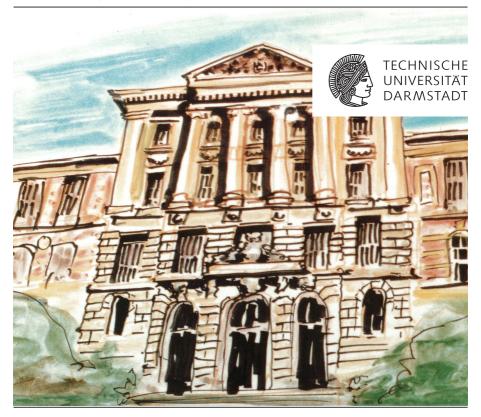# Conference on Modular Forms and Arithmetic Geometry

September 18-22, 2023, TU Darmstadt

Organizers:
Jan Hendrik Bruinier (TU Darmstadt)
Ben Kane (University of Hong Kong)
Steve Kudla (University of Toronto)
Yingkun Li (TU Darmstadt)

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Acknowledgements

Celebrating the 60th birthday of Tonghai Yang

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11:00-11:30		–Coffee Break–
11:30-12:30	Alfes-Neumann	On harmonic weak Maass forms associated to even integer weight newforms
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14:30-15:30	Bringmann	Non-modular forms and their completions
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16:00-17:00	Sankaran	Derivatives of weight one Eisenstein series and the arithmetic Siegel-Weil formula

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11:00-12:00	Mihatsch	0-Cycles on twisted unitary Shimura varieties
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15:15-16:15	C. Li	Kudla-Rapoport conjecture for Krämer models
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17:00-18:00	Lauter	Curves, Cryptography, and Intersection Theory, a long history of collaboration and inspiration with Tonghai

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Thursday, Sept. 21, 2023

Time	Speaker	Title of Talk
09:30-10:30	Zhang	Generic modularity for Shtuka special cycles
10:30-11:00		–Coffee Break–
11:00-12:00	Hu	The subconvexity problem for special values of L-functions
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15:00-16:00	Gan	Triality and Spin lifting for $\mathsf{PGSp}(6)$
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Time	Speaker	Title of Talk
09:00-10:00	Funke	Jacobi-Weierstrass mock modular forms
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10:30-11:30	Rosu	Twists of elliptic curves and their L-functions
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2 Abstracts

Claudia Alfes-Neumann
On harmonic weak Maass forms associated to even integer weight newforms
Bielefeld University, Germany

Abstract: In this talk we review several types of harmonic weak Maass forms that are related to integral even weight newforms. We can relate integer weight harmonic weak Maass forms to these newforms via the so-called xi-operator. I will review three different constructions of such harmonic weak Maass forms via the Weierstrass zeta function and suitable generalizations depending on the weight and the field of coefficients of the corresponding newform. A second construction via theta liftings gives a half-integral weight harmonic weak Maass form whose coefficients are given by periods of certain meromorphic modular forms with algebraic coefficients and periods of the integer even weight newform. This is joint work with Jens Funke, Michael Mertens, and Eugenia Rosu resp. Jan Bruinier and Markus Schwagenscheidt.

Kathrin Bringmann
Non-modular forms and their completions
University of Cologne, Germany

Abstract: In this talk I describe ways how to make functions which are close to being modular forms actually modular. In particular I study mock modular forms, false theta functions and related objects.

Jens Funke
Jacobi-Weierstrass mock modular forms
Durham University, UK

Abstract: The explicit construction of harmonic weak Maass forms associated to holomorphic cusp forms represents one of the central questions in the theory of mock modular forms. For weight 2 newforms associated to rational elliptic curves Alfes, Griffin, Ono, and Rolen gave a construction employing the classical theory of elliptic functions. In this talk we present a construction of such forms for general even weight via a vector-valued generalization of the Weierstrass zeta-function. This is joint work with C. Alfes, M. Mertens, and E. Rosu.

Wee Teck Gan
Triality and Spin lifting for PGSp(6)
National University of Singapore, Singapore

Abstract: In a joint work with Gaetan Chenevier, we exploit theta correspondence and the triality automorphism of PGSO(8) to produce the Spin lifting from PGSp(6) to GL(8). Locally, a similar idea was used in joint work with Gordan Savin to produce a weak local Langlands correspondence for PGSp(6). As a result, we establish some expected analytic properties of the Spin L-function of PGSp(6).

Eyal Goren
Super special stories
McGill University, Canada

Abstract: After introduction to supersingular graphs and their applications, I will discuss three new theorems concerning supersingular isogeny graphs (these are graphs having supersingular j invariants in characteristic p as vertices and r - isogenies as edges, where r is a prime different from p). One is due to Sam Mayo, the other two are joint work with Jonathan Love. The extension of such results to super special graphs associated to super special abelian varieties with real multiplication is work in progress.

Benedict Gross
The elliptic curve $A(p)$
Harvard and UCSD, USA

Abstract: After defining the curve $A(p)$, with complex multiplication by the ring of integers of $\mathbb{Q}(\sqrt{-p})$, I will discuss some analytic results on its L-function, and their arithmetic implications.

Yueke Hu
The subconvexity problem for special values of L-functions
Tsinghua University, China

Abstract: The subconvexity bound for the special values of L-functions is a step toward understanding L-functions on the critical line. It was fully solved for

automorphic forms on GL(1) and GL(2). Results on higher rank groups on the other hand have been sporadic so far. In this talk we report some recent progress on the subconvexity problem in this direction.

Özlem Imamoglu
TBA
ETH, Switzerland

Kristin Lauter
Curves, Cryptography, and Intersection Theory, a long history of
collaboration and inspiration with Tonghai
Meta, USA

Chao Li
Kudla-Rapoport conjecture for Krämer models
Columbia University, USA

Abstract: The Kudla-Rapoport conjecture, proved jointly with Wei Zhang, is a precise identity relating arithmetic intersection numbers of special cycles on unitary Shimura varieties with good reduction and central derivatives of Siegel Eisenstein series. We discuss how to formulate and prove an analogous identity for certain unitary Shimura varieties with bad reduction (Krämer models at ramified places). We will motivate these conjectures, mention applications to L-functions and highlight interesting new phenomena in the presence of bad reduction. This is joint work with Qiao He, Yousheng Shi and Tonghai Yang.

Wanlin Li
Basic reductions of abelian varieties I
Washington University in St. Louis, USA

Abstract: Elkies proved that an elliptic curve over $\mathbb{Q}$ has infinitely many supersingular reductions. The generalization of the 0-dimensional supersingular locus of the modular curve is the so called basic locus of a Shimura curve at a good prime. In this talk, we generalize Elkies's theorem to some abelian varieties over totally real fields parametrized by certain unitary Shimura curves; these Shimura

curves arise from the moduli spaces of cyclic covers of the projective line ramified at 4 points. This is part I of the two talks, whose goal is to explain the main ingredients of the proof. This is our joint work (in progress) with Elena Mantovan and Rachel Pries.

Andreas Mihatsch
0-Cycles on twisted unitary Shimura varieties
University of Bonn, Germany

Abstract: The (unitary) arithmetic Siegel–Weil formula relates the arithmetic degrees of special 0-cycles on integral models of unitary Shimura varieties to Fourier coefficients of the first derivatives of Eisenstein series. In my talk, I will discuss a possible extension of this formula to unitary groups over division algebras. Due to the lack of special divisors in this setting, we define our 0-cycles through RZ uniformization. Following the general framework of Madapusi, we assemble them into a q-series for a unitary group over the division algebra in question. After taking degrees, we expect this series to coincide with the first derivative of an Eisenstein series. This is joint work in progress with Tonghai Yang.

Ken Ono
Variants of Lehmer's Conjecture for Ramanujan's Tau-function
University of Virginia, USA

Abstract: Modular forms are generating functions of important quantities in arithmetic geometry, combinatorics, number theory, and physics. Despite many deep developments in the arit hmetic geometric and analytic aspects (e.g. Deligne's proof of the Weil Conjectures, the development of Galois representations, Birch and Swinnerton-Dyer Conjecture, to name a few), some of the seminal questions about them remain open. Perhaps the most prominent of these is Lehmer's Conjecture on the nonvanishing of modular form coefficients such as Ramanujan's tau-function. In joint work with J. Balakrishnan, W. Craig, and W.-L. Tsai, the speaker has obtained the first results that establish that many integers are never modular form coefficients.

Michael Rapoport
On the quasi-canonical Arithmetic Fundamental Lemma
University of Bonn, Germany

Abstract: The Arithmetic Fundamental Lemma (conjectured by W. Zhang and proved by W. Zhang, W.Zhang-A.Mihatsch and Z. Zhang) gives an analytic expression for the intersection number of a special cycle of valuation zero on a RZ-space with a translate of it. I will discuss the quasi-canonical version which concerns special cycles of valuation one. Joint work with C. Li and W. Zhang.

Eugenia Rosu
Twists of elliptic curves and their L-functions
Leiden University, Netherlands

Abstract: We will first review work on quadratic twists of elliptic curves and explicit formulas for the central values of their L-functions and their applications. We further consider cubic twists of the elliptic curve $y^2 = x^3 + 1$ and compute formulas that relate the central value of their L-functions $L(E, 1)$ to the square of a trace of a modular function evaluated at a CM point. When the value above is non-zero, we recover the order of the Tate-Shafarevich group, and we show that the value is indeed an integer square.

Siddarth Sankaran
Derivatives of weight one Eisenstein series and the arithmetic Siegel-Weil formula
University of Manitoba, Canada

In a classic paper by Kudla, Rapoport and Yang (IMRN 1999) the authors express the Fourier coefficients of the derivative of a certain Eisenstein series in terms of arithmetic divisors on the moduli space of CM elliptic curves. This result is the simplest case of Kudla's conjectural "arithmetic Siegel-Weil formula". In this talk, we will revisit aspects of this paper from a more representation-theoretic viewpoint. In particular, we will see how the usual Siegel-Weil formula, combined with some Hecke algebra considerations, can be applied in this setting; with the benefit of hindsight, this approach may offer a more conceptual reason for the appearance of the derivative of the Eisenstein series. Time permitting, we will also discuss generalizations.

Yunqing Tang
Basic reductions of abelian varieties II
UC Berkeley, USA

Abstract: Elkies proved that an elliptic curve over $\mathbb{Q}$ has infinitely many supersingular reductions. The generalization of the 0-dimensional supersingular locus of the modular curve is the so called basic locus of a Shimura curve at a good prime. In this talk, we generalize Elkies's theorem to some abelian varieties over totally real fields parametrized by certain unitary Shimura curves; these Shimura curves arise from the moduli spaces of cyclic covers of the projective line ramified at 4 points. This is part II of the two talks, whose goal is to explain the main ingredients of the proof. This is our joint work (in progress) with Elena Mantovan and Rachel Pries.

Wei Zhang
Generic modularity for Shtuka special cycles
MIT, USA

Abstract: Join work with Tony Feng and Zhiwei Yun. Previously we formulated a generating series of special cycles on the moduli stack of unitary Shtukas and conjecture its modularity. In this talk I will report a proof of the modularity upon taking the cycle classes on the generic fibers of the moduli spaces.

3 General Information

3.1 Lecture Hall

Location: Technische Universität Darmstadt. The lectures are taking place in building S2|08 Lecture Hall 171, which is situated on Hochschulstrasse 4, 64289 Darmstadt.

Exception: On Thursday (Sept. 21) the morning session will take place in building S2|14 room 24, which is situated on Schlossgartenstrasse 9, 64289 Darmstadt.

3.2 Food & Beverage

The university cafeteria "Mensa" offers a variety of cheap meals for lunch, building S1|11 (next to the Welcome Hotel), Monday to Friday 11:15 to 14:15. Additionally, the bistro LesBar at the university library is open from 12:30 to 20:00, building S1|20 ULB. Furthermore there are lots of good restaurants and bistros near TU Darmstadt, many of them offer additionally vegetarian meals or vegan meals. Please dial 0049 6151 preceding the number given below.

Name	Address	Phone	Cuisine
3klang	Riegerplatz 3	6698843	International
Adega Alentejana	Heinheimer Str. 38	971796	Portuguese
Haroun's	Friedensplatz 6	23487	Arabic
Havana Bar	Lauteschlägerstr. 42	710459	Cuban
Da Nino	Alexanderstr. 29	24220	Italian
Djadoo	Viktoriaplatz 12	1016310	Oriental
Jins Haus	Elisabethenstraße 41	6066136	Asian
La Bodega	Kahlertstr. 34	291674	Spanish
Long Men Zhen	Wilhelminenstraße 9		Chinese
Manjing Hau	Adelungstraße 13	6602720	Asian
Mondo Daily Bistro	Grafenstr. 31	9699161	Oriental
Pizzeria Mono	Riegerplatz 1	7874994	Italian
Ratskeller	Marktplatz 8	26444	German
Shiraz	Dieburger Str. 73	6011640	Oriental
Sitte	Karlstr. 15	22222	German
Tajinerie	Dieburger Str. 70	711981	Moroccan
Zen Noodle	Mathildenpl. 4	9506435	Chinese

3.3 WiFi

Eduroam is available on the campus of the university. If you need help with access, please let us know during registration.

3.4 Guided Tour

On Wednesday afternoon from 16:00 to 17:30, we will go on a guided excursion to the Matildenhöhe. The starting point is Info-Point Mathildenhöhe (Olbrichweg 10). We will meet in front of the lecture hall building (S2|08) and leave there at 15:45.

3.5 Conference Dinner

The conference dinner is scheduled for Wednesday, September 20th at 19:00. The venue for this dinner is the restaurant Sardegna, Kahlertstrasse 1, 64293 Darmstadt, which is in a 10 minutes walking distance from the lecture hall and the mathematics department.

3.6 Contact Information

If there are any questions concerning the workshop, please feel free to contact our secretaries:

- Anja Spangenberg
  Office: S2|15, 4th floor, Room 428
  Phone: +49 (0) 6151 - 16 22466

- Monika Kammer
  Office: S2|15, 4th floor, Room K414
  Phone: +49 (0) 6151 - 16 22460

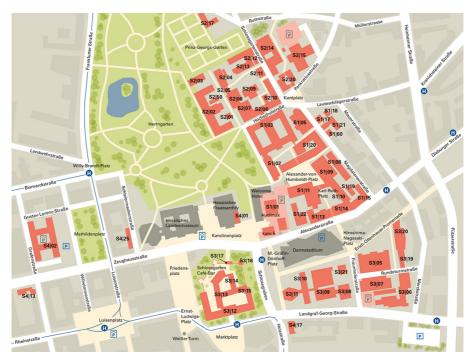- Email: algebra@mathematik.tu-darmstadt.de

Figure 3.1: Map of the TU Darmstadt campus in the city center

For an updated version of this booklet, please check the conference website
    https://www.mathematik.tu-darmstadt.de/algebra/forschung_algebra/
konferenzen_und_workshops_ag_algebra/mod_forms.en.jsp
or scan the following QR code