TECHNISCHE UNIVERSITÄT DARMSTADT

Fachbereich **Mathematik**

Winter seminar of the Darmstadt algebra group

March 01 – March 08, 2015

SCHEDULE

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
08.00 – 08.45	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
09.00 – 13.00	Working/Skiing in groups	Working/Skiing in groups	Working/Skiing in groups	Working/Skiing in groups	Working/Skiing in groups
13.00 – 14.30	Lunch break	Lunch break	Lunch break	Lunch break	Lunch break
14.30 – 15.30	**Bruinier**	**Möller, M.**	**Scheithauer**	**Kramer**	**Li**
15.30 – 16.00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
16.00 – 16.30	**Kappes**	**Veneziano**	**Pippich**	**Alfes**	**Costantini**
16.30 – 16.45	Break	Break		Break	Informal discussions
16.45 – 17.15	**Schwagenscheidt**	**Möller, S.**	Informal discussions	**Opitz**	
17.15 – 18.45	Informal discussions	Informal discussions		Informal discussions	
19.00 – 20.00	Dinner	Dinner	Dinner	Dinner	Dinner
20.00 – 20.30	Evening session	**Völz**	Evening session	Evening session	Evening session
20.30 – 21.00		**Schmid**			

TITLES AND ABSTRACTS

Claudia Alfes

Harmonic Maass forms and periods

In this talk we present some recent work on the connection between coefficients of harmonic Maass forms and periods of associated differentials.

Jan Bruinier

Kudla's modularity conjecture and formal Fourier-Jacobi series

A famous theorem of Gross, Kohnen, and Zagier states that the generating series of Heegner divisors on a modular curve is an elliptic modular form of weight $3/2$ with values in the Picard group. This result can be viewed as an elegant description of the relations among Heegner divisors. More generally, Kudla conjectured that the generating series of codimension g special cycles on an orthogonal Shimura variety of dimension n is a Siegel modular form of genus g and weight $1 + n/2$ with coefficients in the Chow group of codimension g cycles. We report on joint work with Martin Raum on the modularity of formal Fourier-Jacobi series, which, when combined with a result of Wei Zhang, leads to a proof of Kudla's modularity conjecture.

Matteo Costantini

Lyapunov exponents of families of K3 surfaces

Lyapunov exponents are numbers describing the dynamics of some special dynamical systems. In particular, they can be associated to the dynamics of variations of Hodge structures. I will present the relation between Lyapunov exponents associated to one dimensional families of K3 surfaces and some Chern classes, describing in particular the situation in some specific modular examples.

André Kappes

Cutting out arithmetic Teichmüller curves in genus 2 with theta functions

(Joint work with Martin Möller.) A square-tiled surface is a closed surface obtained from finitely many unit squares in the plane by gluing their sides by translations. Affinely deforming the squares yields an algebraic curve in the moduli space of compact Riemann surfaces of genus g, called arithmetic Teichmüller curve. It is a hard problem to determine the number of different Teichmüller curves for a fixed number of squares and gluing combinatorics. For genus 2, we propose a description of these Teichmüller curves using theta functions and their derivatives. As the Jacobians of square-tiled surfaces of genus 2 all have multiplication by a pseudo-quadratic order, they correspond to points in a pseudo-Hilbert modular surface X, which in fact is a quotient of the direct product of two modular curves. To determine the class of a Teichmüller curve in the Picard group of X, we cut out a locus in the universal family of abelian surfaces that projects to it. As one result of our description, we determine the Euler characteristics of the Teichmüller curves (but not the irreducibility) for fixed combinatorics.

Jürg Kramer

Kronecker limit formulae, revisited

In our talk, we will discuss an alternative approach to the Kronecker limit formula for $\mathrm{SL}_2(\mathbb{Z})$, which permits a generalization to higher dimensions.

Yingkun Li

The span of restrictions of coherent Eisenstein series

In the theory of modular forms, Eisenstein series plays an important role because of its explicit Fourier coefficients. It is well-known that the algebra of modular forms on $\mathrm{SL}_2(\mathbb{Z})$ is generated by the classical Eisenstein series E_4 and E_6. For a fixed weight k, Kohnen and Zagier showed that it suffices to consider the span of the products of two Eisenstein series E_ℓ and $E_{k-\ell}$. In this talk, we will look at a related question, first raised by Tonghai Yang, about the span of the restrictions of coherent Eisenstein series. We will discuss the construction of these Eisenstein series, its relationship to special values of L-function, and other related open problems.

Martin Möller

Counting covers with Siegel–Veech weight and quasimodular forms

The generating function for counting torus coverings is known to be a quasimodular form. The proof in the simple case by Kaneko–Zagier relies on product expansions of theta functions and the general case by Bloch–Okounkov on q-brackets of shifted symmetric functions. Counting covers with Siegel–Veech weight is motivated from problems in polygonal billards. The resulting counting functions are no shifted symmetric, and yet the generating functions are quasimodular forms. In the talk we will mainly explain basic properties of shifted symmetric functions, q-brackets and their relation to theta functions.

Sven Möller

Cyclic orbifold construction of holomorphic VOAs

We describe a theory of orbifolds of VOAs by automorphisms of arbitrary finite order. This generalises previous results by Frenkel, Lepowsky and Meurman and Miyamoto on orbifolds of order 2 and 3, respectively.

Sebastian Opitz

Discriminant forms and vector valued modular forms

A given space of vector valued for the Weil representation associated to an even lattice only depends on the discriminant form of the lattice. The dimension of this space can be computed from the representation numbers of the quadratic form of the discriminant form. As the discriminant form decomposes orthogonally into p-adic Jordan components, it suffices to compute the representation numbers of these Jordan components. We give formulas for the representation numbers of small 2-adic Jordan components, generalizing a result of Scheithauer. If p is odd, the representation numbers of the p-adic Jordan components can be obtained by decomposing the discriminant form into orbits with respect to the action of its own othogonal group. It suffices to know the length of each orbit, this length can be computed recursively by formulas of Scheithauer which we simplify.

Anna von Pippich

Kronecker limit formulae and regularized determinants

The classical Kronecker limit formula describes the derivative at $s = 0$ of the non-holomorphic Eisenstein series for $\mathrm{SL}_2(\mathbb{Z})$ in terms of the Dedekind Delta function. In our talk, we will explain how this formula can be used to compute the regularized determinant of the Laplacian on an elliptic curve. Moreover, we will discuss corresponding results for hyperbolic Riemann surfaces.

Nils Scheithauer — **Automorphic products of singular weight**

We give a simple characterisation of Borcherds function Φ_{12} and describe the classification of reflective automorphic products of singular weight on lattices of prime level.

Stefan Schmid — **Singular moduli that are S–units**

It is a well-known fact, that singular moduli are algebraic integers. In 2011 David Masser asked if there were any algebraic units among them and if so, if there were only finitely many. The last part of the question was answered affirmatively by Philipp Habegger. Instead of looking at the algebraic integers, which are integers for all places of $\mathbb{Q}$, we consider subsets S of places of $\mathbb{Q}$ and investigate singular moduli which are S–units.

Markus Schwagenscheidt — **Siegel modular forms of degree 2**

We give a short introduction to the theory of Siegel modular forms of degree 2. We define the spinor zeta function associated to a Hecke eigenform F and formulate Andrianov's fundamental equation relating the Hecke eigenvalues and the Fourier coefficients of F. Further, we state Böcherer's conjecture which roughly says that the central critical value of the twisted spinor zeta function of an eigenform F is proportional to the square of a weighted sum of Fourier coefficients of F. If time permits, we present Böcherer's proof of his conjecture in the case of Saito–Kurokawa lifts.

Francesco Veneziano — **Torsion-anomalous intersections**

Anomalous intersections are a framework introduced by Bombieri, Masser and Zannier, which comprises and generalises a vast body of problems and conjectures in arithmetic geometry. Let V be a variety contained in a group variety G, which is usually taken to be an abelian variety or a torus. When intersecting V with an algebraic subgroup B, if the intersection $V \cap B$ has a component of dimension strictly greater than "expected", then such a component is said to be torsion-anomalous. In analogy with many fundamental results in the field, there are conjectures giving geometrical conditions for the variety V to have only finitely many (maximal) torsion-anomalous subvarieties. I will present partial results in this direction. Joint work with S. Checcoli and E. Viada.

Fabian Völz — **Hyperbolic and elliptic Eisenstein series**

In 1979 Kudla and Millson introduced a hyperbolic Eisenstein series generalising the classical (parabolic) non-holomorphic Eisenstein series. Instead of being associated to cusps these are coming from hyperbolic elements of the underlying modular group. Following this idea Kramer and Jorgenson considered elliptic Eisenstein series being based on elliptic elements.

In this talk, we revisit these two types of generalised Eisenstein series and discuss some of their properties. In particular, we try to establish them as theta lifts by integrating a certain weighted Poincare series against Siegel's theta function of signature $(1, 2)$.