Intersection Theory

7. Exercise sheet

Exercise 1:

Let X be an algebraic scheme and L a line bundle on X. Denote by ν the projection from $L - \{0\}$, the complement of zero section, to X. Prove the following: For all $k \geq 0$, the sequence

$$\mathrm{CH}_{k+1} X \xrightarrow{c_1(L)\cap -} \mathrm{CH}_k X \xrightarrow{\nu^*} \mathrm{CH}_{k+1}(L - \{0\}) \to 0$$

is exact.

Exercise 2:

Let $i : X \to \mathbb{P}^m$ a closed embedding of codimension d. Furthermore, assume that X is smooth and that it is an intersection of divisors $D_1, \cdots, D_d$. Derive an explicit formula for the first Chern class of the tangent bundle of X, $c_1(T_X)$, in terms of the first Chern class of the canonical bundle and the degrees of the divisors D_i.

Hint: Use the exact sequence

$$0 \to T_X \to T_{\mathbb{P}^m} \to N_{X/\mathbb{P}^m} \to 0$$

where $N_{X/\mathbb{P}^m}$ *denotes the normal bundle.*