Modular Forms on Higher Rank Groups

September 17-20, 2019

Organizers **Jan Hendrik Bruinier** **Yingkun Li** **Michalis Neururer**

Graphic on the frontpage by Prof. Dr. Karl H. Hofmann

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Acknowledgements

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S.G.A.

DFG-Forschergruppe

LOEWE

Exzellente Forschung für Hessens Zukunft

Fachbereich
Mathematik

1 Program

	Tuesday	Wednesday	Thursday	Friday
9:30-10:20	Registration S103|123	Furusawa	Pitale	Millson
10:30-11:00		Coffee break	Coffee break	Coffee break
11:00-11:50	Skoruppa	Marzec	Saha	Pollack
12:00-14:00	Lunch break	Lunch break	Lunch break	Lunch break
14:00-14:50	Böcherer	Bouganis	Zemel	Funke
15:00-15:30	Coffee break	Coffee break	Coffee break	
15:30-16:20	Das	Taïbi	Yuen	
		Guided tour of Mathildenhöhe 16:45-18:45	Conference dinner 19:00	

The talks take place in **S103|123** and coffee breaks are in **S103|116**.

Tuesday, Sept. 17, 2019		
Time	Speaker	*Title of Talk*
09:30-11:00		–Registration–
11:00-11:50	Skoruppa	The Macdonald identities and Jacobi forms of lattice index
12:00-14:00		–Lunch Break–
14:00-14:50	Böcherer	On paramodular forms of arbitrary degree
15:00-15:30		–Coffee Break–
15:30-16:20	Das	Fundamental Fourier coefficients of Siegel modular forms

Wednesday, Sept. 18, 2019		
Time	Speaker	*Title of Talk*
09:30-10:20	Furusawa	On Böcherer's conjecture
10:30-11:00		–Coffee Break–
11:00-11:50	Marzec	Some evidence towards the Resnikoff-Saldana conjecture
12:00-14:00		–Lunch Break–
14:00-14:50	Bouganis	Quaternionic modular forms and the Rankin-Selberg method
15:00-15:30		–Coffee Break–
15:30-16:20	Taïbi	Discrete series multiplicities for classical groups over Z and level 1 algebraic cusp forms

Thursday, Sept. 19, 2019		
Time	Speaker	*Title of Talk*
09:30-10:20	Pitale	Critical L-values and congruences for Siegel modular forms, I
10:30-11:00		–Coffee Break–
11:00-11:50	Saha	Critical L-values and congruences for Siegel modular forms, II
12:00-14:00		–Lunch Break–
14:00-14:50	Zemel	Heegner divisors on toroidal compactifications of orthogonal Shimura varieties
15:00-15:30		–Coffee Break–
15:30-16:20	Yuen	Finding all paramodular Borcherds products and applications

Friday, Sept. 20, 2019		
Time	Speaker	*Title of Talk*
09:30-10:20	Millson	Holomorphic forms on quotients of hermitian locally symmetric spaces and generalized special cycles
10:30-11:00		–Coffee Break–
11:00-11:50	Pollack	Modular forms on exceptional groups
12:00-14:00		–Lunch Break–
14:00-14:50	Funke	The construction of Green currents and singular theta lifts for unitary groups

2 Abstracts

Siegfried Böcherer

On paramodular forms of arbitrary degree

Universität Mannheim

We consider paramodular forms of arbitray degree for square-free polarization matrices. They have many properties in common with classical Siegel modular forms of level one. We discuss in particular Siegel Eisenstein series, explicit pullback formulas and the relevant double coset decompositions and Hecke algebras, Siegel's main theorem and the basis problem.

This is joint work with Rainer Schulze-Pillot.

Thanasis Bouganis

Quaternionic modular forms and the Rankin-Selberg method

University of Durham

The properties (analytic, algebraic or p-adic) of special values of the standard L-function attached to Siegel and Hermitian modular forms are of central interest and have been extensively studied. In this talk, we will discuss another family of modular forms, which are associated to the isometry group of a quaternionic skew hermitian form. There are many similarities to the Siegel and Hermitian case but also important differences. We will present some results on the study of their standard L-function using the Rankin-Selberg method. This will lead us to discuss the existence of some theta series, a problem of which, in turn, is related to Howe duality and invariant theory.

Soumya Das

Fundamental Fourier coefficients of Siegel modular forms

Indian Institute of Science, Bangalore

We prove that if F is a non-zero (possibly non-cuspidal) vector-valued level one Siegel modular form of any degree, then it has infinitely many non-zero Fourier coefficients which are indexed by half-integral matrices having odd, square-free (and thus fundamental) discriminant. The proof uses an induction argument in the

setting of vector-valued modular forms. As an application of a variant of our result and building upon the work of A. Pollack, this implies an unconditional proof of the standard analytic properties of the spinor L-function of a holomorphic cuspidal Siegel Hecke eigenform of degree 3 and level one.

Jens Funke

The construction of Green currents and singular theta lifts for unitary groups

University of Durham

In this talk we describe how singular theta lifts for the reductive dual pair $U(p,q) \times U(1,1)$ can be employed to construct two different kinds of Green forms for codimension q-cycles in Shimura varieties associated to unitary groups. In particular, we establish an adjointness result between the singular theta lift and the Kudla-Millson theta lift and discuss further applications in the context of the Kudla Program.

This is joint work with Eric Hofmann.

Masaaki Furusawa

On Böcherer's conjecture

Osaka City University

In the 1980's, Böcherer proclaimed a remarkable conjecture concerning the relationship between a certain finite sum of Fourier coefficients of Siegel cusp forms of degree two which is a Hecke eigenform and a central value of a quadratic twist of its spinor L-function. In this talk, we would like to discuss a proof of the conjecture and some related results.

This is a joint work with Kazuki Morimoto.

Jolanta Marzec

Some evidence towards Resnikoff-Saldana conjecture

TU Darmstadt

The Resnikoff-Saldana conjecture proposes a bound for Fourier coefficients of Siegel modular forms of any degree, generalizing the classical Ramanujan-Petersson conjecture. In the talk we consider the case of degree 2. We show that the conjecture holds for many (to be specified) Fourier coefficients of Siegel modular forms which are not generalized Saito-Kurokawa lifts, as long as it holds for

the ones that are fundamental. To do this we employ relations between Fourier coefficients, local Bessel periods and Satake parameters, ultimately translating a result of Weissauer on the generalized Ramanujan-Petersson conjecture to a bound for Fourier coefficients.

John Millson

Holomorphic forms on quotients of hermitian locally symmetric spaces and generalized special cycles

University of Maryland

I will speak about the 2019 University of Maryland PhD thesis written by my student Yousheng Shi entitled "Locally symmetric spaces and the cohomology of the Weil representation".

This talk, the PhD thesis of Yousheng Shi and the 1980 PhD thesis of Greg Anderson (see below) are concerned with three of the four infinite families of bounded symmetric domains $D = G/K$, those of types $\mathrm{I}_{pq}, \mathrm{II}_n, \mathrm{III}_n$. The corresponding three families of isometry groups of these domains are $G = \mathrm{U}(p,q)$, resp. $\mathrm{Sp}(2n,\mathbb{R})$, resp. $\mathrm{O}^*(2n)$. In 1980, R. Parthasarathy proved that the degrees d for which a nonzero holomorphic form could exist on a compact quotient $M = \Gamma\backslash D$ of one of these domains were severely limited. For $\mathrm{U}(p,q)$ the possible degrees are $d = pq - rs, 0 \le r \le p, 0 \le s \le q$; for $\mathrm{Sp}(2n,\mathbb{R})$ they are $d = \frac{n(n+1)}{2} - \frac{r(r+1)}{2}, 0 \le r \le n$ and for $\mathrm{O}^*(2n)$ they are $d = \frac{n(n-1)}{2} - \frac{r(r-1)}{2}, 0 \le r \le n$. In his Princeton PhD thesis, written in 1980, Greg Anderson constructed nonzero holomorphic forms in the degrees d allowed by Parthasarathy's results. To do this, he constructed holomorphic relative Lie algebra $\mathfrak{g}, K$-cocycles φ of Hodge type $(d,0)$ with values in the Fock model $\mathscr{F}$ of the oscillator/Weil representation. He then applied the theta distribution $\Theta : \mathscr{F} \to \mathbb{C}$ to the values of these cocycles considered as $\mathscr{F}$-valued G-invariant holomorphic differential forms on D to obtain closed nonzero holomorphic d-forms $\Theta \circ \varphi$ on the quotient M.

One of the main points of Shi's thesis is to explain where the degrees d in Parthasarathy's theorem come from. In fact, the above degrees coincide with the *complex* codimensions of natural totally geodesic algebraic cycles in M (for the appropriate Γ) , to be called *generalized* special cycles, associated to families of subgroups H of the groups G above of the same type as G. For $\mathrm{U}(p,q)$ the corresponding subgroups H are given by $H = \mathrm{U}(r,s)$; for $\mathrm{Sp}(2n,\mathbb{R})$ they are $H = \mathrm{Sp}(2r,\mathbb{R})$ and for $\mathrm{O}^*(2n)$ they are $H = \mathrm{O}^*(2r)$. The explanation of the coincidence in the two sets of numbers goes through Anderson's cocycles φ above. To connect with the generalized special cycles it is necessary to "double φ", that is form the wedge product $\Phi = \varphi \wedge \overline{\varphi}$ with values in $\mathscr{F} \otimes \overline{\mathscr{F}}$ which is a closed

$2d$-form of Hodge type (d,d). In his thesis, Shi proved that the form on M obtained by applying the sum of delta functions associated to the Γ-orbit of a lattice vector determining the cycle to the values of the image of Φ in the Schrödinger model gave rise to a closed form representing the Poincaré dual of the generalized special cycle. For the case of $\mathrm{U}(p,q)$ and $s = q$ one obtains the *special cycles* and their Poincaré dual forms of Kudla-Millson. Thus Shi's results can be regarded as a partial generalization of the work of Kudla-Millson.

By restricting the above cocycles to subgroups of G as above, e.g. $\mathrm{O}(p,q)$ (already done in the work of Kudla-Millson for the case s=q), $\mathrm{Sp}(2n,\mathbb{C}) \subset \mathrm{Sp}(4n,\mathbb{R})$ and $\mathrm{O}(n,\mathbb{C}) \subset \mathrm{O}(n,n) \subset \mathrm{U}(n,n)$ considerable further generalization of the work of Kudla-Millson appears to be possible.

Ameya Pitale

Critical L-values and congruences for Siegel modular forms, I

University of Oklahoma

Deligne's conjecture predicts that the special values of L-functions of automorphic forms at critical points are algebraic up to certain prescribed transcendental factors. There is a lot of literature on progress towards this conjecture for various classes of automorphic forms. In this talk, we will present new results for the degree 5 standard L-functions of genus 2 holomorphic Siegel modular forms. The main step is to obtain an integral representation of the L-function, which we are able to do for general genus n. Using results of Ramakrishnan and Shahidi, the genus 2 special value result allows us to obtain the algebraicity of the symmetric fourth L-function of an elliptic cusp form twisted by characters.

This is joint work with Abhishek Saha and Ralf Schmidt.

Aaron Pollack

Modular forms on exceptional groups

Duke University

By a "modular form" for a reductive group G we mean an automorphic form that has some sort of very nice Fourier expansion. Building upon work of Gan, Gross, Savin, and Wallach, I will explain how there is a notion of modular forms on certain real forms of the exceptional groups. Some examples of these modular forms include the minimal representation on $E_{8,4}$, singular and distinguished automorphic forms on $E_{7,4}$ and $E_{6,4}$, and cusp forms on G_2 all of whose Fourier coefficients are integers.

Abhishek Saha
Critical L-values and congruences for Siegel modular forms, II
Queen Mary University of London

I will describe some very recent joint work with Ameya Pitale and Ralf Schmidt where we prove a general result on congruences between Hecke eigenvalues of Siegel cusp forms of degree 2 modulo primes dividing a certain quotient of L-values. The starting point of our result is the explicit pullback formula that gives an integral representation for the twisted standard L-function for a holomorphic vector-valued Siegel cusp form of degree n and arbitrary level, described in Ameya Pitale's talk that precedes mine. In this talk, I will sketch a proof that the pullback of the nearly holomorphic Eisenstein series that appears in this integral representation is actually cuspidal in each variable and has nice arithmetic properties. This directly leads to the first version of the congruence result. I will then describe a second, more refined version of our congruence theorem, that is obtained by looking at Arthur packets and the refined Gan-Gross-Prasad conjecture in this particular setup.

Nils-Peter Skoruppa
The Macdonald identities and Jacobi forms of lattice index
Universität Siegen

We propose a new and short proof for the Macdonald identities using only some easy facts from the theory of Jacobi forms of lattice index and classical root systems. (The needed basic features of these theories will be explained in the talk). We discuss applications and open questions related to the new proof, and we end the talk by deducing from the Macdonald identities, for 42 elliptic curves over the rationals and of rank 1 product identities for the (classical) Jacobi forms attached to elliptic curves by the theory of modular forms.

Olivier Taïbi
Discrete series multiplicities for classical groups over Z and level 1 algebraic cusp forms
ENS Lyon

I will explain a new method for evaluating the multiplicity of a given discrete series in the space of level 1 automorphic forms of a split classical group G over Z, and classification results for level one cuspidal algebraic automorphic representations for general linear groups (of arbitrary dimension) over Q in motivic weight ≤ 24.

A (non-trivial) application is the determination of all level 1 scalar-valued cuspidal Siegel modular forms in weight ≤ 13 and arbitrary genus.

This is joint work with Gaëtan Chenevier.

David Yuen

Finding all paramodular Borcherds products and applications

University of Hawaii

Borcherds products are a useful tool for studying spaces of paramodular forms. We present an algorithm to compute all paramodular cusp forms that are Borcherds products of a given weight and level. We give applications to rigorous computations, including the modularity of abelian surfaces.

Shaul Zemel

Heegner divisors on toroidal compactifications of orthogonal Shimura varieties

Hebrew University of Jerusalem

A well-known result of Borcherds yields the modularity of Heegner divisors on complex orthogonal Shimura varieties (i.e. Grassmannian quotients). These varieties are typically non-compact, and one way of completing them to compact varieties is via toroidal compactifications. However, the boundary components there also contain divisors. We show how to extend the Heegner divisors to such compactifications in a canonical way such that the modularity result of Borcherds still holds.

This is joint work with Jan Bruinier.

3 General Information

3.1 Lecture Hall

Location: Technische Universität Darmstadt. The talks will take place in the “Altes Hauptgebäude” S103, lecture hall 123. The address of the building is Hochschulstraße 1. The coffee breaks take place in the same building, in room 116.

3.2 Food & Beverage

The university cafeteria “Mensa” offers a good variety of cheap meals for lunch, building S1|11 (next to the Welcome Hotel), Monday to Friday 11:15 to 14:00. Additionally, the bistro at the university library is open all day from 08:00 to 22:00, building S1|20 ULB. Furthermore there are lots of good restaurants and bistros near TU Darmstadt. Please dial 0049 6151 preceding the number given below.

Name	Address	Phone	Cuisine
3klang	Riegerplatz 3	6698843	International
Adega Alentejana	Heinheimer Str. 38	971796	Portuguese
Ban Thai	Hügelstraße 21	7379545	Thai
Cafe Chaos	Muehlstraße 36	20635	European
Elisabeth	Schulstraße 14	2787858	European
Haroun‘s	Friedensplatz 6	23487	Oriental
Ichido	Holzstraße 2	8004878	Japanese
Meet and Eat	Mathildenplatz 4	4927451	Chinese
La Bodega	Kahlertstraße 34	291674	Spanish
Linzer Stube	Kiesstraße 32	291674	Austrian
Pizzeria da Nino	Alexanderstr. 29	24220	Italian
Ratskeller	Marktplatz 8	26444	German
Restaurant Sitte	Karlstr. 15	22222	German
Ristorante Sardegna	Kahlertstr. 1	23029	Italian
Shiraz	Dieburger Straße 73	6011640	Persian
Wellnitz	Lauteschlaegerstraße 4	6699255	Café

3.3 Guided tour of Mathildenhöhe

We will organize a guided tour of Mathildenhöhe on Wednesday at 16:45-18:45. The meeting point for the start of the tour is Kantplatz, a square next to S1|03. For a cost of 2 euros, participants get the opportunity to go to the top of Hochzeitsturm.

3.4 Conference Dinner

The conference dinner is scheduled for Thursday, September 19th at 19:00. The venue for this dinner is the restaurant Sardegna, Kahlertstraße 1, 64293 Darmstadt, which is in a 10 minutes walking distance from the lecture hall and the mathematics department.

3.5 Contact Information

If there are any questions concerning the workshop, please feel free to contact our secretaries:

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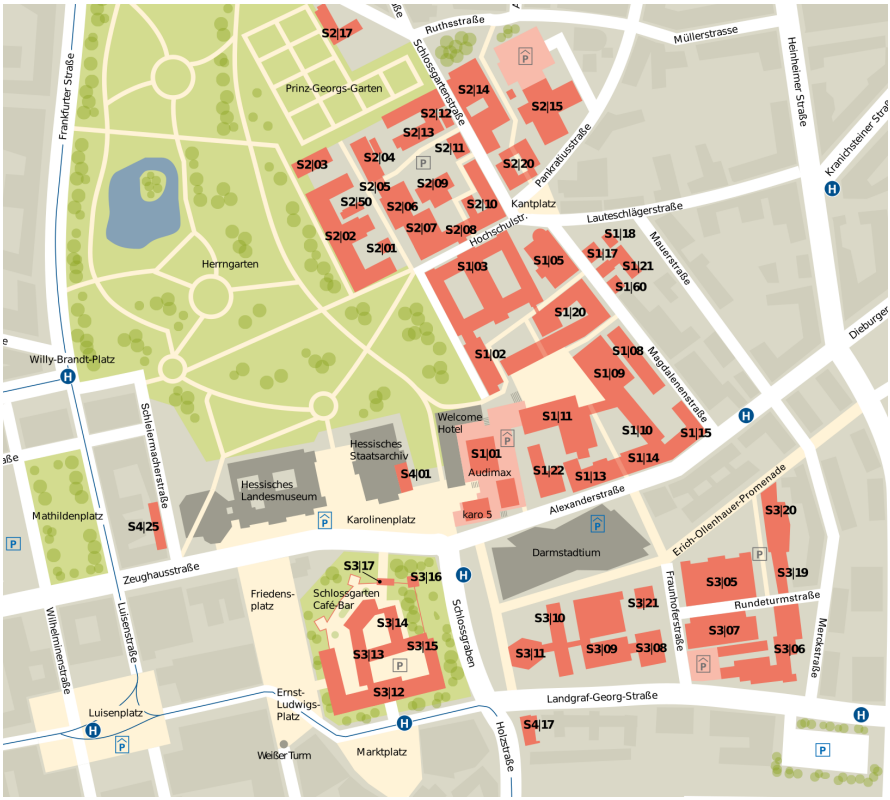

Figure 4.1: Map of the TU Darmstadt campus in the city centre