture in a letter to the editor of Crelle's journal.

Conjecture 1 (E. Catalan, 1844). The only consecutive numbers in the sequence of perfect powers of natural numbers

$$1, 4, 8, 9, 16, 25, 27, 32, 36, 49, 64, 81, 100, 121, 125, 128, 144, 169, \dots$$

are 8 and 9.

In other words, the only non-trivial solution in natural numbers to the equation

$$x^p - y^q = 1$$

with $p, q \geq 2$ and $x, y \geq 1$ is $(p, q, x, y) = (2, 3, 3, 2)$. Similar to other problems in number theory, this simple-looking conjecture stayed unproved for many, many years, until it was proved by Preda Mihăilescu in 2002 [1]. Unlike the proof of Fermat's last theorem (which concerns with the integral solutions to the equation $x^n + y^n = z^n$) by Wiles and Taylor-Wiles in 1994, Mihăilescu's proof used only the arithmetic of cyclotomic fields, and could be understood by bachelor students with basic knowledge in algebra and algebraic number theory. The goal of this weekly seminar is to go through this proof following the excellent and thorough exposition by Schoof [2].

Prerequisite: Introduction to algebra, basic algebraic number theory. Open to both bachelor and master students.

Tasks: Each participant is required to give a talk and submit a short report afterwards. Both can be done in English or German.

Date and Time: Please register on TuCan and send me an email: li@mathematik.tu-darmstadt.de. There will be an organization meeting at 15:20 on Monday **October 1** in room 401, where we will decide the time of the seminar and distribute the talks.

REFERENCES

- [1] Preda Mihăilescu. Primary cyclotomic units and a proof of Catalan's conjecture. *J. Reine Angew. Math.*, 572:167–195, 2004.
- [2] René Schoof. *Catalan's conjecture*. Universitext. Springer-Verlag London, Ltd., London, 2008.