

Fachbereich Mathematik
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TECHNISCHE
 UNIVERSITÄT
 DARMSTADT

14 February 2003

Final examination Analysis I (MCS, CE, CMPE)
 WS 2002/03

LAST NAME:

FIRST NAME: MATRIKEL NO.:

Problem	1	2	3	4	5	6	7	Σ
Points (max.)	5	9	5	10	10	5	6	50
Points obtained								

Grade

before handing in, please fold along this line and enclose solution sheets

Please fill in the head of this problem sheet NOW and IN CLEAR CAPITAL LETTERS. Write your name on all sheets, and number them consecutively. Hand your answers and the prepared two sheets in at the end of the exam. As long as not stated otherwise, you may use everything which was proven in the lecture. Remember that you are expected to give reasons for your results, and that most credit will be given for systematic solutions, not for the bare results. The results of the exam will be posted on Wednesday, Feb. 19, opposite to room 305 in the math building. You can inspect your exam on Monday, Feb. 24, 9:30-11:30 and 13:00-18:00 in Steffen Fröhlich's office, room 324. Then, you may take along your exam on the condition that you agree not to question the marking.

Good luck!

Problem 1:

(5 points)

Prove that 7 divides $8^n + 14n - 1$ for all $n \in \mathbb{N}$.

Problem 2: (3+3+3 points)

Check the following sequences $(a_n)_{n \in \mathbb{N}}$, $(b_n)_{n \in \mathbb{N}}$, and $(c_n)_{n \in \mathbb{N}}$ for convergence. Prove the sequence diverges or compute the limit. Please state all the rules you use.

$$a_n = \frac{3n^2 + 17}{n^2 - 2}, \quad b_n = 1 + (-1)^n, \quad c_n = \frac{\pi^n}{n!}$$

Problem 3: (5 points)

Prove that $|\frac{1-i}{2}| < 1$ and express $\sum_{k=0}^{\infty} (\frac{1-i}{2})^k$ in the form $x + iy$ ($x, y \in \mathbb{R}$).

Problem 4: (10 points)

Are the following series convergent? Provide a proof (you can assume convergence results for standard series).

(i) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+5}}$

(ii) $\sum_{k=1}^{\infty} \frac{k^2}{(3+i)^k}$

Problem 5: (5+5 points)

(a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $0 \leq f(x) \leq x^2$ for all $x \in \mathbb{R}$. Prove that f is continuous at 0.

(b) Show via the $\varepsilon - \delta$ -test that the function

$$f: \mathbb{R} \rightarrow \mathbb{R} \\ f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ \frac{x+2}{x+1} & \text{for } x > 0 \end{cases}$$

is not continuous at 0.

Problem 6: (5 points)

Show that there exists an $x \in \mathbb{R}$ such that $-x^3 = e$.

Problem 7: (6 points)

Prove that the set $A := \{z = x + iy \in \mathbb{C} \mid x \in \mathbb{R}, y > 0\}$ is open.