Workshop
Arithmetic of Eisenstein series
September 22-25, 2014
TU Darmstadt

Organizers **Jan Hendrik Bruinier** **Anna von Pippich**

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Acknowledgements

This workshop is organized within the program of the *DFG-Forschergruppe 1920 "Symmetrie, Geometrie und Arithmetik"*. We gratefully acknowledge financial support by the DFG German Research Foundation and the Department of Mathematics at Technische Universität Darmstadt.

1 Programme

Schedule	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
09.00 – 09.30	Registration			
09.30 – 10.00		**Andreatta**	**Howard**	**Jorgenson**
10.00 – 10.30	**Bertolini**			
10.30 – 11.00		Coffee break	Coffee break	Coffee break
11.00 – 11.30	Coffee break	**Goren**	**Bouganis**	**Funke**
11.30 – 12.00	**Charollois**			
12.00 – 12.30				
12.30 – 14.30	Lunch break	Lunch break	Lunch break	Lunch break
14.30 – 15.30	**Böcherer**	**Freixas**	**Yang**	**Kramer**
15.30 – 16.00	Coffee break	Coffee break	Coffee break	Coffee break
16.00 – 17.00	**Kohnen**	**Sankaran**	**Viazovska**	Informal discussions
18.30 –			Conference dinner	Departure

Monday, Sept. 22, 2014		
Time	Speaker	*Title of Talk*
09:00-10:00		–Registration–
10:00-11:00	Bertolini	*Beilinson–Flach elements and the Birch and Swinnerton-Dyer conjecture*
11:00-11:30		–Coffee Break–
11:30-12:30	Charollois	*Explicit integral cocycles on* GL_n *and special values of p-adic partial zeta functions*
12:30-14:30		–Lunch Break–
14:30-15:30	Böcherer	*On noncuspidal Siegel modular forms of low weight*
15:30-16:00		–Coffee Break–
16:00-17:00	Kohnen	*Two applications of holomorphic Eisenstein series*

Tuesday, Sept. 23, 2014		
Time	Speaker	*Title of Talk*
09:30-10:30	Andreatta	*Orthogonal and CSpin Shimura varieties*
10:30-11:00		–Coffee Break–
11:00-12:00	Goren	*The Bruinier–Yang conjecture*
12:00-14:30		–Lunch Break–
14:30-15:30	Freixas	*On the Riemann–Roch formula in Arakelov geometry and an exotic analytic class number formula*
15:30-16:00		–Coffee Break–
16:00-17:00	Sankaran	*Special cycles on unitary Shimura varieties and Eisenstein series*

Wednesday, Sept. 24, 2014		
Time	Speaker	*Title of Talk*
09:30-10:30	Howard	*Cycles on Shimura varieties and applications to Faltings heights*
10:30-11:00		–Coffee Break–
11:00-12:00	Bouganis	*On special L-values attached to half-integral weight Siegel modular forms*
12:00-14:30		–Lunch Break–
14:30-15:30	Yang	*Coherent and incoherent Eisenstein series*
15:30-16:00		–Coffee Break–
16:00-17:00	Viazovska	*Siegel Eisenstein series and Heegner cycles*

Thursday, Sept. 25, 2014		
Time	Speaker	*Title of Talk*
09:30-10:30	Jorgenson	*Kronecker's limit formula, holomorphic modular functions, and q-expansions on certain moonshine groups*
10:30-11:00		–Coffee Break–
11:00-12:00	Funke	*Cycles in degenerate Hilbert modular surfaces and modular forms*
12:00-14:30		–Lunch Break–
14:30-15:30	Kramer	*Uniform sup-norm bounds on average for cusp forms of higher weights*
15:30-16:00		–Coffee Break–

2 Abstracts

Fabrizio Andreatta
Orthogonal and CSpin Shimura varieties
Università Statale di Milano, Italy

Abstract: Let (V,Q) be a quadratic space over Q of signature $(2,n)$ and let $L \subset V$ be a perfect lattice. I will define the Shimura varieties associated to the algebraic groups $SO(V,Q)$ and $CSpin(V,Q)$ and the lattice L. In the case of $CSpin(V,Q)$ it is a Shimura variety of Hodge type. I will describe its integral canonical model and its special fibers in positive characteristics. Such varieties have a rich supply of arithmetic cycles that I will define.

Massimo Bertolini
Beilinson–Flach elements and the Birch and Swinnerton-Dyer conjecture
Universität Duisburg-Essen, Germany

Abstract: We report on the proof of the equivariant Birch and Swinnerton-Dyer conjecture in analytic rank zero, for the L-series of an elliptic curve twisted by an odd, irreducible, 2-dimensional Artin representation. This proof is based on the so-called Beilinson–Flach elements attached to p-adic families of Eisenstein series. This is joint work with Henri Darmon and Victor Rotger.

Siegfried Böcherer
On noncuspidal Siegel modular forms of low weight
Universität Mannheim, Germany

Abstract: For large weights, the structure of noncuspidal Siegel modular forms is determined by the properties of Klingen Eisenstein series attached to cusp forms. For low weights, the situation is more complicated, in particular for congruence subgroups. The case of weight 2 deserves special attention because of its connection with quaternary theta series (degree one is due to Hecke). I will focus on degree 2 and a precise dimension formula in the case of squarefree level. This talk is based on discussions with T. Ibukiyama and C. Poor and D. Yuen.

Athanasios Bouganis
On special L-Values attached to half-integral weight Siegel modular forms
Durham University, UK

Abstract: In this talk we will discuss algebraic and p-adic properties of special values of L-functions attached to Siegel modular forms of half-integral weight. These L-values, quite similarly to the integral weight situation, can be studied using two different approaches, the doubling method and the Rankin–Selberg method. In both approaches the arithmetic properties of Eisenstein series play a central role. In the integral weight situation there has been considerable work both with respect to the algebraicity of these special L-values (Sturm, Harris, Garrett, Shimura, Böcherer) and to the existence of p-adic measures (Panchishkin, Böcherer and Schmidt). In this talk we will consider the half-integral weight situation, discuss some algebraicity results of Shimura and consider extensions of the above works in this setting.

Pierre Charollois
Explicit integral cocycles on GL_n *and special values of p-adic partial zeta functions*
Université Paris 6, France

Abstract: Building on earlier work by R. Sczech, we construct an explicit integral valued "Eisenstein cocycle" on $\mathrm{GL}_n(\mathbb{Z})$. It allows for a study of the order of vanishing and of the first derivative at $s = 0$ of the p-adic partial zeta functions introduced by Pi. Cassou-Noguès and Deligne–Ribet. This is joint work with S. Dasgupta.

Gerard Freixas
On the Riemann–Roch formula in Arakelov geometry and an exotic analytic class number formula
Université Paris 6, France

Abstract: The first aim of this talk will be to recall the formulation of the arithmetic Riemann–Roch of Gillet–Soulé and give some example of use of arithmetic interest. Then I will move to a degenerate version of it, which is the theme of research of ongoing work with Anna von Pippich. Most notably, we consider the case of the trivial sheaf on the modular curve $X(1)$ equipped with its singular Poincaré metric. This can be seen as an exotic analytic class number formula for the Selberg zeta function, which in turn can be expressed in terms of class numbers and fundamental units of quadratic forms of positive discriminant. I will explain the relation

of this kind of formulas to Eisenstein series. Depending on the time, I will give a glimpse of the proof of our result.

Jens Funke
Cycles in degenerate Hilbert modular surfaces and modular forms
Durham University, UK

Abstract: In this talk I consider cohomological aspects of the classical Hecke correspondences, that is, special cycles in a surface X given by a product of two modular curves. One feature of our approach is the use of a nonstandard compactification of X and a mapping cone realization of its cohomology groups. In particular, we recover the famous Hurwitz class number relation involving the Fourier coefficients of the classical weight 2 Eisenstein series. This is joint work with John Millson.

Eyal Goren
The Bruinier–Yang conjecture
McGill University Montreal, Canada

Abstract: The Bruinier–Yang conjecture is an "extreme case" of expected formulas for the arithmetic intersection numbers between special arithmetic cycles on Shimura varieties of orthogonal type. It deals with the intersection between special points arising from quadratic imaginary fields and special divisors. I will discuss joint work with F. Andreatta, B. Howard and K. Madapusi-Pera in which we prove the conjecture. If time allows, I will discuss also the Bruinier–Kudla–Yang conjecture that deals with special points arising from CM fields of maximal possible degree.

Benjamin Howard
Cycles on Shimura varieties and applications to Faltings heights
Boston College, USA

Abstract: Colmez has conjectured a formula expressing the Faltings heights of CM abelian varieties in terms of Artin L-functions. I will speak about ongoing joint work with Bruinier, Kudla, Rapoport, and Yang toward special cases of this conjecture.

Jay Jorgenson
Kronecker's limit formula, holomorphic modular functions, and q-expansions on certain moonshine groups
City College of New York, USA

Abstract: For any square-free integer N such that the "moonshine group" $\Gamma_0(N)^+$ has genus zero, the Monstrous Moonshine Conjectures relate the Hauptmoduli of $\Gamma_0(N)^+$ to certain McKay-Thompson series associated to the representation theory of the Fischer-Griess monster group. In particular, the Hauptmoduli admits a q-expansion which has integer coefficients. In joint work with Lejla Smajlovic and Holger Then, we study the holomorphic function theory associated to higher genus moonshine groups $\Gamma_0(N)^+$. For all moonshine groups of genus up to and including three, we prove that the corresponding function field admits two generators whose q-expansions have integer coefficients, have lead coefficient equal to one, and have minimal order of pole at infinity. As corollary, we derive a polynomial relation which defines the underlying projective curve, and we deduce whether $i\infty$ is a Weierstrass point.

Winfried Kohnen
Two applications of holomorphic Eisenstein series
Ruprecht-Karls-Universität Heidelberg, Germany

Abstract: In this talk, I will report on two applications of Eisenstein series. The first one is very classical (2005) and goes back to Ö. Imamoglu and myself, it concerns generation of spaces of modular forms by the products of two Eisenstein series. The second one is very recent (joint work with Y.-J. Choie, 2014) and is about a characterization of cusp forms of half-integral weight by the growth of their *squarefree* Fourier coefficients.

Jürg Kramer
Uniform sup-norm bounds on average for cusp forms of higher weights
Humboldt-Universität zu Berlin, Germany

Abstract: Let $\Gamma \subseteq \mathrm{PSL}_2(\mathbb{R})$ be a Fuchsian subgroup of the first kind acting on the upper half-plane $\mathbb{H}$. Let $\{f_1, \ldots, f_d\}$ be an orthonormal basis of the space of cusp forms of weight $2k$ for Γ with respect to the Petersson inner product. In our talk we will show that the sup-norm of the quantity $\sum_{j=1}^{d} |f_j(z)|^2 \,\mathrm{Im}(z)^{2k}$ is bounded as $O_\Gamma(k)$ in the cocompact setting, and as $O_\Gamma(k^{3/2})$ in the cofinite case, where the

implied constants depend solely on Γ. We will also show that the implied constants are uniform if Γ is replaced by a subgroup of finite index.

Siddarth Sankaran
Special cycles on unitary Shimura varieties and Eisenstein series
Universität Bonn, Germany

Abstract: In a long series of work, and inspired by seminal results of Hirzebruch–Zagier and Gross–Zagier, Kudla and others have developped a deep set of conjectures known as Kudla's programme, which seeks to relate certain families of cycles on Shimura varieties with the Fourier coefficients of modular forms.
In this talk I will discuss an aspect of this programme that conjecturally identifies generating series built out of arithmetic cycles with special values of the derivatives of Eisenstein series. In particular, I will focus on some recent progress in the setting of unitary Shimura varieties.

Maryna Viazovska
Siegel Eisenstein series and Heegner cycles
Humboldt-Universität zu Berlin, Germany

Abstract: Gross–Kohnen–Zagier formula expresses the height paring between Heegner divisors on a modular curve in terms of Fourier coefficients of a certain Jacobi modular form. This formula became a starting point of Kudlas programme that relates the Fourier coefficients of Siegel Eisenstein series and arithmetic intersections of Heegner divisors on Shimura varieties. In this talk I would like to present computations that relate Fourier coefficients of certain non-holomorphic Siegel Eisenstein series and height pairing between Heegner cycles on Kuga–Sato varieties.

Tonghai Yang
Coherent and incoherent Eisenstein series
University of Wisconsin, USA

Abstract: In this talk, I briefly describe where these concepts come from, the arithmetic meaning of their Fourier coefficients, their relation, and applications. Along the way, I would also like to raise some questions. This is an informal survey talk.

3 General Information

3.1 Lecture Hall

Location: Technische Universität Darmstadt. The lectures are taking place in the Hörsaal der Kernphysik, Lecture Hall 024, which is situated on the ground floor of building S2|14, Schlossgartenstraße 9, 64289 Darmstadt.

3.2 Food & Beverage

The university cafeteria "Mensa" offers a good variety of cheap meals for lunch, building S1|11 (next to the Welcome Hotel), Monday to Friday 11:15 to 14:00. Additionally, the bistro at the university library is open all day from 08:00 to 22:00, building S1|20 ULB. Furthermore there are lots of good restaurants and bistros near TU Darmstadt. Please dial 0049 6151 preceding the number given below.

Name	Address	Phone	Cuisine
3klang	Riegerplatz 3	6698843	International
Adega Alentejana	Heinheimer Str. 38	971796	Portuguese
Chin-Su	Kranichsteiner Str. 8	9818671	Asian
Habibi	Landwehrstr. 13	6602760	Vegetarian
Haroun's	Friedensplatz 6	23487	Oriental
La Bodega	Kahlertstr. 34	291674	Spanish
Pizzeria da Nino	Alexanderstr. 29	24220	Italian
Ratskeller	Marktplatz 8	26444	German
Restaurant Sitte	Karlstr. 15	22222	German
Ristorante Sardegna	Kahlertstr. 1	23029	Italian
Vis à Vis	Fuhrmannstr. 2	9670806	Bistro

3.3 Conference Dinner

The conference dinner is scheduled for Wednesday, September 24^{th} at 18:30. The venue for this dinner is the restaurant Sardegna, Kahlertstraße 1, 64293 Darmstadt, which is in a 10 minutes walking distance from the lecture hall and the mathematics department.

3.4 Contact Information

If there are any questions concerning the workshop, please feel free to contact our secretaries:

- Karolin Berghaus (in the afternoon)
  Office: S2|15, 4th floor, Room K414
  Phone: +49 (0) 6151 - 16 2089
- Ute Fahrholz (in the morning)
  Office: S2|15, 4th floor, Room K414
  Phone: +49 (0) 6151 - 16 3786
- Email: algebra@mathematik.tu-darmstadt.de

4 Participants

Alfes, Claudia Technische Universität Darmstadt, Germany
alfes@mathematik.tu-darmstadt.de

Andreatta, Fabrizio Università Statale di Milano, Italy
fabrizio.andreatta@unimi.it

Bachmann, Henrik Universität Hamburg, Germany
henrik-bachmann@uni-hamburg.de

Bertolini, Massimo Universität Duisburg-Essen, Germany
massimo.bertolini@uni-due.de

Biberhofer, Sascha Universität Wien, Austria
sascha.biberhofer@univie.ac.at

Böcherer, Siegfried Universität Mannheim, Germany
boecherer@math.uni-mannheim.de

Bouganis, Athanasios University of Durham, UK
athanasios.bouganis@durham.ac.uk

Bruinier, Jan Technische Universität Darmstadt, Germany
bruinier@mathematik.tu-darmstadt.de

Burenko, Ilya National Research University - HSE Moscow, Russia
iburenko@gmail.com

Charollois, Pierre Université Paris 6, France
charollois@math.jussieu.fr

Crawford, Jonathan University of Durham, UK
j.k.crawford@durham.ac.uk

De Gaetano, Giovanni Humboldt-Universität zu Berlin, Germany
degaetano@math.hu-berlin.de

Deividas, Sabonis Technische Universität München, Germany
deividassab@gmail.com

Dickson, Martin University of Bristol, UK
martin.dickson@bristol.ac.uk

Dittmann, Moritz Technische Universität Darmstadt, Germany
mdittmann@mathematik.tu-darmstadt.de

Dzambic, Amir Goethe-Universität Frankfurt, Germany
dzambic@math.uni-frankfurt.de

Freixas, Gerard Université Paris 6, France
freixas@math.jussieu.fr

Frey, Linda Technische Universität Darmstadt, Germany
raabe@mathematik.tu-darmstadt.de

Funke, Jens University of Durham, UK
jens.funke@durham.ac.uk

Goren, Eyal McGill University of Montreal, Canada
eyal.goren@mcgill.ca

Grados, Miguel Humboldt-Universität zu Berlin, Germany
daygoro.grados@gmail.com

Habegger, Philipp Technische Universität Darmstadt, Germany
habegger@mathematik.tu-darmstadt.de

Hattori, Shin Kyushu University, Japan
shin-h@math.kyushu-u.ac.jp

Henn, Andreas Technische Universität Dortmund, Germany
andreas.henn@math.tu-dortmund.de

Hofmann, Eric Ruprecht-Karls-Universität Heidelberg, Germany
hofmann@mathi.uni-heidelberg.de

Howard, Benjamin Boston College, USA
howardbe@bc.edu

Hwang, Jihyun Sungkyunkwan University Seoul, South Korea
jihyun89@skku.edu

Jorgenson, Jay City College of New York, USA
jjorgenson@mindspring.com

Kohnen, Winfried Ruprecht-Karls-Universität Heidelberg, Germany
winfried@mathi.uni-heidelberg.de

Korodi, Tamás RWTH Aachen, Germany
tamaskorodi1988@gmail.com

Kramer, Jürg Humboldt-Universität zu Berlin, Germany
kramer@mathematik.hu-berlin.de

Kwon, Yeong-Wook Sungkyunkwan University Seoul, South Korea
pronesis196884@gmail.com

Maurischat, Kathrin Ruprecht-Karls-Universität Heidelberg, Germany
maurischat@mathi.uni-heidelberg.de

Mohammad Reza, Rahmati CIMAT Guanajuato, Mexico
mrahmati@cimat.mx

Nabardi, Kamran Azarbaijan Shahid Madani University, Iran
nabardi@azaruniv.edu

Neururer, Michael University of Nottingham, UK
pmxmone@nottingham.ac.uk

Opitz, Sebastian Technische Universität Darmstadt, Germany
opitz@mathematik.tu-darmstadt.de

Park, Chol Max-Planck-Institut Bonn, Germany
cpark@math.toronto.edu

Paul, Thorsten Universität des Saarlandes Saarbrücken, Germany
thorstenpaul@math.uni-sb.de

von Pippich, Anna Technische Universität Darmstadt, Germany
pippich@mathematik.tu-darmstadt.de

Rolen, Larry Universität zu Köln, Germany
larryrolen@gmail.com

Sankaran, Siddarth Universität Bonn, Germany
sankaran@math.uni-bonn.de

Scheithauer, Nils Technische Universität Darmstadt, Germany
scheithauer@mathematik.tu-darmstadt.de

Schmid, Stefan Technische Universität Darmstadt, Germany
sschmid@mathematik.tu-darmstadt.de

Schwagenscheidt, Markus Technische Universität Darmstadt, Germany
schwagenscheidt@mathematik.tu-darmstadt.de

Solomatin, Pavel National Research University - HSE Moscow, Russia
pavelsolomatin179@gmail.com

Viazovska, Maryna Humboldt-Universität zu Berlin, Germany
viazovska@gmail.com

Völz, Fabian Technische Universität Darmstadt, Germany
voelz@mathematik.tu-darmstadt.de

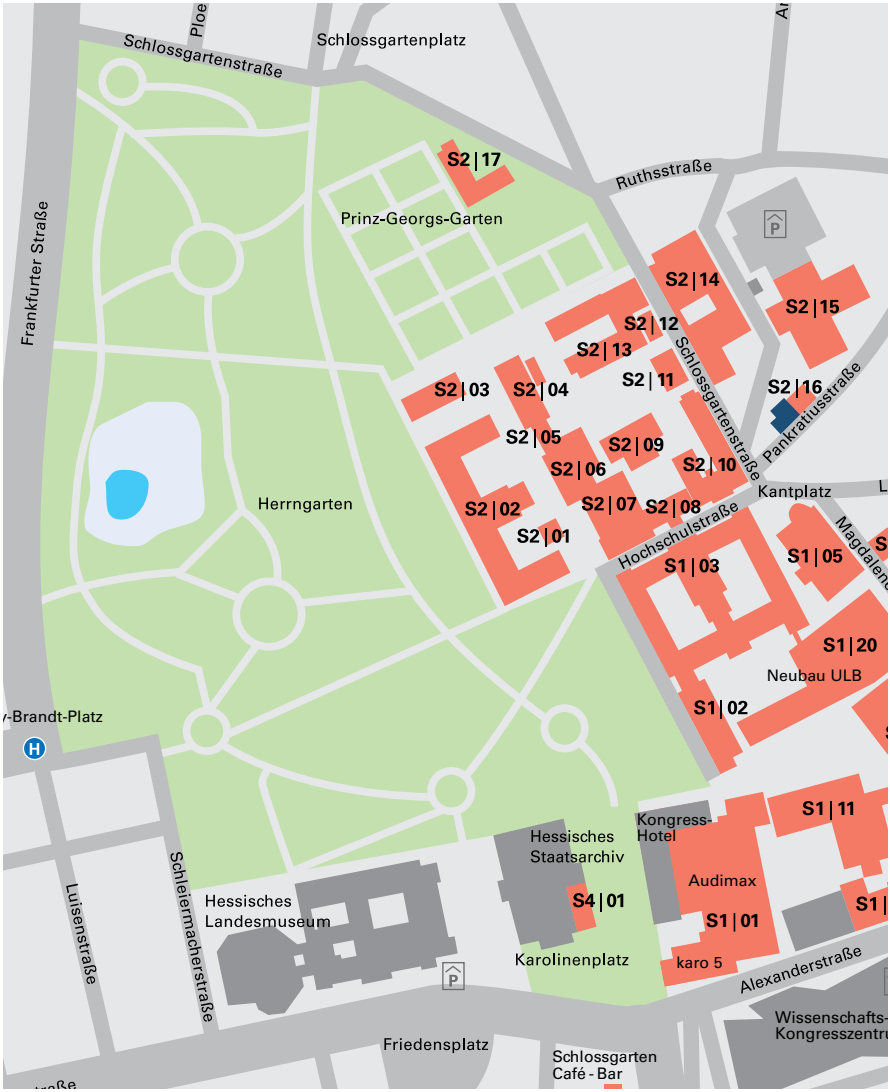
Walls, Patrick McMaster University Hamilton, Canada
pwalls@math.mcmaster.ca

Werner, Fabian Technische Universität Darmstadt, Germany
werner@mathematik.tu-darmstadt.de

Yang, Tonghai University of Wisconsin at Madison, USA
thyang@math.wisc.edu

Zapata Rolon, Jose Miguel Universität zu Köln, Germany
rzapata@math.uni-koeln.de

Zemel, Shaul Technische Universität Darmstadt, Germany
zemel@mathematik.tu-darmstadt.de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Kantplatz
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Kongress-Hotel
Hessisches Staatsarchiv
Audimax
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Hessisches Landesmuseum
Karolinenplatz
Alexanderstraße
Luisenstraße
Schleiermacherstraße
Friedensplatz
Schlossgarten Café - Bar