$(\infty, 2)$-CATEGORIES

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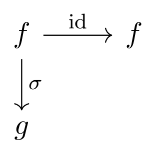
1. From 2 to $(\infty, 2)$-categories

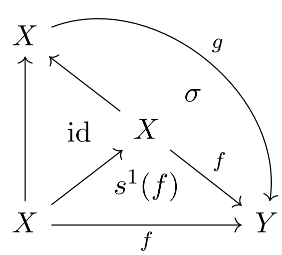
1.1. **Thin simplices.** Let us explain why morally an ∞-category in the sense of Higher Topos Theory [2] is actually an $(\infty, 1)$-category.

Remark 1.1. Let $\mathcal{C}$ be an ∞-category, then for any $X \in \mathcal{C}$ the projection from the slice $\mathcal{C}_{/X} \to \mathcal{C}$ is a right fibration [4, 018F]. In particular, the projection $\mathrm{Hom}(X, Y) \simeq \mathcal{C}_{/X} \times_{\mathcal{C}} \{Y\} \to \Delta^0$ is a right fibration and therefore also a Kan-fibration [2, Prop. 1.2.5.1].

Morally, we can think of it as follows. Let $f, g \in \mathrm{Hom}(X, Y)$ and $\sigma\colon f \to g$ an edge in $\mathrm{Hom}(X, Y)$. Then the horn $\Lambda^2_0 \to \mathrm{Hom}(X, Y)$ given by



admits a lift to Δ^2 if and only if the horn $\Lambda^3_2 \to \mathcal{C}$ represented in the diagram



admits a lift to Δ^3. However, this is clear since $\mathcal{C}$ is an ∞-category.

In the context of 2-categories the fact that $\mathrm{Hom}(X, Y)$ is a Kan complex is not desirable. As we have seen above the main obstruction to obtain a theory of $(\infty, 2)$-categories is that every inner horn Λ^n_i of $\mathcal{C}$ can be lifted for $n \geq 3$ and $0 < i < \infty$. To resolve this problem we have to weaken this condition. But first let us define this notion of having a lift.

Definition 1.2. Let $X_\bullet$ be a simplicial set. Then a 2-simplex $\sigma \in X_2$ is called *thin* if every map $\tau\colon \Lambda^n_i \to X_\bullet$ such that $\tau(\{i-1 < i < i+1\}) = \sigma$ admits a lift to $\Delta^n \to X_\bullet$ for all $n \geq 3$ and $0 < i < n$.

For example in an ∞-category every 2-simplex is thin by definition. Moreover, in a reasonable theory of $(\infty, 2)$-categories every equivalence σ should be thin. We will also observe this once we construct a simplicial set out of a 2-category.

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1.2. **2-categories.** We start by recalling the necessary notions of 2-categories. For an overview with relevant proofs and references, we refer to [4, 007K]. We will not give a detailed definition of 2-categories and refer to *loc. cit.*.

Definition 1.3. A 2-category C consists of the following data:

- A class of objects $\mathrm{Ob}(C) : X, Y, Z, \ldots$.
- For every $X, Y \in \mathrm{Ob}(C)$ a category $\mathrm{Hom}_C(X, Y)$. The objects $f, g \in \mathrm{Hom}_C(X, Y)$ are called 1-*morphisms* and a morphism γ from f to g is denoted by $\gamma\colon f \Rightarrow g$ and is called a 2-*morphism.*
- For every object $X \in \mathrm{Ob}(C)$ an identity 1-morphism $\mathrm{id}_X \in \mathrm{Hom}_C(X, X)$.
- For every triple $X, Y, Z \in \mathrm{Ob}(C)$ a *composition* functor

$$\circ\colon \mathrm{Hom}_C(X, Y) \times \mathrm{Hom}_C(Y, Z) \to \mathrm{Hom}_C(X, Z).$$

- An associativity natural isomorphism $h \circ (g \circ f) \xRightarrow{\sim} (h \circ g) \circ f$ for all composable f, g, h, called *associativity constraints.*

We further have isomorphisms called *unit contraints* $\nu_X\colon \mathrm{id}_X \circ \mathrm{id}_X \xRightarrow{\sim} \mathrm{id}_X$ and the associativity constraints are compatible with the unit constraints, cf. [4, 007Q]. Moreover, left and right composition with the identity yields a fully faithful functor.

Remark 1.4. From a 2-category C, one can construct the Endomorphism category $\mathrm{End}(X) = \mathrm{Hom}_C(X, X)$ for an object X. One can show that composition $\circ$ yields a monoidal structure on $\mathrm{End}(X)$. The unit object in this monoidal category is the identity 1-morphism id_X together with the unit constraints. In particular, the identity 1-morphism and the unit constraints are unique (up to unique isomorphism) [4, 00EV].

We will also need the notion of a 2-functor. There are multiple ways to define 2-functors F between 2-categories C and D. In principle there are three (actually six) different notions arising from the unit constraints and composition. We can ask for 2-morphisms $\mathrm{id}_{F(X)} \Rightarrow F(\mathrm{id}_X)$ resp. $F(h) \circ F(g) \circ F(f) \Rightarrow F(h \circ g \circ f)$ to exists, to exists and be an isomorphism or even equality respectively. The weakest notion is the existence and will be called *lax.* When $\mathrm{id}_{F(X)} = F(\mathrm{id}_X)$ a 2-functor will be called *strictly unitary* but let us make this more precise following [4, 008G].

Definition 1.5. Let C, D be 2-categories. A *lax functor* $F\colon C \to D$ consists of the following data:

- For every object $X \in C$, an object $F(X) \in D$.
- For every pair $X, Y \in C$, a functor

$$F_{X,Y}\colon \mathrm{Hom}_C(X, Y) \to \mathrm{Hom}_D(F(X), F(Y)).$$

  If X and Y are clear, we will drop the subscript on F.
- For every $X \in C$, a 2-morphism $\epsilon_X\colon \mathrm{id}_{F(X)} \Rightarrow F(\mathrm{id}_X)$ in D, referred to as *unit constraints.*
- For every composable pair of morphisms $X \xrightarrow{f} Y \xrightarrow{g} Z$ in C a 2-morphism

$$\mu_{g,f}\colon F(g) \circ F(f) \Rightarrow F(g \circ f),$$

  referred to as the *composition constraint.*

Moreover, there is a compatibility of unit constraints, composition constraints and unit constraints, cf. [4, 008K].

- A lax functor $F\colon C \to D$ is called *unitary*, if the unit constraints $\mathrm{id}_{F(X)} \Rightarrow F(\mathrm{id}_X)$ are isomorphisms and *strictly unitary* if $\mathrm{id}_{F(X)} = F(\mathrm{id}_X)$.

Remark 1.6. Let $F\colon C \to D$ be a lax functor between 2-categories. Consider the endomorphism category $\mathrm{End}(X)$ for $X \in C$. Then we obtain a functor $F_{X,X}\colon \mathrm{End}(X) \to \mathrm{End}(F(X))$. The unit and composition constraints assure that $F_{X,X}$ is in fact a lax monoidal functor, which can again be used for uniqueness statements.

We are now ready to construct a simplicial set out of a 2-category C. For this we will view the linearly ordered set $[n] \coloneqq \{0 < 1 < \cdots < n\}$ as a 2-category, where the 1-morphisms are uniquely given by $i \leq j$ and the 2-morphisms are just the identity. Then $\mathrm{N}^{\mathrm{D}}_n(C)$ will denote the set of strictly unitary lax functors from $[n]$ to C. The construction $[n] \mapsto \mathrm{N}^{\mathrm{D}}_n(C)$ determines a simplicial set

$$\Delta^{\mathrm{op}} \hookrightarrow \mathrm{Cat}^{\mathrm{op}} \hookrightarrow 2\mathrm{Cat}^{\mathrm{op}}_{\mathrm{ULax}} \xrightarrow{\mathrm{Hom}_{2\mathrm{Cat}^{\mathrm{op}}_{\mathrm{ULax}}}(\bullet, C)} \mathrm{Set}$$

Definition 1.7. Let C be a 2-category. The simplicial set $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ is called the *Duskin Nerve* of the 2-category C.

We can make the simplicial set $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ more precise, cf. [4, 00A1].

(0) As for the nerve of a category, $\mathrm{N}^{\mathrm{D}}_0(C)$ consists of a collection of objects $\{X_i\}$ in C,
(1) $\mathrm{N}^{\mathrm{D}}_1(C)$ consists of a collection of morphisms $\{f_{j,i}\colon X_i \to X_j\}$

However, contrary to the Nerve of a 1-category, we obtain more information on 2-simplices.

(2) By the composition constraints, the set of strictly unitary lax functor $[2] \to C$ consists of a collection of 2-morphisms $\{\mu_{k,j,i}\colon f_{k,j} \circ f_{j,i} \Rightarrow f_{k,j}\}$. The strictly unitary condition assures that identity and unit constraines are preserved (in a strict sense). The set $\mathrm{N}^{\mathrm{D}}_3$ encodes the compatibility of the composition, unit and associativity constraints.

In fact, these data already uniquely determine the Duskin Nerve in the following sense.

Proposition 1.8 ([4, 00A3])**.** *Let C be a 2-category. Then the restriction map*

$$\mathrm{Hom}_{\mathrm{Set}_\Delta}(\Delta^n, \mathrm{N}^{\mathrm{D}}_\bullet(C)) \to \mathrm{Hom}_{\mathrm{Set}_\Delta}(\partial\Delta^n, \mathrm{N}^{\mathrm{D}}_\bullet(C))$$

is bijective for $n \geq 4$ and injective for $n = 3$.

Finally, we can give our main example of thin simplices.

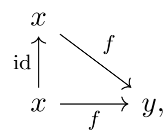
Theorem 1.9 ([4, 00AF])**.** *Let C be a 2-category. A 2-simplex $\sigma\colon g \circ f \Rightarrow h$ in $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ is thin if and only if σ is an isomorphism.*

1.3. $(\infty, 2)$**-categories.** Let C be a 2-category. Theorem 1.9 tells us that in $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ ever map $\Lambda^2_1 \to \mathrm{N}^{\mathrm{D}}_\bullet(C)$ can be extended to a thin 2-simplex by using the identity $g \circ f \overset{\mathrm{id}}{\Longrightarrow} g \circ f$. Moreover, every degenerate 2-simplex, which is an equivalence, is also thin. This justifies the following definition.

Definition 1.10. Let $\mathcal{C}$ be a simplicial set. We say $\mathcal{C}$ is an $(\infty, 2)$-*category* if

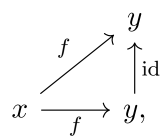
(1) every map $\Lambda^2_1 \to \mathcal{C}$ can be extended to a thin 2-simplex,
(2) every degenerate 2-simplex is thin, and
(3) for $n \geq 3$

(i) every $\tau\colon \Lambda^n_0 \to \mathcal{C}$ such that $\tau_{\{0<1<n\}}$ is left degenerate, i.e. of the form



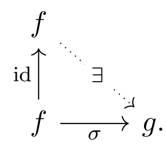
can be extended to $\Delta^n \to \mathcal{C}$, and

(ii) every $\tau\colon \Lambda^n_0 \to \mathcal{C}$ such that $\tau_{\{0<n-1<n\}}$ is right degenerate, i.e. of the form



can be extended to $\Delta^n \to \mathcal{C}$.

Remark 1.11. One way to think of condition (3.i) in the definition of an $(\infty,2)$-category $\mathcal{C}$ is that it ensures a a solution to the lifting problem



Moreover, let us remark that every ∞-category is automatically and $(\infty,2)$-category. While it is clear that every 2-simplex is thin property (3) has to be checked and follows from a theorem of Joal [4, 019F] using that in every degenerate 2-simplex the edge corresponding to $\{0<1\}$ is an isomorphism.

Let us give two important examples of $(\infty,2)$-categories.

Proposition 1.12. *Let C be a 2-category, then $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ is an $(\infty,2)$-category.*

Proof. Let us refer to [4, 01WD] and only say a few words to the proof of (1). We already that a simplex in $\mathrm{N}^{\mathrm{D}}_\bullet(C)$ is thin if and only if the is an isomorphism. In particular, conditions (1) and (2) of Definition 1.10 are automatic. To show (3) one proceeds via explicit computation, using various compatibilities inside the definition of $\mathrm{N}^{\mathrm{D}}_n(C)$. However, one only has to worry about $n=3$ and $n=4$. If $n\geq 5$, then Λ^n_n contains the 3-skeleton of Δ^n and therefore by Proposition 1.8, we can reduce to the previous cases. □

Proposition 1.13 ([4, 01YL])**.** *Let C be a simplicial category and suppose $\mathrm{Hom}_C(X,Y)_\bullet$ is an ∞-category for all objects $X,Y\in C$. Then the homotopy coherent Nerve $\mathrm{N}^{\mathrm{hc}}(C)$ is an $(\infty,2)$-category.*

1.4. **The pith of an $(\infty,2)$-category.** For an ∞-category $\mathcal{C}$ we can define the core $\mathcal{C}^{\simeq}$, a Kan complex, as the simplicial subset of $\mathcal{C}$ where each edge of an n-simplex is invertible. For $(\infty,2)$-categories there is a similar construction.

Definition 1.14. Let $\mathcal{C}$ be an $(\infty,2)$-category. We let $\mathrm{Pith}(\mathcal{C})$ denote the simplicial subset of $\mathcal{C}$ of those simplices $\sigma\colon \Delta^n \to \mathcal{C}$ such every 2-simplex of Δ^2 is mapped to a thin simplex under σ. We refer to $\mathrm{Pith}(\mathcal{C})$ as the *pith of* $\mathcal{C}$.

Perhaps the most interesting result for us is that $\mathrm{Pith}(\mathcal{C})$ is in fact an ∞-category and moreover, its Hom-space can be explicitly computed.

Proposition 1.15. *Let $\mathcal{C}$ be an $(\infty, 2)$-category. Then $\mathrm{Pith}(\mathcal{C})$ is an ∞-category and*

$$\mathrm{Hom}_{\mathrm{Pith}(\mathcal{C})}(X, Y) \simeq \mathrm{Hom}_{\mathcal{C}}(X, Y)^{\simeq}$$

for all $X, Y \in \mathcal{C}$.

2. How to model $\mathrm{Cat}_{(\infty,2)}$ as an ∞-category?

Our definition of $(\infty, 2)$-categories is a modern version presented in [4] by Lurie. However, there are also other approaches realizing $(\infty, 2)$-categories as fibrant/cofibrant objects of certain model categories. All of these approaches are actually equivalent to the one given in the previous section, cf. [3] and [1]. One such way is the theory of *complete Segal spaces*, which admits naturally an appropriate model structure. In fact, we will define (∞, n)-categories for $n \geq 2$ without much effort, though we will ignore all the model theoretic information, which is the actual heart of the theory. To this end, we follow [5].

Let us start by observing that Proposition 1.15 tells us that $(\infty, 2)$-categories should be thought of as Cat_∞-enriched ∞-categories. This can also be made explicit for an $(\infty, 2)$-category $\mathcal{C}$ by defining

$$\mathrm{Hom}_{\mathcal{C}}(X, Y) \coloneqq \mathcal{C}_{/X} \times_{\mathcal{C}} \{Y\}$$

for $X, Y \in \mathcal{C}$. The projection $\mathcal{C}_{/X} \to \mathcal{C}$ is an interior fibration [4, 01WU] and therefore $\mathrm{Hom}_{\mathcal{C}}(X, Y)$ is indeed an ∞-category.

Hence, we need a way to define categories internal to a given theory. We will start with a bit of motivation following [5].

Definition 2.1. Let C be a 1-category with fiber products. A *category internal to C* is a functor

$$X : \Delta^{\mathrm{op}} \to C$$

such that $X_n \to X_1 \times_{X_0} X_1 \times_{X_0} \cdots \times_{X_0} X_1$ is an isomorphism, where we take the n-fold product on the right hand side.

Let us remark that in the above definition the fiber product is actually given relative to the degeneracy maps $X_1 \times_{d^0, X_0, d^1} X_1$. In particular, if $C = \mathrm{Set}$ a map $\Lambda^n_i \to X$ can be uniquely lifted to a map $\Delta^n \to X$ by the universal property of the fiber product, i.e. X is the Nerve of a 1-category.

We cannot generalize this definition immediately to ∞-categories by replacing Set with the ∞-category of Kan complexes $\mathbb{S}$. This already becomes apparent when considering groupoids. Let X a category internal to Grpd. Then composing X with the forgetful functor $\mathrm{Grpd} \to \mathrm{Set}$ yields a 1-category $J(X)$. In particular, one might try to use the model structure on Grpd to obtain a model of 1-categories. But for this we also have to go the other way.

If C is a category, we can construct a category internal to Grpd, denoted by $\mathrm{Disc}(C)$ via the embedding $\mathrm{Set} \to \mathrm{Grpd}$.

We can also define $\max(C)$ via $\max(C)_0 = C^{\simeq}$ and $\max(C)_1 = \mathrm{Fun}([1], C)^{\simeq}$. Note that by design $J(\mathrm{Disc}(C)) \cong C \cong J(\max(C))$. In particular, there could be inequivalent model structures yielding the same 1-category. We are interested in the model for 1-categories obtained via the max-process. The categories internal to Grpd which come from this construction can be characterized via the following *completeness* statement.

Proposition 2.2. *Let C be a 1-category. Let $\max(C)_E \subseteq \max(C)_1$ denote the full subgroupoid of equivalences. Then the functor $\mathrm{id}\colon \max(C)_0 \to \max(C)_E$ mapping X to id_X is an equivalence of groupoids.*

2.1. **Complete Segal spaces.** We will generalize the notion of internal categories to arbitrary ∞-categories.

Definition 2.3. Let $\mathcal{C}$ be an ∞-category. A *category object in* $\mathcal{C}$ is a functor

$$X\colon \mathrm{N}(\Delta^{\mathrm{op}}) \to \mathcal{C}$$

such that $X_n \to X_{\{0,1\}} \times_{X_1} X_{\{1,2\}} \times_{X_2} \cdots \times_{X_{n-1}} X_{\{n-1,n\}}$ is an equivalence.
If $\mathcal{C} = \mathbb{S}$, then we call X a *Segal space*.

Let us remark that any Segal space X may be viewed as a bisimplicial set $X\colon \Delta^{\mathrm{op}} \times \Delta^{\mathrm{op}} \to \mathrm{Set}$

Definition 2.4 (Homotopy category of a Segal space)**.** Let X be a Segal space. We define the *homotopy category of* X, denotes by $\mathrm{Ho}(X)$ as the category with

- $\mathrm{Ob}(\mathrm{Ho}(X)) = X([0])_0$, and
- for $x, y \in X([0])_0$ we define $\mathrm{Hom}_{\mathrm{Ho}(X)}(x,y)$ via the following pullback

$$\begin{array}{ccc} \mathrm{Hom}_{\mathrm{Ho}(X)}(x,y) & \longrightarrow & X_1 \\ \downarrow & & \downarrow{\scriptstyle (d^0,d^1)} \\ \{x,y\} & \longrightarrow & X_0 \times X_0 \end{array}$$

Moreover, we set X_{WE} as the full subspace of X_1 of edges that are isomorphisms in $\mathrm{Ho}(X)$.

We are now ready to define the completeness that we saw earlier for the max construction. Note that the natural map $X_0 \to X_1$ given by degeneration factors over X_{WE}.

Definition 2.5. A Segal space X is called *complete* if the map $X_0 \to X_{WE}$ is an equivalence.

If X is a complete Segal space then regarding X as a bisimplicial set $X\colon \Delta^{\mathrm{op}} \times \Delta^{\mathrm{op}} \to \mathrm{Set}$ we can extract a simplicial set via $X_{\bullet,0}\colon [n] \mapsto X([n])_0$. In fact, this simplicial set is an ∞-category. Going the other way if $\mathcal{C}$ is an ∞-category, then the map $[n] \mapsto \mathrm{Fun}(\Delta^n, \mathcal{C})^{\simeq}$ yields a complete Segal space. Thus, complete Segal spaces yield a model for ∞-categories.

Theorem 2.6 (Joyal-Tierny)**.** *The full sub ∞-category $\mathrm{CSS}_1 \subseteq \mathrm{Fun}(\Delta^{\mathrm{op}}, \mathbb{S})$ of complete Segal spaces is equivalent to Cat_∞.*

The benefit of this viewpoint on the theory of ∞-categories is that we can easily iterate this process and obtain a theory of (∞, n)-categories.

Definition 2.7. An *n-fold complete Segal space* is a category object in $X \in \mathrm{CSS}_{n-1}$ such that

(1) $X_{0,*,\ldots,*}\colon (\Delta^{\mathrm{op}})^{\times(n-1)} \to \mathbb{S}$ factors over $\mathbb{S}^{\simeq}$, and
(2) $X_{*,0,\ldots,0}$ is a complete Segal space.

Theorem 2.8. *The ∞-category CSS_n is a model for (∞, n)-categories.*

Remark 2.9. Let us give a quick remark on the model structure of CSS_2. Lurie shows in [3, §1.5] that for a "nice" combinatorial left proper simplicial model category simplicial category $\mathbf{A}$, one can define a model structure on $\mathrm{Fun}(\Delta^{\mathrm{op}}, \mathbf{A})$ such that the fibrant objects are category objects in $\mathbf{A}$ (this has to be made precise). However, such a model category is given by the category of *marked simplicial sets* $\mathrm{Set}_{\Delta,+}$ [2]. One of the main theorems of [3] is that model structure on $\mathrm{Fun}(\Delta^{\mathrm{op}}, \mathrm{Set}_{\Delta,+}$ yields a model for so called *scaled simplicial sets*, which model $(\infty, 2)$-categories as we have defined it, cf. [1, Rem. 1.21].

2.2. **The functor** op **and** co**.** Next, we want to construct the opposite of a $(\infty, 2)$-category. However, there are two ways to achieve this. First, we can take opposed edges, just as for ∞-categories. However, we could also take the opposite edges in the ∞-category of morphisms, i.e. change direction of the 2-morphisms. The former will give us the functor op and the latter procedure will yield another functor denoted by co.

Let us stat by constructing the opposite of any simplicial set. To this end, consider the functor $\mathrm{O}\colon \Delta \to \Delta$ given by the identity on objects and mapping a morphism $\alpha\colon [n] \to [m]$ to $\mathrm{O}(\alpha)(i) = m - \alpha(n-i)$. Then we define for a simplicial set $\mathcal{S}$ its opposite simplicial set as the composition

$$\mathcal{S}^{\mathrm{op}}\colon \Delta^{\mathrm{op}} \xrightarrow{\mathrm{O}^{\mathrm{op}}} \Delta^{\mathrm{op}} \xrightarrow{\mathcal{S}} \mathrm{Set}$$

We can use this process to obtain the functors op and co on CSS_2. To this end, let $X \in \mathrm{CSS}_2$. Then we can regard X as a functor from $\mathrm{N}(\Delta^{\mathrm{op}}) \to \mathrm{Cat}_\infty$. By definition the Segal space $X_{n,0}\colon [n] \mapsto X([n])_0$ is complete and therefore yields an ∞-category, which we view as the underlying ∞-category of X. Thus precomposition of X with $\mathrm{N}(\mathrm{O})^{\mathrm{op}}$ yields a new 2-fold complete Segal space

$$X^{\mathrm{op}}\colon \mathrm{N}(\Delta^{\mathrm{op}}) \xrightarrow{\mathrm{N}(\mathrm{O}^{\mathrm{op}})} \mathrm{N}(\Delta^{\mathrm{op}}) \xrightarrow{X} \mathrm{Cat}_\infty$$

whose underlying ∞-category is $(X([\bullet])_0)^{\mathrm{op}}$. Since precomposition is functorial, we therefore obtain an endofunctor

$$\mathrm{op}\colon \mathrm{CSS}_2 \to \mathrm{CSS}_2.$$

On the other hand, we could also postcompose with $\mathrm{op}\colon \mathrm{Cat}_\infty \to \mathrm{Cat}_\infty$. This yields another functor

$$\mathrm{co}\colon \mathrm{CSS}_2 \to \mathrm{CSS}_2,\ X \mapsto X^{\mathrm{co}}.$$

Note that each $X([1])$ is an ∞-category and we can consider its edges as the space of 2-morphisms in X seen as an $(\infty, 2)$-category. Therefore, the 2-fold Segal space X^{co} is given by taking opposite 2-morphisms in X. Note that by design $X^{\mathrm{co}}([n])_0 = X([n])_0$. Thus, we do not change the 1-morphisms in the underlying ∞-category of X.

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