ARITHMETIC HIRZEBRUCH-ZAGIER DIVISORS AND MODULAR FORMS

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Abstract. We report on recent work on arithmetic Hirzebruch-Zagier divisors on Hilbert modular surfaces. These divisors can be viewed as the coefficients of an elliptic modular form of weight two with values in an arithmetic Chow group in analogy to the classical result of Hirzebruch and Zagier. The intersection of this modular form with the second power of the line bundle of modular forms equipped with the Petersson metric can be determined. In particular we obtain the arithmetic self intersection number of the line bundle of modular forms.

1. Introduction

In 1976 Hirzebruch and Zagier showed that the intersection numbers of certain special divisors, nowadays called Hirzebruch-Zagier divisors, on Hilbert modular surfaces can be interpreted as the Fourier coefficients of holomorphic elliptic modular forms of weight two [HZ]. Their discovery triggered a lot of further research in that direction. For instance, Oda considered similar cycles for the orthogonal group $\mathrm{O}(2, n)$ [Oda], and Kudla and Millson studied special cycles for the orthogonal group $\mathrm{O}(p, q)$ and the unitary group $\mathrm{U}(p, q)$ in great generality by means of the Weil representation, see e.g. [KM].

Inspired by the work of Gross and Keating [GK] on the arithmetic intersection of three modular correspondences, Kudla proved that the intersection numbers of certain arithmetic divisors on a regular model of a Shimura curve can be interpreted as the coefficients of the derivative of a Siegel Eisenstein series of genus 2 [Ku1]. In subsequent work Kudla, Rapoport, and Yang developed a broad conjectural picture relating arithmetic intersection numbers of special arithmetic divisors on regular models of Shimura varieties of orthogonal type to modular forms, in particular to (derivatives of) Eisenstein series. In special cases they obtained results that strongly support these conjectures [Ku2], [Ku3], [Ku4], [KRY]. For instance, for Shimura curves the results are quite complete.

One conclusion of this work is that the complex geometric results of Hirzebruch and Zagier and their generalizations should have arithmetic analogues over $\mathbb{Z}$. Here the classical intersection theory has to be replaced by Arakelov intersection theory.

In the present note we briefly report on joint work with J. Burgos, J. Kramer, and U. Kühn on arithmetic Hirzebruch-Zagier divisors on Hilbert modular surfaces and their relationship to elliptic modular forms [BBK]. Since these surfaces are examples of *non-compact* Shimura varieties, we have to work with the extended arithmetic intersection

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theory developed in [BKK1], [BKK2] (see the article of U. Kühn in this volume). This causes some technical complications, but, on the other hand, allows us to take advantage of the q-expansion principle, which provides a strong link between analysis and arithmetic.

2. Hilbert modular surfaces

For details we refer to the books [Fr], [Ge], [Go]. Let K be a real quadratic field and write $D > 0$ for its discriminant. Throughout we assume that D is a prime. This implies that the class number h_K is odd. We write $\mathcal{O}_K$ for the ring of integers of K, $\mathfrak{d} = (\sqrt{D})$ for the different, and $x \mapsto x'$ for the conjugation in K. Moreover, we write $\Gamma_K = \mathrm{SL}_2(\mathcal{O}_K)$ for the Hilbert modular group corresponding to K. We may view it as a discrete subgroup of $\mathrm{SL}_2(\mathbb{R})^2$ acting on $\mathbb{H}^2$, the product of two copies of the upper complex half plane $\mathbb{H} = \{z \in \mathbb{C};\ \Im(z) > 0\}$, via Moebius transformations, i.e., $M(z_1, z_2) = (\frac{az_1+b}{cz_1+d}, \frac{a'z_2+b'}{c'z_2+d'})$ for $M = (\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in \Gamma_K$, and $(z_1, z_2) \in \mathbb{H}^2$. The quotient $Y_K = \Gamma_K \backslash \mathbb{H}^2$ is a non-compact complex space, which can be compactified by adding h_K points, the so-called cusps. In that way we obtain a compact normal complex space X_K, the Baily-Borel compactification of Y_K. The spaces Y_K and X_K are actually (quasi-) projective algebraic and defined over $\mathbb{Q}$. The singularities of X_K lie at the cusps and the elliptic fixed points. By the work of Hironaka and Hirzebruch there exists a desingularization $\widetilde{X}_K \to X_K$.

To study the geometry of these *Hilbert modular surfaces* we are interested in rational functions on X_K and a bit more generally in Hilbert modular forms. Recall that a meromorphic (respectively holomorphic) function f on $\mathbb{H}^2$ is called a Hilbert modular form (respectively a holomorphic Hilbert modular form) of weight k for the group Γ_K, if it satisfies the transformation law

$$f\left((\begin{smallmatrix} a & b \\ c & d \end{smallmatrix})(z_1, z_2)\right) = (cz_1 + d)^k (c'z_2 + d')^k f(z_1, z_2) \tag{2.1}$$

for all $(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in \Gamma_K$. Hilbert modular forms of weight 0 can be identified with rational functions on X_K. Hilbert modular forms of weight k can be viewed as rational sections of the line bundle $\mathcal{M}_k(\mathbb{C})$ of modular forms of weight k on $\widetilde{X}_K$.

There exists a family of special divisors on Hilbert modular surfaces. This is typical for Shimura varieties of orthogonal or unitary type. (Recall that $\mathrm{SL}_2(\mathbb{R})^2$ is essentially isomorphic to the real orthogonal group of signature $(2, 2)$. The Hilbert modular group can be viewed as the orthogonal group of a suitable lattice of signature $(2, 2)$.) Let m be a positive integer. Then

$$T(m) = \sum_{\substack{(a,b,\lambda) \in (\mathbb{Z}^2 \times \mathfrak{d}^{-1})/\{\pm 1\} \\ ab - \lambda\lambda' = m/D}} \{(z_1, z_2) \in \mathbb{H}^2; \quad az_1z_2 + \lambda z_1 + \lambda' z_2 + b = 0\}$$

defines a Γ_K-invariant analytic divisor on $\mathbb{H}^2$. It descends to an algebraic divisor on Y_K, the *Hirzebruch-Zagier divisor* of discriminant m. Moreover, we obtain Hirzebruch-Zagier divisors on X_K by taking the closure of $T(m)$, and on $\widetilde{X}_K$ by taking the pullback with respect to the desingularization morphism. It is well known that $T(m) = \emptyset$ if and only if $\chi_D(m) = -1$, where $\chi_D = (\frac{D}{\cdot})$ denotes the quadratic character corresponding to $K/\mathbb{Q}$.

3. Geometric generating series

If X is a regular projective algebraic variety over $\mathbb{C}$, we write $\mathrm{CH}^1(X)$ for its first Chow group, that is, the group of algebraic divisors on X modulo rational equivalence. Moreover, we put $\mathrm{CH}^1(X)_{\mathbb{Q}} = \mathrm{CH}^1(X) \otimes_{\mathbb{Z}} \mathbb{Q}$.

It is our goal to describe the position of the Hirzebruch-Zagier divisors in $\mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}$. To this end we consider the generating series

$$A(\tau) = \mathrm{c}_1(\mathcal{M}_{-1/2}(\mathbb{C})) + \sum_{m>0} T(m) q^m \in \mathbb{Q}[[q]]^+ \otimes_{\mathbb{Q}} \mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}, \tag{3.1}$$

where $q = e^{2\pi i \tau}$ for $\tau \in \mathbb{H}$, and $\mathrm{c}_1(\mathcal{M}_k(\mathbb{C}))$ denotes the first Chern class of the line bundle of modular forms of weight k.

The result of Hirzebruch and Zagier mentioned in the introduction relates this generating series to elliptic modular forms (see e.g. [Sh]). We denote by $M_k(D, \chi_D)$ the space of holomorphic modular forms of weight k for the Hecke group

$$\Gamma_0(D) = \{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}_2(\mathbb{Z}); \quad c \equiv 0 \pmod{D} \} \tag{3.2}$$

with character χ_D. Any such modular form $f \in M_k(D, \chi_D)$ has a Fourier expansion of the form $f = \sum_{n \geq 0} c(n) q^n$. We let $M_k^+(D, \chi_D)$ be the subspace of those $f \in M_k(D, \chi_D)$, whose Fourier coefficients $c(n)$ satisfy the so-called plus space condition, i.e., $c(n) = 0$ whenever $\chi_D(n) = -1$.

Theorem 3.1 (Hirzebruch and Zagier)**.** *The Hirzebruch-Zagier divisors generate a subspace of* $\mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}$ *of dimension* $\leq \dim(M_2^+(D, \chi_D))$. *The generating series* $A(\tau)$ *is a modular form in* $M_2^+(D, \chi_D)$ *with values in* $\mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}$, *i.e.,*

$$A(\tau) \in M_2^+(D, \chi_D) \otimes_{\mathbb{Q}} \mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}.$$

So if λ is a linear functional on $\mathrm{CH}^1(\widetilde{X}_K)_{\mathbb{Q}}$, then

$$\lambda\left(\mathrm{c}_1(\mathcal{M}_{-1/2}(\mathbb{C}))\right) + \sum_{m>0} \lambda(T(m)) q^m \in M_2^+(D, \chi_D).$$

Typical linear functionals are given by the intersection pairing with a fixed divisor on $\widetilde{X}_K$. Hirzebruch and Zagier actually proved the theorem in a slightly different form by explicitly computing the intersection numbers of $T(m)$ with other Hirzebruch-Zagier divisors and with the exceptional divisors above the cusps. In [Bo3] Borcherds provided a different proof using automorphic products.

If we take for λ the intersection with $\mathrm{c}_1(\mathcal{M}_k(\mathbb{C}))$, we find that $A(\tau) \cdot \mathrm{c}_1(\mathcal{M}_k(\mathbb{C})) \in M_2^+(D, \chi_D)$. It turns out that this modular form is precisely the (up to a scalar factor unique) Eisenstein series in the space $M_2^+(D, \chi_D)$ (see [Fra], [Ha]).

Theorem 3.2 (Franke and Hausmann)**.** *We have*

$$A(\tau) \cdot \mathrm{c}_1(\mathcal{M}_k(\mathbb{C})) = -\frac{k}{2} \zeta_K(-1) \cdot E_2^+(\tau),$$

where $\zeta_K(s)$ is the Dedekind zeta function of K, and $E_2^+(\tau)$ denotes the Eisenstein series

$$E_2^+(\tau) = 1 + \frac{2}{L(-1,\chi_D)} \sum_{n\geq 1} \sum_{d|n} d\left(\chi_D(d) + \chi_D(n/d)\right) q^n \in M_2^+(D,\chi_D). \tag{3.3}$$

In particular, the geometric self intersection number of the line bundle of modular forms $\mathcal{M}_k(\mathbb{C})$ on $\widetilde{X}_K$ is given by $\mathcal{M}_k(\mathbb{C})^2 = k^2\zeta_K(-1)$.

4. Arithmetic generating series

It is well known that the Hilbert modular surface Y_K has a moduli interpretation. It parametrizes isomorphism classes of triples (A,ι,ψ), where A is an abelian surface over $\mathbb{C}$, ι is an $\mathcal{O}_K$-multiplication, that is, a ring homomorphism $\mathcal{O}_K \hookrightarrow \operatorname{End}(A)$, and ψ is a $\mathfrak{d}^{-1}$-polarization, that is, an isomorphism of $\mathcal{O}_K$-modules $\mathfrak{d}^{-1} \to \operatorname{Hom}_{\mathcal{O}_K}(A,A^\vee)^{\mathrm{sym}}$ from the inverse different $\mathfrak{d}^{-1}$ to the module of $\mathcal{O}_K$-linear symmetric homomorphisms, taking the totally positive elements of $\mathfrak{d}^{-1}$ to $\mathcal{O}_K$-linear polarizations (see [Go] Chapter 2).

This moduli interpretation can be used to construct a model of Y_K over $\mathbb{Z}$. By the work of Rapoport [Ra], Deligne, and Pappas [DePa] it is known that the moduli problem over an arbitrary base scheme over $\mathbb{Z}$ is represented by a regular algebraic stack $\mathcal{H}$, which is flat and of relative dimension two over $\operatorname{Spec}\mathbb{Z}$. It is smooth over $\operatorname{Spec}\mathbb{Z}[1/D]$. The corresponding complex variety $\mathcal{H}(\mathbb{C})$ is isomorphic to Y_K.

For $k \in \mathbb{Z}$ sufficiently divisible there exists a line bundle $\mathcal{M}_k$ on $\mathcal{H}$ such that the induced bundle on $\mathcal{H}(\mathbb{C})$ can be identified with the line bundle $\mathcal{M}_k(\mathbb{C})$ of Hilbert modular forms of weight k for Γ_K of the previous section. By the q-expansion principle and the Koecher principle, the global sections of $\mathcal{M}_k$ can be identified with holomorphic Hilbert modular forms of weight k for Γ_K with integral rational Fourier coefficients. The scheme

$$\overline{\mathcal{H}} = \operatorname{Proj}\left(\bigoplus_k H^0(\mathcal{H},\mathcal{M}_k)\right)$$

can be regarded as an arithmetic Baily-Borel compactification of the coarse moduli space corresponding to $\mathcal{H}$. It is normal, projective, and flat over $\operatorname{Spec}\mathbb{Z}$ (see [Ch]). The corresponding complex variety $\overline{\mathcal{H}}(\mathbb{C})$ is isomorphic to X_K.

To simplify the exposition, for the rest of this note we make the following

Assumption 4.1. *There exists a desingularization $\pi : \widetilde{\mathcal{X}}_K \to \overline{\mathcal{H}}$ by a regular scheme $\widetilde{\mathcal{X}}_K$, which is projective and flat over $\mathbb{Z}$, such that the regular locus $\overline{\mathcal{H}}^{reg}$ is fiber-wise dense in $\widetilde{\mathcal{X}}_K$, and such that the induced morphism $\widetilde{\mathcal{X}}_K(\mathbb{C}) \to X_K$ is a desingularization as in the previous sections $\widetilde{X}_K$.*

It is not known whether such an arithmetic variety $\widetilde{\mathcal{X}}_K$ exists. However, if we consider an additional level structure to rigidify the moduli problem, then there exist such arithmetic varieties over certain subrings of cyclotomic fields. To get unconditional results one can work with these (as it is done in [BBK]).

An important invariant of an arithmetic variety $\mathcal{X}$ is its first arithmetic Chow group $\widehat{\mathrm{CH}}^1(\mathcal{X})$, that is, the group of arithmetic divisors modulo rational equivalence. Recall that an arithmetic divisor on $\mathcal{X}$ is a pair (y, g_y), where $y \subset \mathcal{X}$ is a divisor, and g_y is a certain Green function for the induced divisor $y(\mathbb{C})$ on the corresponding complex variety $\mathcal{X}(\mathbb{C})$ (see [SABK], [BKK1]).

On our arithmetic Hilbert modular surface we consider the following arithmetic divisors. One can show that $T(m)$ is defined over $\mathbb{Q}$. We obtain a divisor $\mathcal{T}(m)$ on $\widetilde{\mathcal{X}}_K$ by taking the Zariski closure of the divisor on the generic fiber. In [Br1] a certain automorphic Green function $G_m(z_1, z_2)$ for $T(m)$ was constructed. (See [BBK] Definition 2.13 for the appropriate additive normalization.) One can view this Green function as the regularized theta lift of a certain Maass wave form of weight 0 for $\Gamma_0(D)$ with singularities at the cusps [Br2]. The pair

$$\widehat{\mathcal{T}}(m) = (\mathcal{T}(m), G_m)$$

defines an arithmetic divisor on $\widetilde{\mathcal{X}}_K$, called the arithmetic Hirzebruch-Zagier divisor of discriminant m. The Green function G_m has a logarithmic singularity along $T(m)$ and in addition log-log singularities along the exceptional divisor of $\widetilde{X}_K$. Therefore $\widehat{\mathcal{T}}(m)$ naturally lives in the extended Chow group $\widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})$ of $\widetilde{\mathcal{X}}_K$ with pre-log-log growth along the exceptional divisor of $\widetilde{X}_K$ in the sense of [BKK1]. This is indicated by the additional argument $\mathcal{D}_{\mathrm{pre}}$.

We also obtain an element of $\widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})$ by taking the first arithmetic Chern class $\widehat{\mathrm{c}}_1(\overline{\mathcal{M}}_k)$ of the hermitian line bundle $\overline{\mathcal{M}}_k$ of Hilbert modular forms equipped with the Petersson metric. Recall that the Petersson metric of a Hilbert modular form F of weight k is given by

$$\|F(z_1, z_2)\|_{\mathrm{Pet}} = |F(z_1, z_2)| \big(16\pi^2 \Im(z_1)\Im(z_2)\big)^{k/2}.$$

This defines a hermitian metric on $\mathcal{M}_k(\mathbb{C})$ which has logarithmic singularities along the exceptional divisor of $\widetilde{X}_K$.

We would like to describe the positions of the arithmetic Hirzebruch-Zagier divisors in $\widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})_{\mathbb{Q}}$. Similarly as in the previous section we consider the generating series

$$\widehat{A}(\tau) = \widehat{\mathrm{c}}_1(\overline{\mathcal{M}}_{-1/2}) + \sum_{m>0} \widehat{\mathcal{T}}(m) q^m. \tag{4.1}$$

Under the Assumption 4.1 the main results of [BBK] imply the following theorems.

Theorem 4.2. *The arithmetic Hirzebruch-Zagier divisors $\widehat{\mathcal{T}}(m)$ generate a subspace of $\widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})_{\mathbb{Q}}$ of dimension $\dim(M_2^+(D, \chi_D))$. The generating series $\widehat{A}(\tau)$ is a modular form in $M_2^+(D, \chi_D)$ with values in $\widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})_{\mathbb{Q}}$, i.e.,*

$$\widehat{A}(\tau) \in M_2^+(D, \chi_D) \otimes_{\mathbb{Q}} \widehat{\mathrm{CH}}^1(\widetilde{\mathcal{X}}_K, \mathcal{D}_{\mathrm{pre}})_{\mathbb{Q}}.$$

In analogy to the result of Franke and Hausmann it is interesting to study the arithmetic intersection of $\widehat{A}(\tau)$ with $\overline{\mathcal{M}}_k^2$.

Theorem 4.3. *We have the following identities of arithmetic intersection numbers:*

$$\widehat{A}(\tau)\cdot\widehat{\mathrm{c}}_1(\overline{\mathcal{M}}_k)^2=\frac{k^2}{2}\zeta_K(-1)\left(\frac{\zeta_K'(-1)}{\zeta_K(-1)}+\frac{\zeta'(-1)}{\zeta(-1)}+\frac{3}{2}+\frac{1}{2}\log(D)\right)\cdot E_2^+(\tau), \tag{4.2}$$

where $\zeta_K(s)$ denotes the Dedekind zeta function of K, $\zeta(s)$ the Riemann zeta function, and $E_2^+(\tau)$ the Eisenstein series defined in (3.3)*. In particular, the arithmetic self intersection number of $\overline{\mathcal{M}}_k$ is given by:*

$$\overline{\mathcal{M}}_k^3=-k^3\zeta_K(-1)\left(\frac{\zeta_K'(-1)}{\zeta_K(-1)}+\frac{\zeta'(-1)}{\zeta(-1)}+\frac{3}{2}+\frac{1}{2}\log(D)\right). \tag{4.3}$$

Formula (4.3) provides further evidence for conjectures of Kramer, Maillot-Roessler, and Kudla relating arithmetic self intersection numbers on Shimura varieties to logarithmic derivatives of L-functions.

The proofs of these results rely, among other things, on the arithmetic and geometric properties of Borcherds' regularized theta lift [Bo1], [Bo2], its generalization to weak Maass forms in [Br1], [Br2], results on the existence of "many" Borcherds products, the q-expansion principle, and the computation of similar arithmetic intersection numbers on modular curves in [Bost], [Kü].

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