A <u>singular modulus</u> is the j-invariant of a CM EC or equivalently the values of the j-function at imaginary quadratic values

<u>Ex</u>: $j(\sqrt{-1}) = 1728$, $j\left(\frac{1+i\sqrt{3}}{2}\right) = 0$

singular moduli are algebraic integers

For an alg. number α we define

$$h(\alpha) = \frac{1}{[K:\mathbb{Q}]}\left(\sum_{\sigma} \log\max\{1, \sigma(\alpha)\} + \sum_{v} d_v \log\max\{1, |\alpha|_v\}\right)$$

$$d_v = [K_v : \mathbb{Q}_p]$$

Let J be a sing. mod. of EC with CM with endo. ring of discriminant $\Delta < 0$

$$h(J) \geq c_2 \log|\Delta| - c_3 \qquad c_2 = \frac{3}{\sqrt{5}}$$

If J is a unit, then we have

$$h(J) \leq \frac{c_2}{2}\log|\Delta| + c_4$$

~~Let~~ Let S be a set of places on $K/\mathbb{Q}$
~~and~~ $\alpha \in K$ is called S-integer if $v(\alpha) \geq 0$
or $|\alpha|_v \leq 1$ for all $v \notin S$.

Rem.: ~~A~~ alg ~~int~~ number α is an
alg. integer iff S_∞-integer.
$\Leftrightarrow |\alpha|_p \leq 1$ for all p

$E_0: y^2 = x^3 + 1$. Let T a set of primes
for which E_0 has ordinary reduction.

Rem. If E is EC with CM and j-inv.
j_E and v is a place of good red. for E
~~then~~ and if $|j_E|_v \leq 1$, then E has ord. red.
iff E_0 has ord. red.

Prop: $\left\{ j_E \mid E \text{ EC with CM and } j\text{-inv. } j_E \text{, s.t. } p \mid N(j_E) \Rightarrow p \in T \right\}$

is finite

proof ~~then~~ ingredient
$$-\sum_{v|p} \log |\mathfrak{J}|_v \leq \sum_{v|p} \log (p^{-\frac{?}{p-1}})$$

As we see from this we can take
T to be infinite as long as
$$\sum_{p \in T} \frac{\log p}{p-1}$$

Let p be a prime of supersingular reduction mod p for E_0

<u>Problem</u>: We need an estimate for

$$\log |\mathfrak{J}|_v$$

I have $\log|\mathfrak{J}_v| \leq c_5 \log|\Delta| + D$