GAUS AG on Rigid Analytic Motives

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The goal of the seminar is to provide an introduction to the theory of motives in rigid analytic geometry developed by Ayoub, and to apply it to explain the presence of Frobenii and monodromy operators on p-adic cohomology theories following the work of Binda, Gallauer and Vezzani.

Motives are meant to be a geometric avatar for the cohomological data attached to a variety. One approach is via motivic homotopy theory, which combines sheaf theory and classical homotopy theory using the framework developed in [Lur09]. Roughly speaking, the homotopy type of a motive over a variety S is built up by ∞-sheaves. Since every ∞-groupoid is the fundamental infinity groupoid of a topological space, ∞-sheaves simultaneously generalize homotopy types of spaces via constant sheaves and smooth varieties over S via the Yoneda embedding. After $\mathbb{A}^1$-localization (forcing $\mathbb{A}^1$ to be contractible) and $\mathbb{P}^1$-stabilization (which is an analogue of the classical construction of spectra for the "motivic sphere" $\mathbb{P}^1 \cong S^1 \times \mathbb{G}_m$) of ∞-sheaves, one can construct motivic Eilenberg-MacLane spectra associated to sufficiently nice rings Λ, which represents motivic cohomology with Λ-coefficients. The category $\mathrm{DA}(S, \Lambda)$ is defined to be the subcategory of $\mathrm{EM}(\Lambda)$-modules in $\mathbb{P}^1$-spectra, satisfying a 6-functor formalism, which describes the compatibility of the symmetric monoidal structure of $\mathrm{DA}(S, \Lambda)$ with functoriality in S and their right adjoints. This produces a unified framework for studying cohomological "realization" functors $R\Gamma : \mathrm{DA}(S) \to \mathcal{D}$.

There are analogous constructions of motivic homotopy categories for formal schemes and rigid analytic varieties. Morever, there are some quite robust comparison theorems between such categories and DA of the special fibre, which provides a more subtle version of a six functor formalism for rigid analytic motives, developed in [AGV22]. The categories $\mathrm{RigDA}(-)$ can even be defined for analytic adic spaces and hence diamonds via pro-étale descent, allowing for some techniques from perfectoid geometry found in [BV21]. The formalism of motivic homotopy theory also allows a lot of flexibility in nonarchimedean geometry; for instance, rigid analytic motives carry a canonical †-structure up to homotopy, which is used to define an overconvergent de Rham realization functor in [BV21].

The second part of the seminar will focus on [BGV23], which provides a framework for studying and explaining the presence of frobenii and monodromy operator on p-adic cohomology theories. Let p be a prime number. For a finite field extension $K/\mathbb{Q}_p$ and a smooth proper variety $X \to \mathrm{Spec}(K)$, one can consider the de Rham cohomology $H^*_{\mathrm{dR}}(X)$, a finite dimensional K-vector space. Fontaine and Jannsen conjecture that $H^*_{\mathrm{dR}}(X)$ is canonically obtained via extension of scalars from a vector space over the maximal unramified subextension K_0, denoted $H^*_{\mathrm{HK}}(X)$, equipped with a (φ, N)-module structure, consisting of:

- a Frobenius semi-linear isomorphism $\varphi\colon H^*_{\mathrm{HK}}(X) \simeq H^*_{\mathrm{HK}}(X)$ and
- a nilpotent endomorphism $N\colon H^*_{\mathrm{HK}}(X) \to H^*_{\mathrm{HK}}(X)(-1)$, i.e. satisfying $N\varphi = p \cdot \varphi N$.

The modules $H^*_{\mathrm{HK}}(X)$, called *Hyodo-Kato cohomology*, have been constructed in increasing generality (including for rigid analytic varieties X) by Hyodo–Kato, Mokrane, Große–Klönne, Beilinson, Ertl–Yamada, and Colmez–Niziol. The constructions involve choices of suitable integral models of X over $\mathcal{O}_K$ and make nontrivial use of methods from log-geometry, which raises the question of independence and make functoriality properties difficult to prove.

The manuscript [BGV23] gives a construction of the Hyodo–Kato cohomology relying on the theory of rigid analytic motives developed in [Ayo15; AGV22]. A key ingredient in the construction is an equivalence of categories

$$\mathrm{RigDA}(\mathbb{C}_p) \simeq \mathrm{DA}_N(\bar{\mathbb{F}}_p), \quad M \mapsto (\Psi M, N_{\Psi M}\colon \Psi M \to \Psi M(-1)),$$

where $\mathrm{RigDA}(\mathbb{C}_p)$ is the category of rigid analytic motives over $\mathbb{C}_p$, $\mathrm{DA}_N(\bar{\mathbb{F}}_p)$ is the category of algebraic motives over $\bar{\mathbb{F}}_p$ together with an (ind-)nilpotent operator N, and Ψ is the motivic nearby cycles functor of Ayoub; notably, the equivalence is only on the categorical level in the sense that it does not carry effective rigid motives to effective motives over the special fibre. Objects of the target category are canonically equipped with an action under pullback by the absolute Frobenius φ, and the φ-equivariant objects define the category of motivic (φ, N)-modules.

Composing with suitable realization functors (e.g. ℓ-adic or Hodge realizations in case $K = \mathbb{C}((t))$) allows one to extend various algebraic cohomology theories to analytic cohomology theories equipped with frobenius and monodromy data. Using the de Rham realization from Le Bras–Vezzani [BV21], this allows for a canonical construction of rigid cohomology in characteristic p and yields a new definition of Hyodo-Kato cohomology for rigid spaces, naturally defined on the generic fibre and independent of the choice of integral models, as conjectured.

This framework also explains how weight filtrations arise from categorical weight structures on motives, and allows for the construction of a p-adic Clemens–Schmid chain complex, whose exactness would be implied by the p-adic weight monodromy conjecture.

There is a connection with the de Rham–Fargues–Fontaine cohmology of [BV21]: Namely, let Δ be the locus $p \neq 0$ in $(\mathrm{Spf}\,\mathrm{A}_{\mathrm{inf}})^{\mathrm{rig}}$, so $\Delta = \mathcal{Y}_{(0,\infty]}(\mathbb{C}_p^\flat)$ in the notation of Fargues–Scholze. Let x_∞ be its closed point $\mathrm{Spa}(\breve{\mathbb{Q}}_p) \to \Delta$ and Δ^* its complement. In [BV21], the authors construct a functor $\mathrm{RigDA}(\mathbb{C}_p) \to \mathrm{RigDA}(\Delta^*)$. Informally, it associates to (the motive of) a rigid analytic space X the family of relative de Rham cohomology groups $H^*_{\mathrm{dR}}(X_C/C)$ over points $x_C \to \Delta^*$. Any module with φ-action on Δ^* extends uniquely to Δ, and taking the fibre at x_∞ recovers the Hyodo–Kato cohomology, to be thought of as the “limit” of the groups $H^*_{\mathrm{dR}}(X_C/C)$ as $x_C \to x_\infty$. This allows for a reinterpretation of monodromy operators as limit structures on cohomology and generalises phenomena appearing in complex geometry.

Structure of talks and prerequisites:

Roughly, the first half of the talks provides the background material for [BGV23] and the second half studies the manuscript. The program assumes familiarity with algebraic geometry, and to some extent ∞-categories and non-archemedian geometry. Necessary prerequisites are listed below each talk. The talks labelled by * are independant of the other talks and suitable for advanced master’s students. These are **talks 4, 5 and 8**. In case you are interested in participating but not sure what talk to give, please feel free to contact one of the organizers.

Time and place:

The seminar takes place in a hybrid format jointly organized by Darmstadt and Frankfurt.

- Thursdays, 15:00 – 16:30 during the winter term 2023/24. No talk on December 07 (Ruth Moufang Lecture) and at some other date, probably in 2024, due to the GAUS colloquium (t.b.a.).
- Start: October 19, end: February 08
- The last session on either Feb 01 or Feb 08 (depending on the GAUS colloquium) is a double session with 2 talks between 14:00 – 18:00. Afterwards there will be a joint dinner. Details will be announced later.
- The seminar will take place in Darmstadt, Room S215 244.
- Zoom meeting ID: 612 2072 7363, Password: Largest six digit prime number.

1 Leitfaden (Oct 19)

An overview talk (20–30 minutes) given by one of the organizers explaining the interrelation of talks and the structure of the seminar.

2 Infinity sheaves (Oct 19)

Talk 1: (only 60 minutes due to the Leitfaden) The talk collects some results on ∞-sheaves. We restrict to the context used in [AGV22].

Prerequisites: Familiarity with ∞-categories.

- State the universal property of localizations [Lur09, Proposition 5.5.4.20].
- Fix a commutative, unital (ordinary) ring Λ. For a 1-category define the ∞-category of Λ-presheaves $\mathrm{PSh}(\mathcal{C}, \Lambda)$ as functors from $\mathcal{C}$ to $D(\Lambda)$.
- For a 1-site $(\mathcal{C}, \tau)$ define the ∞-category of Λ-hypersheaves $\mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}, \Lambda)$ as the localization of $\mathrm{PSh}(\mathcal{C}, \Lambda)$ with respect to τ-hypercoverings, see [AGV22, Definition 2.3.1]. Discuss the adjunction $\mathrm{PSh}(\mathcal{C}, \Lambda) \underset{\mathrm{incl}}{\overset{L_\tau}{\rightleftarrows}} \mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}, \Lambda)$, see [AGV22, Remark 2.3.3 (2)].
- Sketch that there is a t-exact equivalence $\mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}, \Lambda) \cong D(\mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}, \Lambda)^{\heartsuit})$, see [Lur, Theorem 2.1.2.2, Definition 2.1.0.1].

3 Algebraic étale motives

Review the construction of motives for schemes with a view towards the construction for rigid analytic varieties: Among the different constructions of categories of motives Ayoub's category of étale motives [Ayo14b] seems most convenient for the purposes of the seminar. The talks construct the category from an ∞-categorical (as opposed to a model categorical) view point, the six functors and sketch finer properties of the category like localization and the comparison with Chow groups.

Prerequisites: Familiarity with sheaves and six functors. Ideally, one of the speakers has some familiarity with ∞-categories and/or motives.

3.1 Algebraic étale motives I (Oct 26)

Talk 2: Follow [AGV22, Section 2] but restrict to the case of schemes, compare [Ayo14b, Section 3]: Fix a commutative unital (ordinary) ring Λ. The aim of the talk is to construct the category $\mathrm{DA}(X) = \mathrm{DA}^{\text{ét}}(X, \Lambda)$ of étale motives on a scheme X with coefficients in Λ. The reference [Ayo14b, Section 3] is written in the language of model categories, see also [Ayo14a] for a survey. Fortunately, [AGV22, Section 2] explains the relevant constructions also for schemes.

More precisely, for a scheme X denote by $\mathcal{C} = \mathrm{Sm}_{/X}$ the full subcategory of schemes over X consisting of X-smooth maps. This gives a site $(\mathcal{C}, \tau) := (\mathrm{Sm}_{/X}, \text{ét})$ when equipped with the étale topology. Denote $\mathrm{PSh}(\mathcal{C}) := \mathrm{PSh}(\mathcal{C}; \Lambda)$ and $\mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}) := \mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}; \Lambda)$ as in the previous talk. Follow [AGV22, Section 2] and the references therein to discuss the following points:

- Elaborate on the following diagram to define $\mathrm{DA}(X)$ as time permits (to abbreviate we drop Λ from the notation):
$$\mathrm{PSh}(\mathcal{C}) \underset{\mathrm{incl}}{\overset{L_\tau}{\rightleftarrows}} \mathrm{Sh}^{\wedge}_{\tau}(\mathcal{C}) \underset{\mathrm{incl}}{\overset{L_{\mathbb{A}^1}}{\rightleftarrows}} \mathrm{DA}^{\mathrm{eff}}(X) \underset{\Omega^\infty}{\overset{\Sigma^\infty}{\rightleftarrows}} \mathrm{DA}(X)$$
Here, L_τ and $L_{\mathbb{A}^1}$ denote the localization with respect to étale hypercoverings and the set of maps $\mathbb{A}^1_Y \to Y$ for all objects $Y \in \mathrm{Sm}_{/X}$, respectively. Their right adjoints incl are the inclusion functors. The functor $\Sigma^\infty \colon \mathrm{DA}^{\mathrm{eff}}(X) \to \mathrm{DA}(X)$ is the initial functor in Pr^L sending $\mathbb{G}_m[-1] = \Lambda(1)$ to an invertible object. Its right adjoint is denoted Ω^∞. So $\mathrm{DA}(X)$ is the colimit in Pr^L of the filtered diagram
$$\mathrm{DA}^{\mathrm{eff}}(X) \overset{-\otimes\Lambda(1)}{\to} \mathrm{DA}^{\mathrm{eff}}(X) \overset{-\otimes\Lambda(1)}{\to} \mathrm{DA}^{\mathrm{eff}}(X) \to \cdots .$$
Under the equivalence $(\mathrm{Pr}^R)^{\mathrm{op}} \cong \mathrm{Pr}^L$ one has $\mathrm{DA}(X) \cong \lim_{\underline{\mathrm{Hom}}(\Lambda(1),-)} \mathrm{DA}^{\mathrm{eff}}(X)$.

- The category $\mathrm{DA}(X)$ is a stable ∞-category with a symmetric monoidal closed structure. It is generated under colimits (up to shifts and twists) by the objects $M(U) := \Sigma^\infty L_{\mathbb{A}^1}\Lambda[\mathrm{Hom}_X(-,U))]$ for all qcqs $U \in \mathrm{Sm}_{/X}$. The objects are compact if all U have finite Λ-cohomological étale dimension, see [Ayo14b, Proposition 3.19] and [AGV22, Lemma 2.4.5 ff].

- For any map $f\colon Y \to X$ of schemes there is an adjunction $f^* : \mathrm{DA}(X) \rightleftarrows \mathrm{DA}(Y) : f_*$ where f^* is characterized as the stable colimit-preserving symmetric monoidal functor with $f^*M(U) = M(U \times_X Y)$.

- For a map $f\colon Y \to X$ locally of finite type there is an adjunction $f_! : \mathrm{DA}(Y) \rightleftarrows \mathrm{DA}(X) : f^!$. The functor $f_!$ is characterized by the properties: it satisfies étale descent; if f is an open immersion, then $f_!$ is left-adjoint to f^*; for any factorization $f = \bar{f} \circ j$ with j an open immersion and $\bar{f}$ proper one has $f_! = j_! f_*$.

3.2 Algebraic étale motives II (Nov 02)

Talk 3: Continue the discussion from the previous talk. The speaker is free to make some choices among the following properties:

- The association $X \mapsto \mathrm{DA}^{(\mathrm{eff})}(X), f \mapsto f^*$ extends to a functor $\mathrm{Schemes}^{\mathrm{op}} \to \mathrm{CAlg}(\mathrm{Pr}^L)$ that is finitary (when restricted to $(\Lambda, \text{ét})$-admissible qcqs schemes) and satisfies étale hyperdescent, see [AGV22, Propositions 2.1.21, 2.5.11 (2), Theorem 2.4.3].

- The functors $\mathrm{DA}^{(\mathrm{eff})}(-)$ satisfy localization. In particular, $\mathrm{DA}(-)$ together with the pairs of functors (f^*, f_*), $(f_!, f^!)$ and $(- \otimes -, \underline{\mathrm{Hom}}(-,-))$ satisfy a six functors formalism.

- Let X be a scheme and ℓ a prime number invertible in $\mathcal{O}_X$. The map of sites $(\mathrm{Sm}_{/X}, \text{ét}) \to (\text{Ét}_{/X}, \text{ét}) =: X_{\text{ét}}$ induces a symmetric monoidal equivalence $\mathrm{D}(X_{\text{ét}}, \Lambda)_{\ell\text{-nil}} \cong \mathrm{DA}(X, \Lambda)_{\ell\text{-nil}}$ compatible with the six functors [AGV22, Theorem 2.10.4].

- Let k be a field and $p\colon X \to \mathrm{Spec}(k)$ a separated map of finite type. Assume X is geometrically unibranch (for example, normal) and $\Lambda = \mathbb{Q}$. Then, $\mathrm{H}^0\mathrm{Hom}_{\mathrm{DA}(X)}(\mathbb{Q}(n)[2n], f^!\mathbb{Q}) = \mathrm{CH}^n(X) \otimes \mathbb{Q}$ for all $n \in \mathbb{Z}$. See [CD16, Corollary 5.5.5] for the comparison with étale motives with transfers (respectively, h-motives) and [Cis21, Theorem 1.4.3].

4 Rigid Analytic Geometry

These two talks are meant to serve as an introduction to non-archimedian geometry. The main source is [Bos14]. The first talk presents Tate's rigid analytic varieties over a non-archimedian field. The second talk then aims to compare the classical theory with Raynaud's approach via formal models, in order to motivate and reach the general definition of rigid analytic space.

4.1 ⋆Rigid analytic spaces I - Tate's rigid analytic varieties (Nov 09)

Talk 4: Following [Bos14], introduce the classical theory of rigid analytic varieties over a non-archimedian field.

- Define the Tate algebra T_n and affinoid algebras over non-archimedian fields [Bos14, Section 2.2]. Eventually choose some properties of affinoid algebras in [Bos14, Section 2.2-2.3] to be explained in more or less details.

- Define the set $\mathrm{Sp}(A)$ for an affinoid algebra A [Bos14, Section 3.2], describe $\mathrm{Sp}(T_n)$ [Bos14, Corollary 11, Section 2.2].

- Introduce affinoid subdomains and rational subdomains. State Gerritzen–Grauert's theorem [Bos14, Theorem 20, Section 3.3]. State Tate's acyclicity theorem [Bos14, Section 4.3].

- Define the strong Grothendieck topology on affinoid spaces [Bos14, Definition 4, Section 5.1] and rigid analytic varieties [Bos14, Definition 4, Section 5.3].

- Sketch the construction of the analytification functor $(-)^{\mathrm{an}}$ as in [Bos14, p. 111]. State the analogue of GAGA's theorem [Bos14, Theorem 11-13, Section 6.4].

4.2 ⋆Rigid analytic spaces II - Raynaud's rigid spaces (Nov 16)

Talk 5: Buidling up on the previous talk, discuss formal models of rigid analytic varieties and introduce Raynaud's rigid spaces. The speaker should also explain some statements and constructions appearing in [AGV22, Section 1] that will be used later. We suggest to follow [Bos14] subject to the restrictions [Bos14, p. 162], namely we work either over a formal affine admissible noetherian base, or over $\mathrm{Spf}(\mathcal{O}_K)$ for K a non-archimedean field.

Prerequisites: Familiarity with formal schemes, rigid varieties and/or adic spaces can be useful.

- Recollection on formal schemes. Define admissible formal schemes [Bos14, Section 7.4] and admissible blowups [Bos14, Section 8.2].

- Explain the construction of the rigid analytic fibre $\mathfrak{X}^{\mathrm{rig}}$ of a formal scheme $\mathfrak{X}$ locally of topologically finite type over $\mathrm{Spf}(\mathcal{O}_K)$ [Bos14, Proposition 3, Section 7.3]. Discuss the comparaison $(\hat{X})^{\mathrm{rig}} \hookrightarrow (X_K)^{\mathrm{an}}$ for a scheme $X/\mathcal{O}_K$ [Bos14, p. 161]. Sketch the proof of the equivalence of categories between rigid analytic varieties and admissible formal schemes up to admissible formal blowups [Bos14, Theorem 3, Section 8.4].

- Define the category of rigid analytic spaces [AGV22, Definition 1.1.3]. Explain the relation with uniform adic spaces [AGV22, Corollary 1.2.7]. Define the analytification functor [AGV22, Construction 1.1.15]. Define etale and smooth morphisms of rigid spaces.

5 Rigid analytic motives

Following [AGV22], for an rigid analytic space X, construct the category $\mathrm{RigDA}(X)$ of rigid analytic motives over X. Discuss the 6 functors and comparaison theorems with categories of algebraic and formal motives.

Prerequisites: Familiarity with sheaves and six functor. Knowledge of ∞-categories and/or algebraic motives will help.

5.1 Rigid analytic motives I: definition, analytification, and 6 functors (Nov 23)

Talk 6: The reference is [AGV22, Sections 2 and 4].

- Construct RigDA following [AGV22, Section 2]. The construction is analogous to DA reviewed in Section 3. Discuss the 5 functors and $f_\sharp$.

- Explain the analytification of algebraic motives (2.13). Discuss the remaining sixth functor $f_!$ on the analytification of a scheme (Proposition 2.2.7). Eventually mention the existence for weakly compactifiable morphisms (Definition 4.3.4), and compare to Huber's compactifications as in section 4.2.

- If time allows, discuss the compatibility of the six functor formalism with analytification as in section 4.6.

5.2 Rigid analytic motives II: comparing categories of motives (Nov 30)

Talk 7: The reference is [AGV22, Section 3].

- State and prove the equivalence in [AGV22, Theorem 3.1.10] relating categories of formal motives over a formal scheme and algebraic motives over the special fibre.
- State [AGV22, Theorem 3.3.3] and sketch the proof. This will be important for proving [BGV23, Corollary 4.12] in talk 9. Pay special attention to the case when $\mathcal{S} = \mathrm{Spf}\ \mathcal{O}_K$ is the ring of integers of a complete nonarchimedean field K. Always restrict to the case where Λ is the spectrum associated to an ordinary ring.

6 *Interlude on (Co-)Modules and monodromy operators (Dec 14)

Talk 8: Follow [BGV23, Section 2]. Modules over a split square zero extension $\mathbb{1} \oplus t[-1]$ are identified with the full subcategory of the lax fixed points under $- \otimes t$ of (ind-)nilpotent operators.

Prerequisites: Familiarity with ∞-categories. Some knowledge of Higher Algebra will also be useful.

- State Theorem 2.1 and explain its proof in Sections 2.2–2.5 of the reference.
- Discuss the example of (φ, N)-modules in Section 2.6. If time permits, also discuss the example of unipotent motives in Section 2.7.

7 Motivic (φ, N)-modules and Hyodo–Kato cohomology

Follow [BGV23] to construct the Hyodo–Kato cohomology. A key input is the de Rham realisation functor from [BV21].

7.1 Rigid motives as algebraic motives with monodromy (Dec 21)

Talk 9: The reference is [BGV23, Sections 3.1, 4.1, 4.2].

- Explain the proof of the equivalence $\mathrm{RigDA}_{\mathrm{gr}}(K) \simeq \mathrm{DA}_N(k)$ in Corollary 4.12. Besides the previous talk the main ingredient is Proposition 4.9. Discuss its proof as time permits and remark the identification of $\mathrm{M}^{\mathrm{coh}}(\mathbb{G}_m)$ via cohomological purity [AGV22, Corollary 3.8.32]. Also, mention that all motives have good reduction if K is algebraically closed and discuss the proof as time permits.
- Introduce the notation in Definition 4.13.
- Identify the motive of the Tate curve, see Proposition 4.19. This will require a brief discussion on the Kummer motive from section 3.1.

7.2 Weight structures (Jan 11, prelim)

Talk 10: The reference is [BGV23, Sections 3.3, 4.3].

- Prove Corollary 3.29. Give the example 3.30. For this introduce the heart of a weight structure in Definition 3.20. Elaborate on Remarks 3.21, 3.22 as time permits.
- Introduce the Chow weight structure on categories of motives and prove Proposition 4.22.
- If time permits, discuss Corollary 3.29.

7.3 Extending realization functors (Jan 18, prelim)

Talk 11: Following [BGV23, Sections 3.2, 4.5], the speaker should present the general principle of extending cohomological realisation functors from the category of algebraic motives in characteristic p to the category of rigid analytic motives. This is needed in future talks for the discussion around ℓ-adic cohomology and the construction of Hyodo–Kato cohomology. The speaker should coordinate with the speaker from talk 13 (7.5).

- Explain the construction of $\widehat{F}$ and prove the uniqueness result Corollary 3.17.
- Prove Corollaries 4.29 and 4.34. If time permits mention Corollary 4.33 which uses (φ, N)-modules.
- If time permits, mention the ℓ-adic case in Remark 4.49 and the Hodge case in Remark 4.50, especially the weight filtrations.

7.4 The de Rham realisation functor (Feb 1, prelim)

Talk 12: The reference is [BV21]. Sketch the construction of the de Rham realisation $\mathrm{dR}\colon \mathrm{RigDA}(K)^\omega \to \mathcal{D}(K)^{\mathrm{op}}$, see also [BGV23, Definition 4.36]. It would be nice to explain the relation with de Rham cohomology of algebraic varieties which seems to be implicitly contained in [BV21, Theorem 6.10].

Prerequisites: Knowledge of the theory of adic spaces, diamonds, and solid modules is required. This will be a very difficult talk to prepare and give since many details will have to be abbreviated or omitted.

7.5 The Hyodo–Kato cohomology (Feb 8, prelim)

Talk 13: Follow [BGV23, Sections 4.6, 4.7]. The speaker should coordinate with the speaker from talk 11 (7.3).

- Introduce the Hyodo–Kato cohomology $\widehat{R\Gamma}_{\mathrm{rig}}$ and prove Theorem 4.42.
- Prove corollary 4.46 and explain the identification of the monodromy action on the cohomology of the Tate curve.
- If it has not been done yet in talk 11 (7.3), mention the ℓ-adic case in Remark 4.49 and the Hodge case in Remark 4.50, especially the weight filtrations.

8 (Optional) The de Rham–Fargues–Fontaine cohomology (Feb 8, prelim)

Talk 14: Following [BV21], discuss the de Rham–Fargues–Fontaine cohomology and the relation with the paper [BGV23].

Prerequisites: Knowledge of the adic Fargues–Fontaine curve is required.

- For $K/\mathbb{Q}_p$ algebraically closed, sketch the construction of the de Rham–Fargues–Fontaine cohomology
$$\mathrm{dR}^{\mathrm{FF}}_{\mathrm{K}}\colon \mathrm{RigDA}(K) \cong \mathrm{RigDA}(K^\flat) \xrightarrow{\mathcal{D}} \mathrm{RigDA}(\mathcal{X}_K) \xrightarrow{\mathrm{dR}} \mathrm{Qcoh}(\mathcal{X}_K)^{\mathrm{op}}.$$
- Explain the comparison with overconvergent de Rham cohomology [BV21, Corollary 5.14]

$$\begin{array}{ccc} \mathrm{RigDA}(K) & \xrightarrow{\mathrm{dR}_K} & \mathrm{Qcoh}(\mathrm{Spa}(K)) \\ & {\scriptstyle \mathrm{dR}^{FF}_K} \searrow \quad \nearrow {\scriptstyle \infty^*} & \\ & \mathrm{Qcoh}(\mathcal{X})) & \end{array}$$

and with rigid cohomology [BV21, Section 6.3]

$$\begin{array}{ccccc} \mathrm{DA}(k) & \xrightarrow{\xi} & \mathrm{RigDA}(K) & \xrightarrow{H^i_{FF}(-/\mathcal{X}_K)} & \mathrm{VB}(\mathcal{X}_K) \\ & \searrow_{H^i_{\mathrm{rig}}(-)} & & \nearrow & \\ & & \mathrm{Isoc}(\breve{\mathbb{Q}}_p, \varphi) & & \end{array}$$

- Explain the relation with Hyodo–Kato cohomology, building on the comparaison with rigid cohomology. Compare with [BGV23, Remark 1.2].

References

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