Celebrating Erich

Berlin 2018

ABC

A	**amalgamation**	**piecing together fitting parts**
B	**bisimulation**	**games & strategies**
C	**cycles**	**rounding it out**

content

- **A is for amalgamation**
  from small- to large-scale local acyclicity
  or from local to global consistency
  or from local to global symmetries

- **B is for bisimilar coverings**
  for the local unravelling of cycles
  in graphs or hypergraphs

- **C is for cycles**
  in graphs and hypergraphs or
  in groups and groupoids

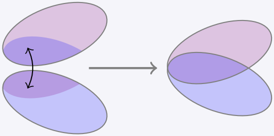
with new constructions to locally avoid cycles

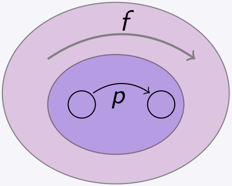
really, A is for B, B is for C, C is for A, . . .

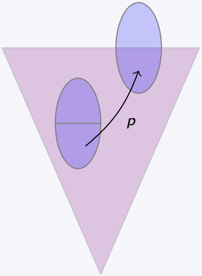
amalgamation?

identification in local overlaps, as in

- Fraïssé limits
- extending local to global symmetries
  Hrushovski/Herwig/Herwig–Lascar EPPA
- Erich's elegant proof of fmp for GF





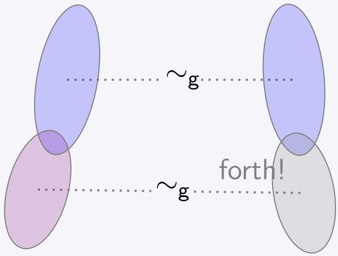


bisimulation?

the quintessential b&f equivalence :

$\sim$ bisimulation for transition systems/Kripke structures
b&f between graph-like structures

$\sim_{\mathbf{g}}$ guarded bisimulation for relational structures
b&f between hypergraph-like structures



walks and cycles in graphs & hypergraphs?

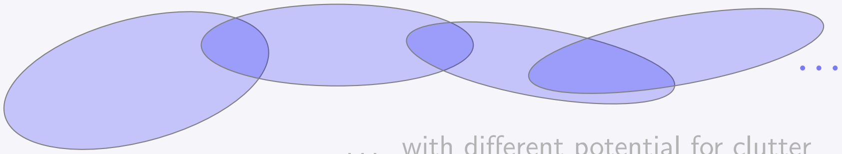
walk/path in graph

edges

⇝

overlaps

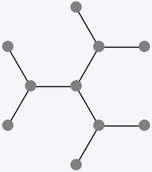
walk/path in hypergraph



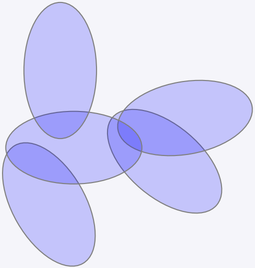
. . . with different potential for clutter

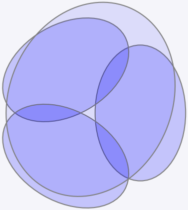
acyclicity = tree-likeness

in graphs: obvious



in hypergraphs: maybe less so





(I) cycles in graphs/cycles in hypergraphs

cycles in graphs (and graph-like relational structures)

– not governed by bisimulation

– avoidable in (infinite) bisimilar tree unfoldings

– acyclicity preserved in weak substructures

cycles in hypergraphs (and relational structures)

– not governed by (guarded) bisimulation

– avoidable in (infinite, guarded) bisimilar unfoldings: *generalised tree model property* for GF (Grädel 99)

– acyclicity *not* preserved in weak substructures (!)

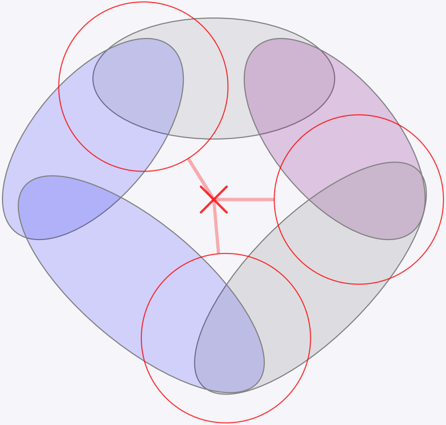
aside 1: what even is a hypergraph cycle?

- hypergraph acyclicity (α-acyclicity, Fagin et al)
- tree-decomposability of hypergraphs (Graham)
- conformality & chordality (Berge, Fagin et al)

all equivalent, as notions of hypergraph acyclicity
but what are the relevant cycles?

Julian Bitterlich: new notion of *hypergraph cycle*,
characterising hypergraph acyclicity

aside 1: what even is a hypergraph cycle?



a naive cycle
is a **B-cycle** if

no three overlaps
covered together!

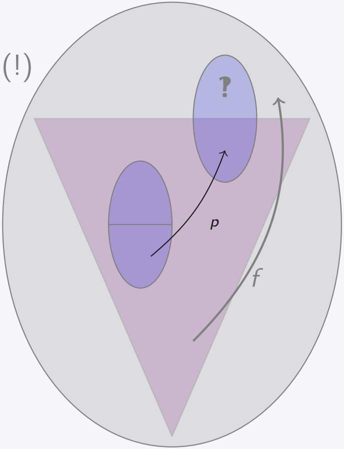
Bitterlich 2018: hypergraph acyclicity $\Leftrightarrow$ no such cycles
with nice structure theory to match

aside 2: tree models & finite models (ML)

- ML has tree model property
  $\sim$-invariance, tree unfolding
- ML has finite model property
  $\sim^{\ell}$-invariance, $\sim^{\ell}$-quotients
- ML has finite tree model property
  $\sim^{\ell}$-invariance, ℓ-tree unfolding & $\sim^{\ell}$-pruning

aside 2: tree-like models vs. finite models (GF)

- GF has generalised tree model property (Grädel 99)
  $\sim_{\mathrm{g}}$-invariance, hypergraph tree unfolding
- GF has finite model property (Grädel 99)
  $\sim_{\mathrm{g}}^{\ell}$-invariance does not support quotients (!)



Erich's use of Herwig's EPPA thm:

truncated hypergraph tree unfolding,
completed in Herwig EPPA extension

- GF has locally tree-like finite models
  in combination with (O_12)

aside 3: uses of locally acyclic (finite) models

finite and algorithmic model theory, esp. modal and guarded:
bisimulation invariance in non-elementary settings

- van Benthem–Rosen thms over natural frame classes
  and also for substantial extensions (like GF, ML[CK])
  O_04, Dawar–O_09, O_13, Ciardelli–O_18,
  O_12, Canavoi–O_17

- analysis of guarded negation and guarded constraints
  Bárány–ten Cate–O_12 , Bárány–Gottlob–O_14

$\longrightarrow$ **The freedoms of (guarded) bisimulation** Grädel–O_14

(II) cycles in groups and groupoids

Cayley group
with generators $e \in E$

$\mathbb{G} = (G, \cdot, 1, (e)_{e \in E})$ $\qquad$ $w \in E^* \longmapsto w^{\mathbb{G}} \in G$

Cayley graph

$\mathbb{G} = (G, (R_e)_{e \in E})$, $R_e = \{(g, ge) \colon g \in G\}$ $\qquad$ $w \in E^*$: walks in $\mathbb{G}$

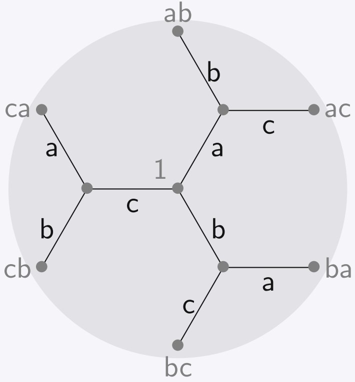
first shot: cycles = generator cycles (= graph cycles)

$\rightsquigarrow$ finite groups of large girth

aside: no short generator cycles = large girth

t.f.a.e.:
- $w^{\mathbb{G}} \neq 1$ for (reduced) generator words $|w| \leqslant 2\ell + 1$
- Cayley graph $\mathbb{G}$ has girth $\geqslant 2\ell + 1$
- is ℓ-locally acyclic
- $\mathbb{G} \restriction N^{\ell}(1)$ a tree

e.g. with involutive generators a, b, c



cycles in groups and groupoids

Cayley group
with (involutive) generators $e \in E \ \ (e^{-1} = e)$

$\mathbb{G} = (G, \cdot, 1, (e)_{e \in E})$ $\qquad w \in E^* \longmapsto w^{\mathbb{G}} \in G$

Cayley graph

$\mathbb{G} = (G, (R_e)_{e \in E})$, $R_e = \{(g, ge) \colon g \in G\}$ $\qquad w \in E^*$: walks in $\mathbb{G}$

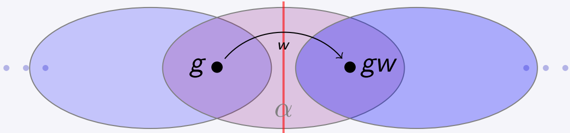
first shot: cycles = generator cycles (= graph cycles)

second shot: cycles = coset cycles (= hypergraph cycles)

cyclic configurations formed by cosets
$g\langle\alpha\rangle^{\mathbb{G}} = g\{w^{\mathbb{G}} \colon w \in \alpha^*\}$ for generator subsets $\alpha \subseteq E$

no short coset cycles – much more than large girth

forbid short cyclic configurations

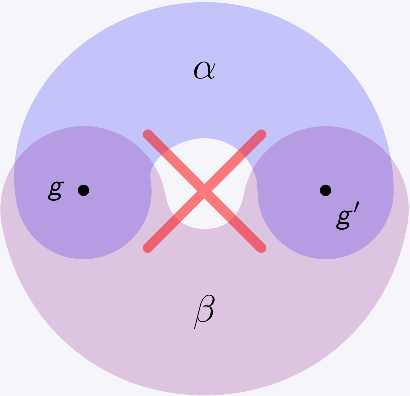


of overlapping hyperedges
without internal shortcuts

NB: walk in $g\langle\alpha\rangle$ $\leftrightsquigarrow$ single coset-step in α

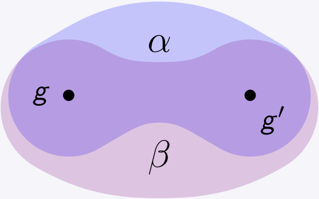
aside: e.g. coset 2-acyclicity

no coset 2-cycles:



$\boldsymbol{g'} \in \boldsymbol{g}\langle\boldsymbol{\alpha}\rangle^{\mathbb{G}} \cap \boldsymbol{g}\langle\boldsymbol{\beta}\rangle^{\mathbb{G}}$

implies $\boldsymbol{g'} \in \langle\boldsymbol{\alpha} \cap \boldsymbol{\beta}\rangle^{\mathbb{G}}$



cycles in groups and groupoids

motivation for coset acyclicity:

transitions in graph-like structures are memory-less — vertex-to-vertex

transitions in hypergraphs preserve elements in overlap — hyperedge-to-hyperedge

$\rightsquigarrow$ different generators (overlaps) fix same element and form cosets of related group elements

coset cycles of $\mathbb{G}$ can be seen as hypergraph cycles in

Cayley hypergraph

a hypergraph dual of $\mathbb{G}$ with cosets as elements

. . . and analogously for groupoids

groupoid: many-sorted with partial, sort-dependent operation

instead of arbitrary generator words $w \in E^*$:
walks w in fixed directed graph $\mathbb{I} = (S, E)$

$\rightsquigarrow$ elements of sorts $G[s, s']$ for $s, s' \in S$
concatenation/products in matching sites/sorts

$\mathbb{G} = (G, (G[s, s']), \cdot, (1_s), (e))$ $\qquad$ $w \in \mathbb{I}[s, s] \longmapsto w^{\mathbb{G}} \in G[s, s']$

motivation: distinct extensions/operations in different sites
e.g. in hypergraph coverings

Cayley groups and groupoids (summary)

Cayley group/groupoid

with generators $e \in E$ over $\mathbb{I} = (S, E)$ $\qquad w \in \mathbb{I}^* \longmapsto w^{\mathbb{G}} \in G$

with **Cayley graph**

$\mathbb{G} = (G, (R_e)_{e \in E})$, $R_e = \{(g, ge) \colon g \in G\}$ $\qquad w \in \mathbb{I}^*$: walks in $\mathbb{G}$

and **Cayley hypergraph of cosets**

$g\langle\alpha\rangle^{\mathbb{G}}$ for $g \in G$, $\alpha \subseteq E$ $\qquad w \in \mathbb{I}^*(\alpha)$: walks preserving $\bigcap_\alpha \cdot$

. . . hopefully avoiding short generator or even coset cycles

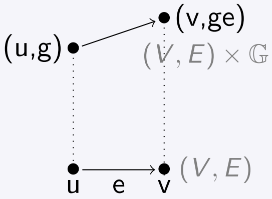
(III) finite cover constructions

use (reduced) products with the right kind of group(oid):

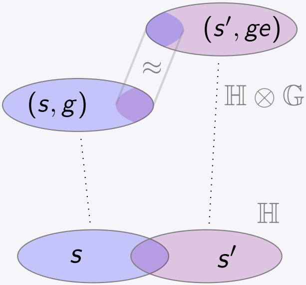
- tree unfolding of graph (V, E) as a weak subgraph of direct product with free Cayley group with generator set E
- finite graph coverings with local acyclicity inherited from Cayley group of **large girth**
- (finite) graph coverings with local acyclicity even w.r.t. some specific transitive closures from Cayley group w/o short **coset** cycles
- finite hypergraph coverings with local acyclicity inherited from Cayley group**oid** w/o short **coset** cycles

direct and reduced products: basic idea

finite graph coverings in direct product with Cayley group: large girth $\rightsquigarrow$ local acyclicity



finite hypergraph coverings in reduced product with Cayley groupoid: coset-acyclicity $\rightsquigarrow$ local acyclicity



$e = \{s, s'\}$

(IV) local acyclicity in Cayley group(oid)s

constructions:

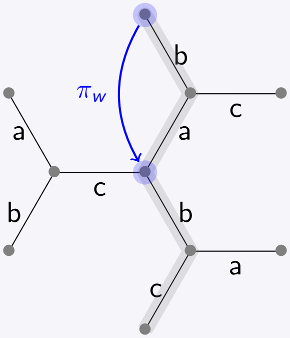
- first shot: groups of large girth (Biggs 89)
- second shot: coset acyclicity in group(oid)s
- new: coset acyclicity for groupoids in groups

Cayley groups of large girth (first shot): basic idea

Biggs' construction, example:

involutive generators a, b, c
find $\mathbb{G}$ of girth > 9

need $w^{\mathbb{G}} \neq 1$ for $|w| \leqslant 9$
e.g. for $w = ababcbacb$



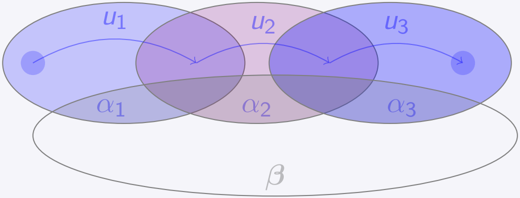
in $\{a, b, c\}$-coloured tree of depth $d = 2$
look at permutations π_e for $e \in \{a, b, c\}$,
where π_e swaps vertices within e-edges

$\mathbb{G} := \langle \pi_a, \pi_b, \pi_c \rangle \subseteq \mathrm{Sym}(V)$ has girth $\geqslant 4d + 2$

coset acylicity (second shot) in Cayley groups

Biggs' idea interleaved with amalgamation:

π_e, π_w act on amalgamated cosets (unfolded coset cycles)
e.g., to force $\pi_w \neq \mathrm{id}$ in $\mathrm{Sym}(V)$ for $w = u_1 u_2 u_3$, $u_i \in \alpha_i^*$



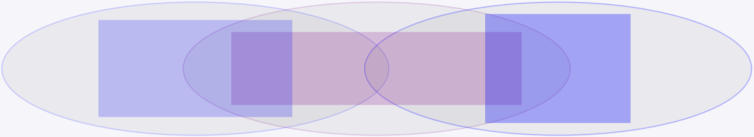
challenge: upgraded $\mathbb{G} \subseteq \mathrm{Sym}(V)$ must not mess up these $\langle \alpha_i \rangle$
by induction w.r.t. size can avoid new $\beta \cap \alpha_i$

acylicity in Cayley groupoids: old and new

- (2012 . . .): adaptation of amalgamation idea for groups
  to more challenging setting of groupoids
- **new:** ramified acyclicity condition for groups can cover
  interpretation of groupoid patterns in group

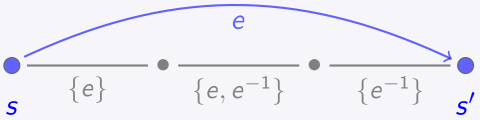
instead of all walks $w \in \alpha^*$ in cosets $\langle\alpha\rangle$:

- walks w.r.t. given groupoid pattern $\mathbb{I} = (S, E)$ (reg. constr.)
- focus on corresponding weak subgraphs of the cosets $\langle\alpha\rangle$
- eliminate small cyclic configurations among those



interpretation of groupoids inside groups

- can interpret directed (groupoid) edges of $\mathbb{I} = (S, E)$ as paths of undirected (involutive, group) edges in E''



- extract $\mathbb{I}$-groupoid $\mathbb{G}$ from E''-group $\mathbb{G}''$

  get coset acyclicity in groupoid $\mathbb{G}$
  from $\mathbb{I}$-coset acyclicity in group $\mathbb{G}''$

theorems

get constructions of finite group(oid)s for given E and N that are

- coset N-acyclic
- compatible with given finite (hyper)graph
- generic in respecting all symmetries of the given data

major applications:

- finite graph and hypergraph coverings that are locally acyclic
- extensions of local to global symmetries:
  strengthening Hrushovski, Herwig–Lascar EPPA thms
- Cayley structures as $\sim$-generic common knowledge models:
  characterisation thms (with Felix Canavoi)
- constructions in semi-group theory:
  proof of Henckell–Rhodes conjecture (Julian Bitterlich)

references

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