Workshop p-adic Riemann Hilbert Correspondence

Darmstadt, June 6-8, 2018

In this workshop we will study aspects of the p-adic Riemann Hilbert correspondence following Ruochuan Liu and Xinwen Zhu ([LZ]) and some applications.

To approach the topic let us recall the Riemann Hilbert correspondence over the complex numbers.

Let X be a complex manifold. Then sending a local system $\mathbb{L}$ to $(\mathscr{O}_X \otimes_{\mathbb{C}} \mathbb{L}, d_X \otimes 1)$ yields an equivalence between the category of local systems of finite-dimensional $\mathbb{C}$-vector spaces and the category of vector bundles on X equipped with an integrable connection.

The goal is to construct for a smooth rigid analytic variety X over a p-adic local field k an analogous functor $\mathcal{RH}$ from the category of $\mathbb{Q}_p$-local systems on X_{et} to a certain category of vector bundles with connections. As there is a much richer structure on the category $\mathbb{Q}_p$-local systems, in the p-adic setting one cannot expect these vector bundles simply to live over X itself. Instead it is no surprise that Fontaine's period ring B_{dR} enters the picture. More precisely, the functor $\mathcal{RH}$ will take values in the category of finite locally free $\mathscr{O}_X \hat{\otimes} B_{\mathrm{dR}}$-modules equipped with a semi-linear action of $\mathrm{Gal}(\bar{k}/k)$, and with a filtration and integrable connection satisfying Griffiths transversality.

Conjecturally, this construction should just be a very special case of a general cohomology theory with values in the category of vector bundles in the Fargues-Fontaine curve (see Scholze's ICM talk [Sch3], §6).

One already very interesting step towards a p-adic Riemann Hilbert correspondence in [LZ] is a special case of a p-adic Simpson correspondence.

We will mainly follow [LZ], with some minor additions. The goal of the workshop is to delve deeply into technical details! The speakers are encouraged to explain interesting techniques thoroughly. All talks will be 90 minutes, which should allow to explain many aspects in detail.

The speakers are invited to consult additional literature if necessary. If in doubt which material to cover, please do not hesitate contact the organizers for any questions. Please also contact other speakers to agree upon what you should cover, what has been explained previously, and what is needed in later talks.

We assume that the participants are familiar with basic notions of p-adic geometry such as adic spaces. We will consider the category of rigid analytic spaces as a full subcategory of the category of adic spaces.

Talk 1. Perfectoid spaces. [Sch1, §2,6,7]

Talk 2. The pro-étale site of a rigid analytic space. [Sch2] (see also [BMS, §5]). In particular explain the completed structure sheaf $\widehat{\mathscr{O}}_X$ ([Sch2, §4]) and the de Rham period sheaf on X_{proet} ([Sch2, §6]). After this talk we should understand all objects that appear in [LZ] (2.1) – (2.3).

Talk 3. Statement of a p-adic Simpson correspondence and preliminaries for the proof. [LZ, 2.1 and 2.2]. Feel free to omit Remark 2.4, 2.5, and 2.6.

Talk 4. Proof of p-adic Simpson modulo Proposition 2.8, Part I. Show (i) and (ii) of Theorem 2.1 ([LZ, 2.3], until Lemma 2.11 and following lines)

Talk 5. Proof of p-adic Simpson modulo Proposition 2.8, Part II. Show (iii) – (v) of Theorem 2.1 ([LZ, 2.3], starting on p.311)

Talk 6. Decompleting Towers. This is a preparation for the next talk. Try to explain some reasonable portion of [KL, §5], in particular Theorem 5.7.4; see also Definition 4.4.2, and §6.1 for the theorem of Cherbonnier-Colmez as a classical example.

Talk 7. Proof of Proposition 2.8. [LZ, 2.4], and [KL, §7] as necessary.

Talk 8. Geometric Riemann-Hilbert correspondence [LZ, 3.1]

Talk 9. Arithmetic Riemann-Hilbert correspondence [LZ, 3.2]

Talk 10. Application I: Rigidity of geometric p-adic representations [LZ, 4.1]. Deduce first the statement of Theorem 1.3 from the arithmetic Riemann-Hilbert correspondence, also it would be nice to explain Remark 1.4 and 1.5.

Talk 11. Application to Shimura varieties [LZ, 4.2].

References

[BMS] B. Bhatt, M. Morrow, P. Scholze: Integral p-adic Hodge theory. arXiv:1602.03148

[KL] K. Kedlaya, R. Liu: Relative p-adic Hodge theory, II: Imperfect period rings, arXiv:1602.06899

[LZ] R. Liu, X. Zhu: Rigidity and a Riemann-Hilbert correspondence for p-adic local systems. Inv. Math. **207** (2017), 291–343.

[Sch1] P. Scholze: Perfectoid spaces. Publ. Math. IHES **116** (2012), 245–313

[Sch2] P. Scholze: p-Adic Hodge theory for rigid-analytic varieties. Forum of Mathematics, Pi (2013), Vol. 1, e1, 77 pages

[Sch3] P. Scholze: p-adic geometry. arXiv: 1712.03708