

# Addendum to ‘Effective metastability of Halpern iterates in CAT(0) spaces’ (Adv.Math. 231 (2012) 2526-2556)

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## Abstract

We show that one proposition in our original paper allows for a much simpler proof resulting in certain improvements of the bounds in the main theorems.

Without being noticed by us, the quantitative analysis of Saejung’s proof [3] (using Banach limits) given in [1] actually succeeded in eliminating any contribution of the use of Banach limits altogether so that even the finitary residuum of the latter determined in section 8 is not needed. This is due to the fact that Proposition 9.1 (iii) easily follows from Proposition 9.1(i) and the first inequality in the proof of Proposition 9.1(ii) leading to the simpler rate  $\psi(\varepsilon, t, M, \varphi) := \varphi(t\varepsilon/3M)$ .

As a consequence of this, the bounds in Theorem 4.2 and in Corollary 4.4 can be improved by simplifying  $\chi_k^*(\varepsilon)$  to  $\chi_k^*(\varepsilon) := \Phi(\varepsilon/3M(k+1))$  (in Theorem 4.2) and to

$$\chi_k^*(\varepsilon) := \left\lceil \frac{12M^2(k+1)}{\varepsilon} + \frac{144M^4(k+1)^2}{\varepsilon^2} \right\rceil - 1$$

(in Corollary 4.4). Note that  $\tilde{\Phi}$  is still needed as it enters the definition of  $\Phi$  but that the term  $\tilde{P}_k$  has been essentially removed.

Completely analogous improvements apply to the main results in [2, 4] (again one may take  $\chi_k^*(\varepsilon) := \Phi(\varepsilon/3M(k+1))$  in [2](Theorem 3.2) and [4](Theorems 4.1 and 4.2).

While our arithmetization of Saejung’s proof (replacing the limit  $z$  of the resolvent  $(z_t^u)$  just by the sequence elements themselves) here had the consequence to eliminate the use of the Banach limit completely, we still expect in the general case that the finite residuum of the latter expressed in Lemmas 8.3 and 8.4 will be needed.

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## References

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