WASEDA University

TECHNISCHE UNIVERSITÄT DARMSTADT

THE UNIVERSITY OF TOKYO

International Research Training Group 1529

Mathematical Fluid Dynamics

Conference on Mathematical Fluid Dynamics

Bad Boll, Germany
May 7-11, 2018

Main Speakers

Peter Constantin
Eduard Feireisl
Franco Flandoli
Yoshikazu Giga
Herbert Koch
Felix Otto
Gregory Seregin
Yoshihiro Shibata
Edriss Titi
Zhouping Xin

Invited Speakers

Helmut Abels
Herbert Amann
Volker Betz
Dieter Bothe
Raphaël Danchin
Francesco De Anna
Robert Denk
Paul Deuring
Karoline Disser
Herbert Egger
Reinhard Farwig
Tadahisa Funaki
Matthias Geissert
Mi-Ho Giga
Olivier Glass
Bernhard Haak
Horst Heck
Amru Hussein
Pen-Yuan Hsu
Tsukasa Iwabuchi
Yoshiyuki Kagei
Takahito Kashiwabara
Hajime Koba
Takayuki Kobayashi
Matthias Köhne
Peter Korn
Peer Kunstmann
Mads Kyed
Jinkai Li
Chun Liu
Mária Lukáčová
Yasunori Maekawa
Paolo Maremonti
Anna Mazzucato
Hana Mizerová
Sylvie Monniaux
Piotr Mucha
Miho Murata
Šárka Nečasová
Hirofumi Notsu
Wojciech Ożański
Anupam Pal Choudhury
Jan Prüss
Reinhard Racke
Michael Renardy
Elisabetta Rocca
Michael Růžička
Jürgen Saal
Martin Saal
Hirokazu Saito
Jonas Sauer
Okihiro Sawada
Gieri Simonett
Kohei Soga
Florian Steinberg
Ryo Takada
Patrick Tolksdorf
Erika Ushikoshi
Emil Wiedemann
Mathias Wilke
Masahiro Yamamoto
Ruizhao Zi
Martin Ziegler

Organizers:
M. Hieber
H. Kozono

For further information please visit:
http://www.mathematik.tu-darmstadt.de/~igk/badboll2018/
or contact: igk@mathematik.tu-darmstadt.de

Japan Society for the Promotion of Science · Founded 1932

Deutsche Forschungsgemeinschaft DFG

Contents

1 Program

Conference on Mathematical Fluid Dynamics, Bad Boll, May 7 - 11, 2018

Time	Monday		Tuesday		Wednesday		Thursday		Friday
8:55h - 9:00h	Opening								
9:00h - 9:45h	Giga		Constantin		Shibata		Titi		Feireisl
9:45h - 10:15h	*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*
10:15h - 10.45h	Glass		Notsu		Ogawa		Kobayashi		Danchin
10:45h - 11:15h	Kagei		Rocca		Korn		Mazzucato		Yamamoto
11:15h - 11:45h	*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*
11:45h - 12:30h	Short Presentations		Short Presentations		Xin		Otto		Closing
12:30h - 14:00h	*Lunch*		*Lunch*		*Lunch*		*Lunch*		*Lunch*
14:00h - 14:45h	Koch		Flandoli		Seregin		Short Presentations		
14:45h - 15:00h	*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*		
15:00h - 15:30h	Li	Egger	Betz	Heck	Tolksdorf	Zi	Bothe	Wiedemann	
15:30h - 16:00h	Kashiwabara	Soga	Ziegler	Saal, M.	Murata	Hsu	Köhne	Racke	
16:00h - 16:30h	Hussein	Mizerová	Sauer	Takada	Disser	Nečasová	Ożański	Rivera	
16:30h - 16:45h	*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*		
16:45h - 17:15h	Kyed	Deuring	Abels	Denk	Iwabuchi	Sawada	Ulbrich	Lukáčová	
17:15h - 17:45h	De Anna	Nguyen	Saito	Kunstmann	Saal, J.	Okabe	She	Koba	
17:45h - 18:00h	*Coffee Break*		*Coffee Break*		*Coffee Break*		*Coffee Break*		
18:00h - 18:30h	Pal Choudhury	Ushikoshi	Wilke	Giorgini	Růžička		Barker	Steinberg	
18:30h - 19:00h	Simonett		Mucha				Renardy		
19:00h - 20:00h	*Dinner*		*Dinner – Long Night of MFD –*		*Conference Dinner*		*Dinner*		
20:00h - 21:00h			Short Presentations						
21:00h			*Refreshments*						

short presentations				
Time	**Monday**	**Tuesday**	**Thursday**	
11:45 – 12:00	Mathis Gries	Marc Wrona		
12:00 – 12:15	Naoto Kajiwara	Sebastian Zaigler		
12:15 – 12:30	Elisabeth Reichwein	Andreas Schmidt		
14:00 – 14:15			Tobias Tolle	
14:15 – 14:30			Pascal Hobus	
14:30 – 14:45			Keiichi Watanabe	
20:00 – 20:12		Thomas Eiter		
20:12 – 20:24		Klaus Kress		
20:24 – 20:36		Laura Westermann		
20:36 – 20:48		Anton Seyfert		
20:48 – 21:00		Aday Celik		

Monday, 7. May 2018

Time	Speaker	*Title of Talk*
09:00-09:45	Yoshikazu Giga	*Geometric regularity criteria for the Navier-Stokes equations as an application of L^∞-theory*
10:15-10:45	Olivier Glass	*On a solid body immersed in a fluid: asymptotic limits and control*
10:45-11:15	Yoshiyuki Kagei	*Large time behavior of solutions to the compressible Navier-Stokes equations in a cylinder under the slip boundary condition*
11:45-12:00	Mathis Gries	*The hydrostatic Stokes semigroup and primitive equations with Dirichlet boundary conditions on spaces of bounded functions*
12:00-12:15	Naoto Kajiwara	*Well-posedness for the phase-field Navier-Stokes equations in the maximal regularity space*
12:15-12:30	Elisabeth Reichwein	*Wellposedness of the Tornado-Hurricane Equations in L_p-spaces*
14:00-14:45	Herbert Koch	*Renormalization of stochastic nonlinear wave equations*
15:00-15:30	Jinkai Li	*Some mathematical analysis on the dynamical models for atmosphere with moisture*
	Herbert Egger	*Structure preserving numerical approximation of dissipative evolution problems*
15:30-16:00	Takahito Kashiwabara	*Semigroup and maximal regularity approach to the primitive equations*
	Kohei Soga	*On convergence of Chorin's projection method to a Leray-Hopf weak solution*
16:00-16:30	Amru Hussein	*Primitive equations in the scaling invariant space $L^\infty(L^1)$*

	Hana Mizerová	*Numerical solution of a dumbbell-based model for dilute polymer solutions*
16:45-17:15	Mads Kyed	*Occurrence of resonance in a thin elastic structure interacting with a viscous fluid*
	Paul Deuring	*Stability of viscous incompressible flow around an obstacle*
17:15-17:45	Francesco De Anna	*On the non-isothermal dynamics of nematic liquid crystals*
	Thieu Huy Nguyen	*Boundedness and Stability of Certain Fluid Flows on Unbounded Domains*
18:00-18:30	Anupam Pal Choudhury	*Nematic Liquid Crystals in Lipschitz domains*
	Erika Ushikoshi	*Hadamard variational formula for the multiple eigenvalues of the Stokes equations with friction slip boundary conditions*
18:30-19:00	Gieri Simonett	*On the motion of a rigid body with a cavity filled with a viscous liquid*

Tuesday, 8. May 2018

Time	Speaker	*Title of Talk*
09:00-09:45	Peter Constantin	*The surface quasi-geostrophic equation (SQG) in bounded domains*
10:15-10:45	Hirofumi Notsu	*The gradient flow structure of the Maxwell viscoelastic model and a structure-preserving finite element scheme*
10:45-11:15	Elisabetta Rocca	*On a hyperbolic system arising in liquid crystals modeling*
11:45-12:00	Marc Wrona	*Global strong convergence of the scaled Navier Stokes equations to the primitive equations*

12:00-12:15	Sebastian Zaigler	*Regularity structures for the primitive equations*
12:15-12:30	Andreas Schmidt	*Strong solutions of the Navier-Stokes equations with the Coulomb boundary condition*

14:00-14:45	Franco Flandoli	*Distributional solutions of 2D Euler equations*
15:00-15:30	Volker Betz	*Hydrodynamic limits of nearest neighbour particle systems*
	Horst Heck	*Stability for Calderon-type inverse problems*
15:30-16:00	Martin Ziegler	*Computer Science of Numerics*
	Martin Saal	*The primitive equations with only horizontal viscosity*
16:00-16:30	Jonas Sauer	*Parabolic equations with rough coefficients and singular forcing*
	Ryo Takada	*Strongly stratified limit for the 3D inviscid Boussinesq equations*
16:45-17:15	Helmut Abels	*Sharp Interface Limit for Two-Phase Flows of Incompressible Fluids*
	Robert Denk	*Maximal regularity for a fluid-structure interaction model*
17:15-17:45	Hirokazu Saito	*Decay properties of solutions for some linearized system of the Navier-Stokes equations with a free surface*
	Peer Kunstmann	*L^q-Helmholtz decomposition for periodic domains and applications to Navier-Stokes equations*
18:00-18:30	Mathias Wilke	*Rayleigh-Taylor instability for the Verigin problem*
	Andrea Giorgini	*Uniqueness and regularity for a diffuse interface system modeling Hele-Shaw flows*
18:30-19:00	Piotr Bogusław Mucha	*Bi-fluid: existence of weak solutions*

20:00-20:12	Thomas Eiter	*Falling drop in an unbounded liquid reservoir: Steady-state solutions*

20:12-20:24	Klaus Kress	*Strong time-periodic solutions to the bidomain equations with arbitrary large forces*
20:24-20:36	Laura Westermann	*Some interesting behavior of the Stokes operator in L^1 and L^∞ spaces*
20:36-20:48	Anton Seyfert	*The Helmholtz-Hodge Decomposition in Exterior Domains*
20:48-21:00	Aday Celik	*Resonance in nonlinear acoustics*

Wednesday, 9. May 2018

Time	Speaker	*Title of Talk*
09:00-09:45	Yoshihiro Shibata	*On some free bounary problem for the Navier-Stokes equations*
10:15-10:45	Takayoshi Ogawa	*Ill-posedness issue for the compressible Navier-Stokes equation*
10:45-11:15	Peter Korn	*Numerical Modelling of Ocean and Atmosphere - Mathematical, Computational and Experimental Aspects*
11:45-12:30	Zhouping Xin	*On Incompressible Inviscid Resistive MHD Surface Waves in 2D*
14:00-14:45	Gregory Seregin	*Liouville type theorems for Navier-Stokes equation*
15:00-15:30	Patrick Tolksdorf	*Beyond the classical Calderón–Zygmund Theory — An alternative approach to maximal regularity*
	Ruizhao Zi	*Dispersive effect and global well-posedness of the compressible viscoelastic fluids*
15:30-16:00	Miho Murata	*The global well-posedness for the Navier-Stokes-Korteweg system*
	Pen-Yuan Hsu	*Swirling flow of the Navier-Stokes equations near a saddle point and no-slip flat boundary*
16:00-16:30	Karoline Disser	*Extrapolation of maximal regularity for non-autonomous parabolic problems*

	Šárka Nečasová	*Viscous compressible fluids in time dependent domain*
16:45-17:15	Tsukasa Iwabuchi	*Besov spaces on open sets with the Dirichlet boundary condition and their application to the fractional Laplacian*
	Okihiro Sawada	*A real analyticity of solutions to the Navier-Stokes equations around straining flows*
17:15-17:45	Jürgen Saal	*Non-regular diffusion vs. regular fluid flow in a 2D wedge type domain*
	Takahiro Okabe	*Remark on the local solvability of the Navier-Stokes equations in weak L^n-space*
18:00-18:30	Michael Růžička	*On the regularity of the steady p-Stokes problem*

Thursday, 10. May 2018

Time	Speaker	*Title of Talk*
09:00-09:45	Edriss S. Titi	*On Recent Advances of the 3D Euler Equations by Means of Examples*
10:15-10:45	Takayuki Kobayashi	*On local energy decay estimates of solutions to the hyperbolic type Stokes equations*
10:45-11:15	Anna Mazzucato	*On boundary layers for incompressible flows under no-slip boundary condition*
11:45-12:30	Felix Otto	*tba*
14:00-14:15	Tobias Tolle	*Capillary accuracy of a hybrid Level Set / Front Tracking method on unstructured meshes*
14:15-14:30	Pascal Hobus	*Triebel-Lizorkin-Lorentz spaces and the Navier-Stokes equations*

14:30-14:45	Keiichi Watanabe	*Local unique solvability for compressible-incompressible two-phase flows with phase transitions*
15:00-15:30	Dieter Bothe	*A kinematic evolution equation for the dynamic contact angle and some consequences*
	Emil Wiedemann	*Renormalisation of active scalar equations*
15:30-16:00	Matthias Köhne	*On the Analysis of Basic Contact Line Models*
	Reinhard Racke	*Hyperbolic variations in fluid dynamics*
16:00-16:30	Wojciech Ożański	*Partial regularity for a surface growth model*
	Jaime E. Muñoz Rivera	*About the essential type and the exponential stability of C_0 semigroups*
16:45-17:15	Stefan Ulbrich	*Optimal boundary control of entropy solutions for nonlinear hyperbolic balance laws with state constraints*
	Mária Lukáčová-Medvidová	*The role of measure-valued solutions in compressible flows*
17:15-17:45	Bangwei She	*Numerical analysis of finite difference MAC scheme for compressible Navier-Stokes equations*
	Hajime Koba	*On compressible fluid system on an evolving surface with boundaries*
18:00-18:30	Tobias Barker	*Weak-Strong Uniqueness Results for the 3D Navier-Stokes Equations*
	Florian Steinberg	*Verified interval computation in Coq*
18:30-19:00	Michael Renardy	*On controllability of linear viscoelastic flows*

Friday, 11. May 2018

Time	Speaker	*Title of Talk*
09:00-09:45	Eduard Feireisl	*On singular limits for inviscid fluid flows*
10:15-10:45	Raphaël Danchin	*The incompressible Navier-Stokes equations with vacuum*
10:45-11:15	Masahiro Yamamoto	*Inverse problems for compressible viscous fluids*

Sharp Interface Limit for Two-Phase Flows of Incompressible Fluids

Helmut Abels

University of Regensburg

helmut.abels@ur.de

We consider the sharp interface limit of a coupled Stokes/Allen-Cahn system, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain. For sufficiently small times we prove convergence of the solutions of the Stokes/Allen-Cahn system to solutions of a sharp interface model, where the interface evolution is given by the mean curvature equation with an additional convection term coupled to a two-phase Stokes system with an additional contribution to the stress tensor, which describes the capillary stress. To this end we construct a suitable approximation of the solution of the Stokes/Allen-Cahn system, using three levels of the terms in the formally matched asymptotic calculations, and estimate the difference with the aid of a suitable refinement of a spectral estimate of the linearized Allen-Cahn operator. Moreover, we will discuss recent extensions of this results e.g. to the Navier-Stokes/Allen-Cahn system or the Stokes/Cahn-Hilliard system.

Weak-Strong Uniqueness Results for the 3D Navier-Stokes Equations

Tobias Barker

University of Oxford

tobiasbarker5@gmail.com

In the first part of the talk, we provide background into some uniqueness results provided by Leray and others in the spirit 'weak-strong uniqueness'. In the second

part of the talk, we give new classes of initial data that ensure that the associated weak Leray-Hopf solutions of the three-dimensional Navier-Stokes equations (in the whole space) coincide on some time interval.

Hydrodynamic limits of nearest neighbour particle systems

Volker Betz

TU Darmstadt

betz@mathematik.tu-darmstadt.de

I will present the result of two different, but related project. The first, based on the PhD thesis of Stefan Walter, explores hydrodynamic limits for a particular type of models (zero activity spatial random permutations) for one-dimensional surfaces in two-dimensional space. The aim is to derive mean curvature motion for these systems. The results are similar in spirit to those obtained for the Ising droplet at zero temperature, but the resulting hydrodynamic equations are different.

The second, related, topic will be about results obtained in the thesis of Alexander Dalinger during the IRTG. Here, we consider Brownian particles interacting via an attractive nearest neighbour potential. They can be thought of as a model for density fluctuations in an elastic string or medium. We will discuss hydrodynamic limits for one- dimensional and multi-dimensional systems, and fluctuations for one-dimensional systems.

A kinematic evolution equation for the dynamic contact angle and some consequences

Dieter Bothe[1], Mathis Fricke[1], Matthias Köhne[2]

[1]TU Darmstadt, [2]HHU Düsseldorf

bothe@mma.tu-darmstadt.de

We investigate the moving contact line problem for two-phase incompressible flows by a kinematic approach, where the key idea is to derive an evolution equation for the contact angle assuming the transporting velocity field to be given. The resulting equation expresses the time derivative of the contact angle in terms of the gradients of the fluid velocity and the local mass transfer rate at the solid wall. Combined with the additionally imposed boundary conditions for the velocity and, furthermore, exploiting the interfacial transmission condition for the viscous stress, we derive an explicit form of the contact angle evolution for sufficiently smooth solutions to a large class of models. In the *absence* of phase change it reads

$$\dot{\theta} = \frac{V_\Gamma}{2L},$$

which only involves the contact line velocity and the slip length from the Navier condition. From this equation we can read off the qualitative behavior of the contact angle evolution for this class of models, which turns out to be unphysical. In particular, if the contact angle is modeled as a function of the contact line velocity, the contact angle is monotonically increasing or decreasing during the evolution. We discuss consequences from this observation and possible generalizations of the model.

Resonance in nonlinear acoustics

Aday Celik

Technische Universität Darmstadt

celik@mathematik.tu-darmstadt.de

We investigate acoustic wave propagation in a medium in which both nonlinear and dissipative effects are taken into account. Specifically, we shall address the question whether the dissipative effects are sufficient to avoid resonance. Results based on the Kuznetsov and Blackstock-Crighton models are presented.

The surface quasi-geostrophic equation (SQG) in bounded domains

Peter Constantin

Princeton University

const@math.princeton.edu

We start with a brief background review of SQG. We then describe results on the global existence of regular solutions of critical SQG in bounded domains. The results are based on lower bounds on the fractional Laplacian with Dirichlet boundary conditions and on commutator estimates. Some of these estimates have independent interest. We describe also results on global weak solutions, and the inviscid limit for SQG.

The incompressible Navier-Stokes equations with vacuum

Raphaël Danchin

LAMA, Université Paris-Est Créteil, France

danchin@u-pec.fr

In this joint work [1] with Piotr B. Mucha, we investigate the existence and uniqueness issue for the inhomogeneous incompressible Navier-Stokes equations supplemented with H^1 initial velocity and only bounded nonnegative density. We prove global existence and uniqueness for general two-dimensional data, and in the three-dimensional case, if the velocity satisfies a suitable scaling invariant smallness condition. Let us underline that, in contrast with prior works, no regularity or strict positivity for the density is required, and we do not need compatibility conditions for the data either. As an application, we solve a question that has been raised by P.-L. Lions in [2] concerning the evolution of a drop of incompressible viscous liquid in the vacuum.

[1] R. Danchin and P. B. Mucha: The incompressible Navier-Stokes equations in vacuum, arXiv:1705:06061, to appear in *Communications on Pure and Applied Mathematics*.

[2] P.-L. Lions : *Mathematical topics in fluid mechanics. Vol. 1. Incompressible models.* Oxford Lecture Series in Mathematics and its Applications, **3**, Oxford University Press, 1996.

On the non-isothermal dynamics of nematic liquid crystals

Francesco De Anna

Pennsylvania State University

fzd16@psu.edu

In this talk we address a well-posedness result for a model describing the dynamics of nematic liquid crystals. The main peculiarity of these media is a privileged orientation of the constitutive molecules, we investigate in the director theory proposed by Ericksen and Leslie. We deal with a material subjected to a variable temperature, the entire physics of which is uniquely determined by the formulations of the free energy and the entropy production. This fact can be viewed as a consequence of the unified framework of the energetic variational approach. The well-posedness is then determined within the functional setting of Besov spaces, by means of Fourier-analysis techniques and above all the Littlewood-Paley decomposition.

Maximal regularity for a fluid-structure interaction model

Robert Denk

University of Konstanz, Germany

robert.denk@uni-konstanz.de

We study a one-phase fluid-structure interaction model in $\mathbb{R}^n$ which has the form of a free boundary value problem for the Navier-Stokes equation combined with an elastic response at the boundary. For the linearized problem, we show maximal regularity in L^p-Sobolev spaces with respect to time and space. This is the basis for local-in-time well-posedness of the nonlinear equation. The proof is based on mapping properties of the operator related to the Lopatinksii matrix of the problem. Due to the inhomogeneous symbol structure, standard parabolic results cannot be applied. However, the theory of the Newton polygon can be used to describe the (non-standard) Sobolev spaces related to the Lopatinskii matrix and to prove that the corresponding operator induces an isomorphism in these spaces.

This talk is based on joint work with Jürgen Saal (Düsseldorf).

Stability of viscous incompressible flow around an obstacle.

Paul Deuring

Université du Littoral "Côte d'Opale"

Paul.Deuring@lmpa.univ-littoral.fr

Let (U, Π) be a solution of the stationary incompressible Navier-Stokes system with Oseen term, under Dirichlet boundary conditions and a decay condition at infinity,

$$
\begin{aligned}
&-\Delta U + \tau\, \partial_1 U + \tau\, (U \cdot \nabla) U + \nabla \Pi = F, \ \ \operatorname{div} U = 0 \ \text{ in } \Omega, \\
&U | \partial\Omega = U_0, \ \ U(x) \to 0 \, (|x| \to \infty),
\end{aligned}
$$

where $\Omega \subset \mathbb{R}^3$ is an exterior domain ($\mathbb{R}^3 \backslash \Omega$ compact). We are interested in a condition on the spectrum of the linear operator $\mathfrak{L}(v) := -\Delta v + \tau\, \partial_1 v + \tau\, (U \cdot \nabla) v + \tau\, (v \cdot \nabla) U$, defined for $v \in \mathscr{D}(\mathfrak{L}) := \{ w \in W^{2,2}_{loc}(\Omega)^3 \cap H^6(\Omega)^3 \ : \nabla w, \ D^2 w \ \ L^2\text{-integrable}, \ \operatorname{div} w = 0 \}$. By "stability of U" we mean that for any L^2-strong solution (u, π) of the the initial-boundary value problem

$$
\begin{aligned}
&u'(t) - \Delta_x u(t) + \tau\, \partial_{x_1} u(t) + \tau\, \big(u(t) \cdot \nabla \big) U + \tau\, (U \cdot \nabla_x) u(t) + \tau\, \big(u(t) \cdot \nabla_x \big) u(t) \\
&\quad + \nabla \pi(t) = 0, \ \ \operatorname{div}_x u(t) = 0 \quad \text{in } \Omega \times (0, \infty), \\
&u(t) | \partial\Omega = 0, \ \ u(x, t) \to 0 \, (|x| \to \infty), \ \ u(0) = u_0,
\end{aligned}
$$

the relation $\| \nabla u(t) \|_2 \to 0 \ (t \to \infty)$ holds if $\| u_0 \|_{1,2}$ is small. J. Neustupa (JMFM 18 (2016)) showed that stability in this sense holds if $\lambda = 0$ is not a generalized eigenvalue of $\mathfrak{L}$, and if there are numbers δ, $a \in (0, \infty)$ such that

$$
\operatorname{Re} \lambda < \max\{ -\delta, \, -a \, (\Im \lambda)^2 \} \quad \text{for any eigenvalue } \lambda \in \mathbb{C} \backslash \{0\} \text{ of } \mathfrak{L}.
$$

Our aim is to replace that latter condition by the requirement that $\operatorname{Re}\lambda < 0$ for any eigenvalue $\lambda \in \mathbb{C}\setminus\{0\}$ of $\mathfrak{L}$.

Extrapolation of maximal regularity for non-autonomous parabolic problems

Karoline Disser

TU Darmstadt and WIAS Berlin

kdisser@mathematik.tu-darmstadt.de

We consider linear inhomogeneous non-autonomous parabolic PDEs, with discontinuous dependence of time. In particular, we show that for these problems, the property of maximal L^p-regularity can be extrapolated from one exponent p to an open interval of exponents. We consider applications to operators given by sesquilinear forms, to simultaneous extrapolation of spatial regularity, to quasilinear problems and to a time-dependent Stokes problem.
This is joint work with Tom ter Elst (University of Auckland) and Joachim Rehberg (WIAS Berlin).

Structure preserving numerical approximation of dissipative evolution problems

Herbert Egger

TU Darmstadt

egger@mathematik.tu-darmstadt.de

Abstract evolution problems that are equipped with an entropy–dissipation structure are considered. We present a general framework for the systematic construction of numerical approximation schemes for such problems that automatically inherit the underlying entropy–dissipation structure. The versatility of the approach is illustrated by application to several problems governed by nonlinear partial differential equations.

Falling drop in an unbounded liquid reservoir: Steady-state solutions

Thomas Eiter

Technische Universität Darmstadt

eiter@mathematik.tu-darmstadt.de

The equations of motion of a liquid drop in an unbounded liquid reservoir are discussed. The fluid behavior inside and outside the drop is modeled by the Navier–Stokes equations, which are coupled by certain boundary conditions. The unknown free boundary of the drop is described by a height function η on the unit sphere.

One of the boundary conditions includes the mean curvature of the free boundary, which leads to a differential equation for η, whose linearization is given by a resolvent problem for the Laplace–Beltrami operator on the unit sphere. Moreover, the height function η provides a corresponding coordinate transformation, which allows to formulate the equations in a fixed domain and to investigate the problem in a setting of Sobolev-type spaces.

The only prescribed (smallness) parameter is the difference $\rho_1 - \rho_2$ of the densities of the fluids. In particular, the Reynolds number is a free parameter of the problem. To obtain a well-posed linearization, the system is linearized around an appropriate steady state, which leads to an Oseen-type system. An application of the contraction mapping principle then yields a solution for the nonlinear problem when $|\rho_1 - \rho_2|$ is sufficiently small.

On singular limits for inviscid fluid flows

Eduard Feireisl

Institute of Mathematics CAS

feireisl@math.cas.cz

We discuss singular several limits in the context of inviscid fluid flows modeled by the full Euler system. We prosose a new approach to these problems based on generalized (measure valued) solutions. The technique of relative energy is employed. Examples are shown, where the limit sensitively depends on the choice of the initial data.

Distributional solutions of 2D Euler equations

Franco Flandoli

Scuola Normale Superiore of Pisa

franco.flandoli@sns.it

Similarly to what done in recent years for dispersive equations and following previous researches of S. Albeverio and collaborators, we consider the 2D Euler equations for an incompressible inviscid fluid with random initial condition. The measure on initial conditions is Gaussian, associated to the enstrophy and sometimes to the energy; it is supported on a negative order Sobolev space which, talking in terms of the vorticity, is of order $-1-\epsilon$. Hence the degree of distribution is analogous to the one of point vortices; but typical realizations are not signed measure. We prove that, for almost every realization of the initial condition, there exists at least one solution, limit of point vortices and limit of L^∞-solutions. We may extend the result to a stochastic version of 2D Euler equations, with noise of transport type. However, in both the deterministic and stochastic case uniqueness remains open, although in the stochastic case we can prove a gradient type estimate for the solution of the associated Kolmogorov equation, a hint to an additional regularity at the level of laws on function spaces. Open problems and perspectives will be outlined.

Geometric regularity criteria for the Navier-Stokes equations as an application of L^∞-theory

Yoshikazu Giga

The University of Tokyo

labgiga@ms.u-tokyo.ac.jp

Geometric regularity criteria via vorticity direction originally introduced by P. Constantin and C. Fefferman (1993) are discussed. It roughly says that a solution is regular if the vorticity direction is Lipschitz in space uniformly-in-time. In 2011,

H. Miura (2011) and the author gave completely another proof based on the blow-up argument which only requires uniform continuity of vorticity direction in space uniformly-in-time, although the blow-up is assumed to be type I.

It is interesting to generalize this theory for a solution in a domain with the no-slip boundary condition. A half-space case has been established by the author jointly with P. Hsu and Y. Maekawa (2014). It turns out that to discuss a domain with curved boundary one needs L^∞-theory, which has been studied by the author with K. Abe and others based on IRTG project.

In this talk, it is explained why L^∞-theory of the Stokes and the Navier-Stokes equations is useful to extend the geometric regularity criteria to a solution satisfying with the Dirichlet boundary condition in a curved domain. Moreover, several key points L^∞-theory itself is discussed. Several extension of L^∞-theory for the Stokes equations will be also discussed with their applications. This is a kind of survey talk based on the author's activities related to IRTG from 2009 to 2018.

Uniqueness and regularity for a diffuse interface system modeling Hele-Shaw flows

Andrea Giorgini

Università di Pavia

andrea.giorgini@unipv.it

We consider a diffuse interface model describing topological changes of two incompressible and viscous fluids confined between two flat plates separated by an infinitesimally small gap (Hele-Shaw cell). This model is derived from the Navier-Stokes-Cahn-Hilliard system by performing a formal asymptotic expansion. The resulting two-dimensional system consists of a modified Darcy's law for the fluid velocity, which is coupled through the Korteweg force with the Cahn-Hilliard equation, associated to the Ginzburg-Landau free energy with logarithmic potential. Letting the interface thickness go to zero, the related free boundary (or sharp interface) problem for Hele-Shaw flows with surface tension can be formally recovered. In this talk I will discuss the uniqueness of weak solutions, their instantaneous propagation of regularity and the validity of the separation property in both matched and unmatched viscosity cases.

On a solid body immersed in a fluid: asymptotic limits and control

Olivier Glass

Université Paris-Dauphine

glass@ceremade.dauphine.fr

We consider the dynamics of a solid immersed in a perfect incompressible fluid. The fluid is driven by the Euler equation for inviscid incompressible fluids with impermeable boundaries, and the solid is driven by the Newton equations and evolves under the influence of the fluid pressure. We study the system in two directions. First, we consider the possible limits of the system as the solid shrinks to a point. We obtain in the limit various vortex models. Second, we study the possibility of controlling the motion of the body by using suitably chosen boundary conditions on some open part of the boundary.

The hydrostatic Stokes semigroup and primitive equations with Dirichlet boundary conditions on spaces of bounded functions

Mathis Gries

TU Darmstadt

gries@mathematik.tu-darmstadt.de

The primitive equations and its linearized form, called the *hydrostatic Stokes equations*, are presented in spaces with norms of the L^∞-type. This approach yields the existence of strong, global solutions to the primitive equations for initial data lacking any degree of differentiability. The impact of Dirichlet boundary conditions is discussed with an emphasis on semigroup estimates, which are used to construct local solutions.

Stability for Calderon-type inverse problems

Horst Heck

Bern University of Applied Sciences

horst.heck@bfh.ch

In this talk we consider stability issues for Calderon-type inverse problems with partial data. We show, that a reflection technique can be used to improve stability estimates for certain types of underlying domains. In particular, we shall present two recent results on stability estimates. The first result deals with increasing stability in the identification of the potential in the Schrödinger equation. The other result describes log-type stability estimates in the identification of the zeroth order perturbation (the potential) of the biharmonic operator.

The presented results are joint work with Anupam Pal Choudhury.

Triebel-Lizorkin-Lorentz spaces and the Navier-Stokes equations

Pascal Hobus

Heinrich-Heine-Universität Düsseldorf

pascal.hobus@uni-duesseldorf.de

The scale of Triebel-Lizorkin-Lorentz spaces provides a unification of many important function spaces like Lebesgue spaces, Lorentz spaces, Sobolev-Slobodeckii spaces and Bessel-potential spaces. It is therefore natural to ask, whether this scale is suitable in the treatment of partial differential equations, since a corresponding outcome would yield results simultaneously in all spaces listed above. In this talk we present basic properties of Triebel-Lizorkin-Lorentz spaces important in the treatment of PDE. This is finally applied to derive local strong well-posedness for the Navier-Stokes equations on corresponding Triebel-Lizorkin-Lorentz ground spaces as well as maximality of the solution.

Swirling flow of the Navier-Stokes equations near a saddle point and no-slip flat boundary

Pen-Yuan Hsu

The University of Tokyo

pyhsu@ms.u-tokyo.ac.jp

As one of the violent flow, tornadoes occur in many place of the world. In order to reduce the loss of human lives and material damage caused by tornadoes, there are many research methods. One of the effective methods is numerical simulation. The swirling structure is significant both in mathematical analysis and the numerical simulations of tornadoes. In this work we try to clarify the swirling structure. More precisely, we do numerical computations on axi-symmetric Navier-Stokes flows with no-slip flat boundary. We compare a hyperbolic flow with swirl and one without swirl and observe some phenomenons occur only in the swirl case. Our main purpose in this work is to combine the point of view from mathematical analysis (especially regularity results) and numerical approach to observe phenomenons which related to the structure of tornadoes.

Primitive equations in the scaling invariant space $L^\infty(L^1)$

Amru Hussein

TU Darmstadt

hussein@mathematik.tu-darmstadt.de

The primitive equations for ocean and atmospheric dynamics serve as a fundamental model for many geophysical flows. For initial data in H^1 global well-posedness is well-established by the seminal work of Cao and Titi. However, it is not clear how 'rough' initial data are allowed to be. This question is related to the still open question of the uniqueness of weak solutions.

Some results in this direction have been obtained by regularity criteria for weak solutions, following in a certain sense the spirit of Serrin's condition in the theory of the Navier-Stokes equations and methods of weak-strong uniqueness.

Our approach to rough initial data results for the primitive equations is very different: it considers the primitive equation as an evolution equation in an anisotropic function space of the form $L^\infty(\mathbb{R}^2; L^1(J))$. This space is invariant under the scaling

$$v_\lambda(t, x_1, x_2, x_3) = \lambda v(\lambda^2 t, \lambda(x_1, x_2, x_3)), \quad \lambda > 0,$$

and it reflects the anisotropic character of the primitive equations.

The approach depends crucially on mapping properties of the hydrostatic Stokes semigroup, i.e. the semigroup of the linearized primitive equations, in the $L^\infty(L^1)$-setting and can thus be seen as the counterpart of the classical iteration schemes for the Navier-Stokes equations for the situation of the primitive equations. We are able to prove global well-posendess for initial data a of the form $a = a_1 + a_2$, where

$$a_1 \in BUC_\sigma(\mathbb{R}^2; L^1(z_0, z_1)) \quad \text{and} \quad a_2 \in L^\infty_\sigma(\mathbb{R}^2; L^1(z_0, z_1))$$

and where $BUC_\sigma(L^1)$ and $L^\infty_\sigma(L^1)$ denote the space of all solenoidal, bounded uniformly continuous and all solenoidal, bounded functions on $\mathbb{R}^2$, respectively, which take values in $L^1(z_0, z_1)$. Due to the lack of strong continuity of the hydrostatic Stokes semigroup in $L^\infty_\sigma(L^1)$ we must assume $\|a_2\|_{L^\infty_\sigma(L^1)}$ to be sufficiently small.

Besov spaces on open sets with the Dirichlet boundary condition and their application to the fractional Laplacian

Tsukasa Iwabuchi

Tohoku University

t-iwabuchi@m.tohoku.ac.jp

In this talk, we introduce the Besov spaces associated with the Dirichlet Laplacian on an arbitrary open set. Basic properties such as the completeness, the embedding theorem, e.t.c. are shown. As an application, we will study the semigroup generated by the Dirichlet Laplacian of fractional order.

Large time behavior of solutions to the compressible Navier-Stokes equations in a cylinder under the slip boundary condition

Yoshiyuki Kagei

Kyushu University

kagei@math.kyushu-u.ac.jp

In this talk, we consider the compressible Navier-Stokes equations in a cylinder under the slip boundary condition. It is shown that if the initial data is smooth and sufficiently close to the motionless state, then there exists a unique solution globally in time. Furthermore, we show that the large time behavior of the solution is described by a superposition of one-dimensional nonlinear diffusion waves and a diffusive rigid rotation.

Well-posedness for the phase-field Navier-Stokes equations in the maximal regularity space

Naoto Kajiwara

The University of Tokyo

kajiwara@ms.u-tokyo.ac.jp

We study the phase-field Navier-Stokes equations in the maximal regularity space. This equation models the dynamics of vesicle membranes in incompressible viscous fluids (e.g. red blood cells in the body). The description of the membrane is given the terms of a phase field function φ, i.e. $\varphi \approx +1$ inside the vesicle membrane, $\varphi \approx -1$ outside and the thin transition layer of width is characterized by a small parameter ε. We transform the equations into a quasi-linear parabolic evolution equation and use the maximal L_p regularity theory to prove local well-posedness and global well-posedness if the initial data is close to the variational strict stable solution.

Semigroup and maximal regularity approach to the primitive equations

Takahito Kashiwabara

The University of Tokyo

tkashiwa@ms.u-tokyo.ac.jp

We review the results so far obtained for the primitive equations (PEs), which describe large-scale motion of ocean or atmosphere. Most of them are concerned with the global-in-time existence and uniqueness of a strong solution provided by an analytic semigroup approach or by a maximal-regularity theory. In particular, investigation of the linear part of the PEs, i.e., the *hydrostatic Stokes operator*, has a central importance. We will first present the L^p-theory where the strong solution is constructed for initial data belonging to $H^{2/p,p}$. We show that the solution becomes C^∞ (even real analytic) in x and t after initial time. Then the endpoint case $p = \infty$ (more precisely, an anisotropic space $L^\infty_{xy} L^p_z$ will be considered) is discussed, which requires more delicate arguments due to the lack of boundedness in L^∞ of the hydrostatic Helmholtz projector. If time permits, justification of hydrostatic approximation in the L^p-setting, that is, convergence from the Navier–Stokes equations to the PEs in the zero aspect-ratio limit, will also be mentioned.

On compressible fluid system on an evolving surface with boundaries

Hajime Koba

Osaka University

iti@sigmath.es.osaka-u.ac.jp

We consider the governing equations for the motion of compressible fluid on an evolving surface with boundaries from an energetic point of view. We employ both an energetic variational approach and the first law of thermodynamics to make a mathematical model of compressible fluid flow on the evolving surface with boundaries.

We are interested in the boundary conditions for our compressible fluid system. We apply the Gauss divergence theorem on evolving surfaces with boundaries to

study the boundary conditions for our system. We state the boundary conditions in co-normal direction that the system satisfies both conservation and energy laws.

We also study the enthalpy, entropy, and free energy of the fluid on the evolving surface with boundaries, and investigate conservative forms of our compressible fluid system.

On local energy decay estimates of solutions to the hyperbolic type Stokes equations

Takayuki Kobayashi

Osaka University

kobayashi@sigmath.es.osaka-u.ac.jp

In this talk, we discuss a local energy decay estimate of solutions to the initial-boundary value problem for the hyperbolic type Stokes equations of incompressible fluid flow in an exterior domain and a perturbed half space. Their equations are linearized version of the hyperbolic Navier-Stokes equations introduced by Racke and Saal [1, 2], which are obtained as a delayed case for the deformation tensor in the incompressible Navier-Stokes equations. Our proof of the local energy decay estimate is based on Dan and Shibata [3]. In [3], they treated the dissipative wave equations in an exterior domain and discussed the local energy decay estimate. Our approach uses the fact that applying the Helmholtz projection to the hyperbolic type Stokes equations, we obtain a similar equations of the dissipative wave ones. This talk is based on joint works with K. Nakamura (University of Tsukuba) and T. Kubo (University of Tsukuba).

References

[1] R. Racke, J. Saal, Hyperbolic Navier-Stokes equations I: Local well-posedness, Evol. Equ. Control Theory. 1 (2012) 195-215.

[2] R. Racke, J. Saal, Hyperbolic Navier-Stokes equations II: Global existence of small solutions, Evol. Equ. Control Theory. 1 (2012) 217-234.

[3] W. Dan, Y. Shibata, On a local energy decay of solutions of a dissipative wave equation, Funkcial. Ekvac. 38 (1995) 545-568.

Renormalization of stochastic nonlinear wave equations

Herbert Koch

Bonn University

koch@math.uni-bonn.de

In this talk on joint work with M Gubinelli and H. Oh I explain local existence for a renormalized wave equation with white noise forcing in 2 spatial dimensions, with an outlook on the quadratic case in 3d and on global existence in the defocusing cubic case in 2d.

On the Analysis of Basic Contact Line Models

Matthias Köhne

Heinrich-Heine-Universität Düsseldorf

koehne@math.uni-duesseldorf.de

The aim of this talk is to discuss basic models for contact line dynamics and, in particular, the corresponding equilibria, their stability, and the qualitative behavior of smooth solutions. We will consider models in two as well as in three spatial dimensions for an incompressible liquid droplet that is surrounded by another incompressible fluid and that moves on a planar solid wall. The two fluids are assumed to be immiscible, to be separated by a sharp interface, whose intersection with the solid wall forms the contact line, and to be subject to partial/perfect slip boundary conditions on the solid wall.

Numerical Modelling of Ocean and Atmosphere - Mathematical, Computational and Experimental Aspects

Peter Korn

Max Planck Institute for Meteorology

peter.korn@mpimet.mpg.de

In recent years a new generation of general circulation models for the ocean and the atmosphere has been developed. These models contain innovative discretization approaches and are much deeper rooted in Mathematics than previous generations of circulation models. In this talk I describe one of these models that treats the ocean primitive equations in more detail. I investigate the implications of the models design principle on the theoretical analysis of model properties and on the solution of numerical experiments. I conclude with an outlook on generalizing the numerical method to non-hydrostatic and compressible equations.

Strong time-periodic solutions to the bidomain equations with arbitrary large forces

Klaus Kress

Technische Universität Darmstadt

kkress@mathematik.tu-darmstadt.de

In this talk, we consider the bidomain equations which are describing the electrical wave propagation in the heart. First, we construct a weak time-periodic solution to the bidomain equations subject to a large class of models for the ionic transport and corresponding to periodic external forces of arbitrary size. Then, we focus on the ionic model by FitzHugh–Nagumo. We use a a result on global existence of a strong solution to the initial value problem and a weak-strong uniqueness argument to obtain the existence of a strong time-periodic solution for this model.

This talk is based on a joint work with with Prof. Yoshikazu Giga and Naoto Kajiwara (The University of Tokyo).

L^q-Helmholtz decomposition for periodic domains and applications to Navier-Stokes equations

Peer Kunstmann

Karlsruhe Institute of Technology (KIT)

peer.kunstmann@kit.edu

In this talk, we show the existence of the Helmholtz decomposition in $L^q(\Omega)$ for domains $\Omega \subseteq \mathbb{R}^d$ that are invariant under translations in $\mathbb{Z}^d$. The range of q depends on the regularity of the boundary. The proof is based on Bloch multiplier results due to B. Barth. We give applications to Stokes and Navier-Stokes equations on such domains. This is joint work with Jens Babutzka.

Occurrence of resonance in a thin elastic structure interacting with a viscous fluid

Mads Kyed

Technische Universität Darmstadt

kyed@mathematik.tu-darmstadt.de

Resonance may occur in an undamped elastic solid when subject to a periodic forcing term. I will address the question of whether or not interaction with a viscous fluid can provide for a sufficient energy dissipation to prevent occurrence of resonance. Specifically, I will consider the interaction of a thin (two-dimensional) elastic plate with a viscous fluid governed by the Navier-Stokes equations and show that the corresponding hyperbolic-parabolic coupled, free boundary problem has a non-resonant solution regardless of the forcing frequency.

Some mathematical analysis on the dynamical models for atmosphere with moisture

Jinkai Li

The Chinese University of Hong Kong

jklimath@gmail.com

In this talk, we will present some recent mathematical results, mainly the global well-posedness and convergence of the relaxation limit, on two kinds of dynamical models for the atmosphere with moisture. In the first part of this talk, which is a joint work with Edriss S. Titi [1], we will consider a tropical atmosphere model introduced by Frierson, Majda, and Pauluis (Commum. Math. Sci. 2004); for this model, we will present the global well-posedness of strong solutions and the strong convergence of the relaxation limit, as the relaxation time ε tends to zero. It will be shown that, for both the finite-time and instantaneous-relaxation systems, the H^1 regularities on the initial data are sufficient for both the global existence and uniqueness of strong solutions, but slightly more regularities than H^1 are required for both the continuous dependence and strong convergence of the relaxation limit. In the second part of this talk, which is a joint work with Sabine Hittmeir, Rupert Klein, and Edriss S. Titi [2], we will consider a moisture model for warm clouds used by Klein and Majda (Theor. Comput. Fluid Dyn. 2006), where the phase changes are allowed, and we will present the global well-posedness of this system.

[1] Jinkai Li; Edriss S. Titi: *A tropical atmosphere model with moisture: global well-posedness and relaxation limit*, Nonlinearity, 29 (2016), no. 9, 2674–2714.

[2] Sabine Hittmeir; Rupert Klein; Jinkai Li; Edriss S. Titi: *Global well-posedness for passively transported nonlinear moisture dynamics with phase changes*, 30 (2017), no. 10, 3676–3718.

The role of measure-valued solutions in compressible flows

Mária Lukáčová-Medvidová

Johannes-Gutenberg University of Mainz

lukacova@uni-mainz.de

In this talk we discuss the convergence of suitable numerical schemes for both viscous and inviscid compressible flows. A standard paradigm for the existence of solutions in fluid dynamics is based on the construction of sequences of approximate solutions or numerical schemes. However, if the underlying model does not provide enough information for the required regularity of the approximate sequence, we are facing the problem to show the scheme convergence. In particular, for multidimensional problems fine scale oscillations persist, which prevents us to obtain compactness results. Consequently, the standard framework of integrable functions seems not to be appropriate in general. To overcome this problem we introduce the class of *dissipative measure-valued solutions*, which allows us to show the convergence of finite volume or combined finite volume-finite element schemes for multidimensional isentropic Euler and Navier-Stokes equations, respectively. On the other hand, using the weak-strong uniqueness result for the above systems we know, that the dissipative measure-valued solution coincides with the strong solution, if the latter exists. Consequently, our results show convergence of suitable numerical schemes to the strong solutions. These results have been obtained in collaboration with Eduard Feireisl (Academy of Sciences, Prague) and Hana Mizerová (Comenius University, Bratislava).

Global existence of solutions with non-decaying initial data 2D(3D)-Navier-Stokes IBVP in half-plane(space)

Paolo Maremonti

Università degli Studi della Campania "L. Vanvitelli"

paolo.maremonti@unicampania.it

In paper [1], we investigate the Navier-Stokes initial boundary value problem in the half-plane $\mathbb{R}^2_+$ with initial data $u_0 \in L^\infty(\mathbb{R}^2_+) \cap J^2_0(\mathbb{R}^2_+)$ or with non decaying initial data $u_0 \in L^\infty(\mathbb{R}^2_+) \cap J^p_0(\mathbb{R}^2_+)$, $p > 2$. We introduce a technique that allows us to solve the two-dimesional problem, further, but not least, it is also employed to obtain weak solutions, as regards the non decaying initial data, to the three-dimensional Navier-Stokes Cauchy problem and IBVP in the half-space. The two-dimensional result is in the wake of a recent literature (see e.g.[2-4]). Instead, the three-dimensional one, apart from the particular result by Sawada in [5], is the first of its kind.

[1] Maremonti and P. Shimizu S., *Global existence of solutions to 2-D Navier-Stokes flow with non-decaying initial data in half-plane* https://arxiv.org/abs/1801.08411v1

[2] Abe K., *Global well-psedeness of the two-dimensional exterior Navier-Stokes equations for non-decaying data*, Arch. Ration. Mech. Anal. **227** (2018) 69-104.

[3] Giga Y., Matsui S. and Sawada O., *Global existence of two-dimensional navier-Stokes flow with nondecaying initial velocity*, J. math. fluid mech., **3** (2001), 302–315.

[4] Maremonti P. and Shimizu S., *Global existence of solutions to 2-D Navier-Stokes flow with non-decaying initial data in exterior domains*, J. Math. Fluid Mech., (2017) doi.org/10.1007/s00021-017-0348-z.

[5] Sawada O., *A remark on the Navier-Stokes flow with bounded initial data having a special structure*, Hokkaido Math. J., **43** (2014) 1-8.

On boundary layers for incompressible flows under no-slip boundary condition

Anna Mazzucato

Penn State University, University Park, USA

alm24@psu.edu

I will discuss recent results on the analysis of the vanishing viscosity limit for incompressible fluids, that is, whether solutions of the Navier-Stokes equations converge to solutions of the Euler equations, when walls are present. At small viscosity, a viscous boundary layer arises near the walls where large gradients of velocity and vorticity may form and propagate in the bulk (if the boundary layer separates). A rigorous justification of Prandtl approximation, in absence of analyticity or monotonicity of the data, is available essentially only in the linear or weakly linear regime under no-slip boundary condition. I will present in particular a detailed analysis of the boundary layer for an Oseen-type equation (linearization around a steady Euler flow) in general smooth domains.

Numerical solution of a dumbbell-based model for dilute polymer solutions

Hana Mizerová

Comenius University in Bratislava

hana.mizerova@fmph.uniba.sk

We propose a conservative multiscale scheme for the numerical simulation of a high-dimensional Navier-Stokes-Fokker-Planck system for dilute polymer solutions. The incompressible Navier-Stokes equations model the unsteady motion of the Newtonian solvent, while the Fokker-Planck equation describes the evolution of the probability density function of infinitely extensible polymer dumbbell molecules. This leads to a problem of unbounded domain. Our method combines a Lagrange-Galerkin and a Hermite spectral methods together with a space splitting approach. We present the performance of the solver and confirm the discrete conservation of mass by several numerical experiments. Further, for the macroscopic closure of the proposed kinetic model we consider a scheme that is a combination of the method of characteristics and the Brezzi- Pitkäranta stabilization method for the conforming linear elements. This leads to an efficient computation with a small number of degrees of freedom. We show the error estimates and prove optimal convergence orders under mild stability conditions for the nonlinear and linear schemes corresponding to the implicit and semi-implicit time discretization, respectively. Theoretical convergence orders are confirmed by the numerical experiments.

Based on a joint work with B. She, and M. Lukáčová, H. Notsu, M. Tabata.

Bi-fluid: existence of weak solutions

Piotr Bogusław Mucha

University of Warsaw

p.mucha@mimuw.edu.pl

I plan to talk about the issue of existence of weak solutions to a simple model of a bi-fluid. In rough words, we consider a fluid characterized by two densities of ingredients, and the mathematical model is set by two continuity equations and the Stokes law defining a potential velocity in terms of pressure described by two densities of the fluid.

Adapting a brand new tool of compactness developed by D Bresch and PE Jabin we are able to construct solutions, using a novel approach based on the Lagrangian coordinates. What is important to underline, there is no parabolic regularization of continuity equations, and the form of the pressure does not allow to use the standard Lions-Feireisl approach.

The talk based on resutls joint with Didier Bresch and Ewelina Zatorska.

The global well-posedness for the Navier-Stokes-Korteweg system

Miho Murata

Kanagawa University

m-murata@kanagawa-u.ac.jp

In this talk, we consider the compressible fluid model of Korteweg type which was introduced by J. E. Dunn and J. Serrin in 1985. It is shown that the system admits a unique, global strong solution for small initial data in $\mathbb{R}^N$, $N \geq 2$. For the purpose, the main tools are the maximal L_p-L_q regularities and L_p-L_q decay properties to the linearized equations. This talk is based on a joint work with Professor Yoshihiro Shibata in Waseda University.

Viscous compressible fluids in time dependent domain

Šárka Nečasová

Institute of Mathematics, Academy of Sciences of the Czech Republic

matus@math.cas.cz

We consider the compressible Navier-Stokes system on time-dependent domains, supplemented with slip boundary conditions. First the global-in-time existence of weak solutions are obtained, see [1]. For both the no-slip boundary conditions as well as slip boundary conditions we prove local-in-time existence of strong solutions. These results are obtained using a transformation of the problem to a fixed domain and an existence theorem for Navier-Stokes like systems with lower order terms and perturbed boundary conditions. We also show the weak-strong uniqueness principle for slip boundary conditions which remained so far open question, see [2]. Finally, we will consider the full system where the existence of global-in-time weak solutions are obtained [3], [4].

[1] E. Feireisl, O. Kreml, Š. Nečasová, J. Neustupa, J. Stebel: Weak solutions to the barotropic Navier-Stokes system with slip boundary conditions in time dependent domains. *J. Differential Equations* **254** (2013), no. 1, 125–140.

[2] O. Kreml, Š. Nečasová, T. Piasecki: Local existence of strong solutions and weak-strong uniqueness for the compressible Navier-Stokes system on moving domains, Submitted

[3] O. Kreml, S. Mácha, Š. Nečasová, A. Wrb́lewska-Kaminska: Weak solutions to the full Navier-Stokes-Fourier system with slip boundary conditions in time dependent domains. J. Math. Pures Appl. (9) 109 (2018), 67–92

[4] O. Kreml, S. Mácha, Š. Nečasová, A. Wrb́lewska-Kaminska: Flow of heat conducting fluid in a time dependent domain, Submitted

Boundedness and Stability of Certain Fluid Flows on Unbounded Domains

Thieu Huy Nguyen

Hanoi University of Science and Technology

huy.nguyenthieu@hust.edu.vn

In this talk, we present some of our recent results on bounded and polynomial stability of certain fluid flows in unbounded domains such as Navier-Stokes, Navier-Stokes-Oseen flows, Oldroyd-B flows...etc. Our methods rely on interpolation functors for operators satisfying certain smoothing $L^p - L^q$ properties, and on energy estimates for operators on L^2 spaces.

The gradient flow structure of the Maxwell viscoelastic model and a structure-preserving finite element scheme

Hirofumi Notsu

Kanazawa University

notsu@se.kanazawa-u.ac.jp

The Maxwell viscoelastic model is studied from mathematical and numerical points of view. It is shown that the model has a gradient flow structure with respect to a viscoelastic energy. A P1/P0 finite element scheme is presented and its stability in the sense of energy is proved by using a corresponding discrete gradient flow structure. The talk is given based on the following paper.

[1] M. Kimura, H. Notsu, Y. Tanaka and H. Yamamoto. The gradient flow structure of an extended Maxwell viscoelastic model and a structure-preserving finite element scheme. arXiv:1802.05566[math.NA], available from https://arxiv.org/pdf/1802.05566.pdf

Ill-posedness issue for the compressible Navier-Stokes equation

Takayoshi Ogawa

Research Alliance Center of Mathematical Science, Tohoku University

ogawa@math.tohoku.ac.jp

We consider the Cauchy problem of the compressible Navier-Stokes system of barotropic type. Applying the Fujita-Kato principle, it is known that the system is uniquely solvable in the scaling critical Besov spaces under $p < 2n$ (Danchin (2007), (2014), Haspot (2011)), and it is ill-posed when $p > 2n$ (Chen-Miao-Zhang (2015)). We show the strong discontinuity for the solution corresponding integral equation occurs and it is ill-posed in the critical homogeneous Besov space with $p = 2n$. The argument depends on finding an initial data that develops the diverging second order iteration for the related integral equation. This is a joint work with Tsukasa Iwabuchi.

Remark on the local solvability of the Navier-Stokes equations in weak L^n-space

Takahiro Okabe

Hirosaki University

okabe@hirosaki-u.ac.jp

In this talk, we consider the existence and the uniqueness of the local weak mild solutions of the Navier-Stokes equations on the whole space with non-trivial external forces. In the framework of weak Lebesgue spaces, we have a difficulty for the existence and the uniqueness of the local solutions due to the lack of the density of smooth functions and the strong continuity of the Stokes semigroup. However, introducing the subspace where the Stokes semigroup is strongly continuous and controlling the singularity of the forces, we discuss the existence of the local solutions and its uniqueness. Finally, we find some necessary conditions for initial data so that the solution is continuous at $t = 0$.

This article is based on the joint work with Professor Yohei Tsutsui.

tba

Felix Otto

Max-Planck-Institut für Mathematik in den Naturwissenschaften

Felix.Otto@mis.mpg.de

Partial regularity for a surface growth model

Wojciech Ożański

University of Warwick

w.s.ozanski@warwick.ac.uk

Consider a 1D *surface growth model* (SGM),

$$\partial_t u + u_{xxxx} + \partial_{xx} u_x^2 = 0$$

on the torus $\mathbb{T}$. Surprisingly, the mathematical theory of this equation shares a number of striking similarities to the theory of the 3D incompressible Navier–Stokes equations (NSE), including the results regarding existence and uniqueness of solutions. In particular, the issue of the existence of finite-time blow-ups of strong solutions to SGM remains unsolved. However, similarly as in the case of NSE, one can estimate from above the *box-counting dimension* d_B of the putative set $\mathscr{T}$ of blow-up times,

$$d_B(\mathscr{T}) \leq 1/4. \tag{2.1}$$

(The corresponding upper bound in the case of NSE is $1/2$.)

During the talk we will discuss the recently developed partial regularity theory for SGM (joint work with J. Robinson), which is an analogue of the theorem of Caffarelli, Kohn & Nirenberg (1982) for NSE. This theory gives sufficient conditions on local (in space-time) behaviour of a weak solution which guarantee local smoothness. Thus, except for bounding the dimension of singular times (2.1), we are now able to bound the dimension of the space-time singular set

$$S := \{(x,t) \colon u \text{ is not Hölder continuous in any neighbourhood of } (x,t)\}$$

by

$$d_H(S) \leq 1, \qquad d_B(S) \leq 7/6,$$

where d_H denotes the *Hausdorff dimension* and d_B denotes the *box-counting dimension*.

Perhaps surprisingly, the approach of Caffarelli, Kohn & Nirenberg (1982) does not seem well adapted to SGM, and the proof of the partial regularity is based on a different approach and, crucially, uses a certain *nonlinear parabolic Poincaré inequality*, which seems to be a novel tool in this context.

Nematic Liquid Crystals in Lipschitz domains

Anupam Pal Choudhury

Johann Radon Institute for Computational and Applied Mathematics (RICAM), Linz

anupampcmath@gmail.com

In this talk, we shall discuss the simplified Ericksen–Leslie model for nematic liquid crystals in three dimensional bounded Lipschitz domains. Applying a semilinear approach, we shall discuss the proof of local and global well-posedness (assuming a smallness condition on the initial data) in critical spaces for initial data in L^3_σ for the fluid and $W^{1,3}$ for the director field. The analysis of such models, so far, has been restricted to domains with smooth boundaries.
This is based on a joint work with Amru Hussein and Patrick Tolksdorf.

Hyperbolic variations in fluid dynamics

Reinhard Racke

University of Konstanz

reinhard.racke@uni-konstanz.de

We present a *survey* on initial and initial-boundary value problems for systems of pde, where a kind of relaxation introduces a hyperbolic, wave type feature into the linear or nonlinear equations. Motivated from thermoelasticity (strings, plates), the resulting effects are discussed for incompressible and for compressible Navier-Stokes equations. Mainly well-posedness and the asymptotic behavior of solutions will be addressed.

Wellposedness of the Tornado-Hurricane Equations in L_p-spaces

Elisabeth Reichwein

Heinrich-Heine-Universität Düsseldorf

Elisabeth.Reichwein@hhu.de

In this talk, we give a short introduction to the tornado-hurricane equations. Furthermore, we explain our strategy to prove the existence of strong solutions for the tornado-hurricane equations in an L_p-space setting.

On controllability of linear viscoelastic flows

Michael Renardy

Virginia Tech

mrenardy@math.vt.edu

Controllability is the ability to steer a system from a given initial state to a desired final state. In this talk, we consider linear viscoelastic flow in a bounded domain. The control is a body force acting in a subdomain. We consider Maxwell or Jeffreys models with several, possibly infinitely many, relaxation modes. Results on approximate null controllability of the stresses as well as the motion are established. We also show that exact null controllability is not possible.

Nonlinear viscoelastic flows are not controllable. It then becomes a challenging and mostly unsolved problem how to characterize the states to which a flow can be controlled.

About the essential type and the exponential stability of C_0 semigroups

Jaime E. Muñoz Rivera

National Laboratory for Scientific Computation

jemunozrivera@gmail.com

We establish a necessary and sufficient condition that a semigroup, defined over a Banach space, must verify, to be exponentially stable. We make this characterization in terms of *the essential type* of the semigroup which is defined by using the Calkin Algebra. We show applications to linear and semilinear elastic models using this method proving the exponential and the lack of exponential stability.

On a hyperbolic system arising in liquid crystals modeling

Elisabetta Rocca

University of Pavia

elisabetta.rocca@unipv.it

In this talk we present the results contained in the paper [E. Feireisl, E. Rocca, G. Schimperna, A. Zarnescu, On a hyperbolic system arising in liquid crystals modeling, preprint arXiv:1610.07828, 1–22, Journal of Hyperbolic Differential Equations, to appear (2018)].

We consider a model of liquid crystals, based on a nonlinear hyperbolic system of differential equations, that represents an inviscid version of the model proposed by Qian and Sheng. A new concept of *dissipative solution* is proposed, for which a global-in-time existence theorem is shown. The dissipative solutions enjoy the following properties:

(i) they exist globally in time for any finite energy initial data;

(ii) dissipative solutions enjoying certain smoothness are classical solutions;

(iii) a dissipative solution coincides with a strong solution originating from the same initial data as long as the latter exists.

On the regularity of the steady p-Stokes problem

Michael Růžička

Albert-Ludwigs-University Freiburg
rose@mathematik.uni-freiburg.de

In this talk, we discuss the question of full natural regularity up to the boundary of the p-Stokes problem.

Non-regular diffusion vs. regular fluid flow in a 2D wedge type domain

Jürgen Saal

Heinrich-Heine-University Düsseldorf

juergen.saal@hhu.de

We are concerned with the question, whether Laplace and Stokes operator subject to perfect slip on a two dimensional wedge type domain in L^p admit optimal Sobolev regularity, i.e., whether their domains are included in $W^{2,p}$. It is known that this holds true for $1 < p < 1+\delta$ with a small $\delta > 0$, depending on the opening angle of the wedge type domain. The main purpose of this talk is to answer the question, whether the optimal regularity extends to the full range $1 < p < \infty$. We will show that for the Laplacian this does only hold on a suitable subspace, but not for every $p \in (1, \infty)$ on the entire L^p space. On the other hand, for the Stokes operator in the space of solenoidal fields L^p_σ we obtain optimal Sobolev regularity for the full range $1 < p < \infty$. Roughly speaking, this relies on the fact that the "bad" part of L^p is complementary to the space of solenoidal vector fields L^p_σ.

The primitive equations with only horizontal viscosity

Martin Saal

Technische Universität Darmstadt

msaal@mathematik.tu-darmstadt.de

We consider the $3D$ primitive equations with only horizontal viscosity

$$\begin{aligned} \partial_t v + v \cdot \nabla_{\mathrm{H}} v + w \partial_z v - \Delta_{\mathrm{H}} v + \nabla_{\mathrm{H}} p = 0, \\ \partial_z p = 0, \\ \mathrm{div}_{\mathrm{H}} v + \partial_z w = 0 \end{aligned}$$

in $\Omega = (-1,1) \times (-h,h)$ $(h > 0)$ where $v = (v_1, v_2)$ is the horizontal velocity, w the vertical one and p the pressure; by $\Delta_{\mathrm{H}}, \nabla_{\mathrm{H}}$ and $\mathrm{div}_{\mathrm{H}}$ we denote the horizontal Laplacian, gradient and divergence. We consider periodic boundary conditions in the horizontal directions and $w(z = \pm h) = 0$ and show that these equations are well-posed for initial data in $H^2(\Omega)$ without imposing any boundary condition for the horizontal velocity components on the vertical boundaries, thereby extending results by Li and Titi. The first step is to solve a linearised equation by a Galerkin approach, where we treat the term $w v_z$ as a transport term with a coefficient vanishing on the boundary. By adopting the Galerkin approach to the inhomogeneous structure we obtain a solution which is weak only with respect to the horizontal variables. From this result we then deduce the well-posedness of the nonlinear problem. Additionally we comment on extensions of that result to cases with even less viscosity.

Decay properties of solutions for some linearized system of the Navier-Stokes equations with a free surface

Hirokazu Saito

Waseda University

hsaito@aoni.waseda.jp

In this talk, we consider some linearized system of the Navier-Stokes equations with free boundary condition and show decay properties of solutions to the linearized system. The approach is based on techniques developed in the paper of Saito and Shibata (2016). More precisely, we investigate the zero points of the Lopatinski determinant in the Fourier space, and then use the residue theorem in order to derive the decay properties mentioned above.

Parabolic equations with rough coefficients and singular forcing

Jonas Sauer

Max-Planck-Institut für Mathematik in den Naturwissenschaften
sauer@mis.mpg.de

Singular stochastic partial differential equations have a rather short yet intense history, sparked by the development of regularity structures due to Martin Hairer. In the meantime, several alternative approaches to regularity structures have been proposed, in particular the method by Otto-Weber inspired by rough path techniques, which works particularly well for quasi-linear SPDEs but so far can deal only with mildly singular noise.

In my talk, I will discuss recent efforts to place the work of Otto-Weber in an abstract framework close to the theory of regularity structures. This leads to general tools on integration, reconstruction, and multiplication which constitute partial progress towards a general theory of singular SPDEÃćÂĂÂŹs with variable diffusion coefficients. As a first application, we use these tools to establish existence and uniqueness results for rough diffusion equations driven by a more singular forcing than in Otto-Weber.

This is joint work with Felix Otto, Scott Smith, and Hendrik Weber.

A real analyticity of solutions to the Navier-Stokes equations around straining flows

Okihiro Sawada

Gifu University

okihiro@gifu-u.ac.jp

The Cauchy problem of the incompressible Navier-Stokes equations with linearly growing initial velocity (given by $Mx + u_0(x)$ with a constant trace-free matrix M) is investigated. When $u_0 \in L^p_\sigma(\mathbb{R}^n)$ for $p \geq n$, the locally-in-time well-posedness was proved, using the smoothing properties of the Ornstein-Uhlenbeck semigroup. In addition, for pure rotating flows (skew-symmetric M), it has been known that the mild solutions are real analytic in spacial variables. In this talk, the condition of M is relaxed for general cases, including the straining flows. The radius of convergence of Taylor series is estimated from below, via calculations for higher-order derivatives. This is a joint work with Seiya Hattori.

Strong solutions of the Navier-Stokes equations with the Coulomb boundary condition

Andreas Schmidt

Technische Universität Darmstadt

aschmidt@mathematik.tu-darmstadt.de

We will consider the Navier-Stokes equations with the Coulomb friction law boundary condition. This condition means that the solution can slip at the boundary once the tangential stress exceeds a given threshold. If that is the case, the tangential component of the solution has the opposite direction of the tangential stress. We will discuss the existence of strong solutions $u \in W^1_2(0,T;L^2(\Omega)) \cap L^2(0,T;H^1(\Omega))$ in a bounded domain Ω.

Liouville type theorems for Navier-Stokes equation

Gregory Seregin

University of Oxford

seregin@maths.ox.ac.uk

Various Liouville type theorems for the Navier-Stokes equations will be stated in the talk. Some of them can be proved but others are still conjectures.

The Helmholtz-Hodge Decomposition in Exterior Domains

Anton Seyfert

Technische Universität Darmstadt

seyfert@mathematik.tu-darmstadt.de

We consider the Helmholtz-Hodge decomposition of vector fields in Lebesgue spaces $L^p(\Omega)$ on exterior domains $\Omega \subset \mathbb{R}^3$. In the case of smooth bounded domains it is known that this decomposition exists and is unique for any $p \in (1, \infty)$. This changes in the case of exterior domains. Depending on the prescribed boundary conditions of the summands of the decomposition and the parameter p, the decomposition might not be unique anymore or even fail at all. These phenomena are strongly related to the well-posedness of the weak Dirichlet problem in exterior domains.

Numerical analysis of finite difference MAC scheme for compressible Navier-Stokes equations

Bangwei She

Institute of Mathematics, Czech Academy of Sciences

she@math.cas.cz

We propose a new finite difference MAC scheme based on the staggered mesh for the compressible Navier-Stokes equations. The scheme is designed for both Dirichlet and periodic boundary conditions. The key features are the application of upwind flux and artificial diffusion. We show that the scheme is entropy stable and convergent to the dissipative measure-valued solution of the system. Using the argument of weak-strong uniqueness [E. Feireisl, P. Gwiazda, A. Świerczewska-Gwiazda, and E. Wiedemann. Dissipative measure-valued solutions to the compressible Navier-Stokes system. *Calc. Var.*, 55–141, 2016.], we further conclude that the numerical solution convergences to the strong solution as long as the latter exists on a periodic domain.

On some free bounary problem for the Navier-Stokes equations

Yoshihiro Shibata

Waseda University

yshibata325@gmail.com

I will talk about the maximal L_p-L_q regularity for the Stokes equations with free boundary conditions. And as an application, I will talk about the local and global well-posedness of free boundary problem for the Navier-Stokes equations with surface tension. The key tool to prove the maximal L_p-L_q regularity is the $\mathscr{R}$ bounded solution operators to the generalized resolvent problem for the Stokes equations with free boundary conditions. The reference domain is a uniformly C^3 one and the main assumption is that the weak Dirichle problem is uniquely solvable.

On the motion of a rigid body with a cavity filled with a viscous liquid

Gieri Simonett

Vanderbilt University

gieri.simonett@vanderbilt.edu

In this talk, I will consider the motion of a rigid body with an interior cavity that is completely filled with a viscous fluid, in the absence of exterior forces. The equilibria of the system will be characterized and their stability properties are analyzed. It will be shown that equilibria associated with the largest moment of inertia are normally stable, while all other equilibria are normally hyperbolic.

From the work of Disser, Galdi, Mazzone, and Zunino follows that every weak Leray-Hopf solution for this system becomes strong at some time and then persists as a strong L_2-solution. Moreover, it also follows that the relative velocity goes to zero. Using recent work by the author, Prüss, Wilke, and Zacher, we present a rather short and direct proof that any of these solutions converges to an equilibrium at an exponential rate. The result reflects the well-known stabilizing effect that the viscous fluid exerts on the motion of the rigid body.

In addition, we will characterize the critical spaces for the governing evolution equation, and we will show how parabolic regularization in time-weighted spaces affords great flexibility in establishing regularity and stability properties for the system. Our approach is based on the theory of L_p-L_q maximal regularity. (Joint work with G. Mazzone and J. Prüss.)

On convergence of Chorin's projection method to a Leray-Hopf weak solution

Kohei Soga

Keio University

soga@math.keio.ac.jp

Finite difference approximation is an effective approach to prove existence of Leray-Hopf weak solutions to the incompressible Navier-Stokes equations on a bounded domain. Ladyzhenskaya demonstrates a lot about this issue in her book. Then, Chorin ('69) shows the so-called Chorin's projection method, where he incorporates a discrete version of Helmholtz-Hodge decomposition with a finite difference method. Chorin proves convergence of his method, assuming existence of a C^5-solution. Temam ('69) further develops discrete approximation of a Leray-Hopf weak solution in the framework of a finite element method.

In this talk, I slightly modify Chorin's original finite difference scheme, and prove that the modified scheme converges to a Leray-Hopf weak solution in a L^2-framework. Through this discussion, I also show a new simple method to prove strong L^2-convergence from weak L^2-convergence, which works effectively with a discrete divergence-free condition.

Verified interval computation in Coq

Florian Steinberg

INRIA Sophia Antipolis

florian.steinberg@inria.fr

In numerical computations where reliability of results is crucial, interval arithmetics can be used to avoid the uncontrolled error propagation of floating point computations. The proof assistant Coq provides a library for inverval computation. This means that each interval enclosure comes with a machine checked proof that this enclosure holds. The talk gives a brief overview over what the current state of interval arithmetics executable in the proof assistant Coq is.

Strongly stratified limit for the 3D inviscid Boussinesq equations

Ryo Takada

Faculty of Mathematics, Kyushu University

takada@math.kyushu-u.ac.jp

We consider the initial value problem for the 3D inviscid Boussinesq equations with the effect of stable stratification:

$$\begin{cases} \partial_t v + (v \cdot \nabla) v = -\nabla q + \theta e_3 & t > 0,\ x \in \mathbb{R}^3, \\ \partial_t \theta + (v \cdot \nabla)\theta = -N^2 v_3 & t > 0,\ x \in \mathbb{R}^3, \\ \nabla \cdot v = 0 & t \geqslant 0,\ x \in \mathbb{R}^3, \\ v(0,x) = v_0(x), \quad \theta(0,x) = \theta_0(x) & x \in \mathbb{R}^3. \end{cases} \tag{B_N}$$

Here, $N > 0$ is the Brunt-Väisälä (buoyancy) frequency for the constant stratification. The system (B_N) is derived from the original Boussinesq equations by considering the perturbation about a mean state $(v, \theta) = (0, N^2 x_3)$ in hydrostatic balance.

In this talk, we consider the singular limit as $N \to \infty$, and show that the classical solution v^N of (B_N) converges to that of the 2D Euler equations. The key ingredients of the proof are to introduce the modified linear dispersive solutions, and the inhomogeneous Strichartz estimates for the linear propagator related to the stable stratification.

On Recent Advances of the 3D Euler Equations by Means of Examples

Edriss S. Titi

Texas A&M University

and

The Weizmann Institute of Science

titi@math.tamu.edu

In this talk we will use a basic example of shear flow to demonstrate some of the recent advances in the three-dimensional Euler equations. Specifically, this example was introduced by DiPerna and Majda to show that weak limit of classical solutions of Euler equations may, in some cases, fail to be a weak solution of Euler equations. We use this shear flow example to provide non-generic, yet nontrivial, examples concerning the immediate loss of smoothness and ill-posedness of solutions of the three-dimensional Euler equations, for initial data that do not belong to $C^{1,\alpha}$. Moreover, we show by means of this shear flow example the existence of weak solutions for the three-dimensional Euler equations with vorticity that is having a nontrivial density concentrated on non-smooth surface (vortex sheet). This is very different from what has been proven for the two-dimensional Kelvin-Helmholtz (Birkhoff-Rott) problem where a minimal regularity implies the real analyticity of the interface. Furthermore, we use this shear flow to provide explicit examples of non-regular solutions of the three-dimensional Euler equations that conserve the energy, an issue which is related to the Onsager conjecture. Eventually, we will discuss the recent remarkable work of De Lellis and Székelyhidi concerning the wild weak solutions of Euler equations and their non-uniqueness. In particular, we propose the following ruling out criterion for non-physical weak solutions of Euler equations: "Any weak solution which is not a vanishing viscosity limit of weak solutions of the Navier-Stokes equations should be ruled out". We will use this shear flow, and other solutions of Euler equations with certain spatial symmetry, to provide nontrivial examples for the use of this ruling out criterion.

This is a joint work with Claude Bardos.

Beyond the classical Calderón–Zygmund Theory — An alternative approach to maximal regularity

Patrick Tolksdorf

Technische Universität Darmstadt

tolksdorf@mathematik.tu-darmstadt.de

One way to obtain maximal L^p-regularity of elliptic operators or the Stokes operator on L^q-spaces is to employ techniques of classical Harmonic Analysis like Mikhlin's theorem to the corresponding resolvent problem on the half-space followed by a localization procedure. These classical results always ensure the validity of appropriate estimates on the whole L^q-scale where $1 < q < \infty$. Many interesting applications, however, involve systems of equations, rough coefficients of elliptic operators, and/or rough boundaries of the underlying domains. In these situations, it is already known since quite a while, that the maximal L^p-regularity property on the ground space L^q is in general wrong for special values of $1 < q < \infty$. Thus, it is impossible to invoke these classical theorems of Harmonic Analysis in order to investigate the underlying equations for maximal L^p-regularity.

In this talk, I will present one q-sensitive result from Harmonic Analysis due to Zhongwei Shen, which is in some sense a modern version of the classical Calderón–Zygmund theorem and I will show, how one can exploit this theorem in order to derive the maximal L^p-regularity property for operators of the type described above.

Capillary accuracy of a hybrid Level Set / Front Tracking method on unstructured meshes

Tobias Tolle

Technical University of Darmstadt

tolle@mathematik.tu-darmstadt.de

Advances in the modeling of surface tension allow the simulation of two-phase flows characterized by increasingly smaller capillary numbers. Yet, non-physical velocities caused by numerical errors still impose limits on configurations dominated by surface tension that are close to equilibrium state. To evaluate the accuracy of a numerical two-phase method with regard to capillary flows, the translating droplet has been suggested. In this case a spherical droplet, for which the Laplace equilibrium holds, is advected by a constant, uniform velocity field. Thus, the deviation from this flow field is directly related to the quality of the interface advection and surface tension modeling.

In this talk, recent results for this test case obtained with a method we call LENT are presented. LENT is a hybrid Level Set / Front Tracking method which uses a finite volume discretization on unstructured meshes. The focus of the talk is on the influence of different combinations of algorithms related to interface advection and surface tension, namely the velocity interpolation scheme, the phase indicator model and the curvature approximation.

This talk is based on joint work with Dr.-Ing. Tomislav Maric and Prof. Dieter Bothe from the *Mathematical Modelling and Analysis* group, TU Darmstadt.

Optimal boundary control of entropy solutions for nonlinear hyperbolic balance laws with state constraints

Stefan Ulbrich

TU Darmstadt

ulbrich@mathematik.tu-darmstadt.de

In this talk we consider the treatment of state constraints in the context of optimal boundary control of nonlinear hyperbolic scalar balance laws. The problem is motivated by the optimal control of hyperbolic PDE networks, e.g. water, gas or traffic networks. We study initial and boundary controls that switch between continuously differentiable functions, where the switching times as well as these functions serve as controls. The appearence of state constraints presents a challenge, since entropy solution my develop shock discontiuities and thus the control-to-state operator is not continuous to L^{∞}. We present a sensitivity and adjoint calculus that shows the differentiability of shock locations and of the smooth solution parts at the observation time under generic conditions. This enables us to prove Robinson's Constraint Qualification after a transformation of the state and to derive first order necessary optimality conditions, which can also be stated in terms of the original state. Then we show that Moreau-Yosida type regularization yields a differentiable optimal control problem and that the obtained optimale controls of the regularized problems converge strongly and the corresponding Lagrange multiplier estimates weak* in the space of regular Borel measures to the Lagrange multiplier of the original state constrained problem as the regularization parameter tends to zero. We briefly discuss the convergence of numerical discretizations as well as some extensions.

Hadamard variational formula for the multiple eigenvalues of the Stokes equations with friction slip boundary conditions

Erika Ushikoshi

Yokohama National University

ushikoshi-erika-ng@ynu.ac.jp

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with a smooth boundary $\partial\Omega$. For any $\varepsilon \geq 0$ and for any $\rho \in C^\infty(\partial\Omega)$, we define a perturbed domain by Ω_ε, whose boundary is $\partial\Omega_\varepsilon := \{x + \varepsilon\rho(y)\nu_y \ ; \ y \in \partial\Omega\}$. Under such a smooth perturbation, we consider the eigenvalue problem of the Stokes equations with the slip friction boundary condition;

$$\begin{cases} \Delta\boldsymbol{\Phi}_\varepsilon + \lambda(\varepsilon)\boldsymbol{\Phi}_\varepsilon = \nabla q_\varepsilon & \text{in } \Omega_\varepsilon, \\ \operatorname{div}\boldsymbol{\Phi}_\varepsilon = 0 & \text{in } \Omega_\varepsilon, \\ (2e(\boldsymbol{\Phi}_\varepsilon)\nu_x + \kappa\boldsymbol{\Phi}_\varepsilon)_\tau = 0 & \text{on } \partial\Omega, \end{cases} \tag{2.2}$$

where $\boldsymbol{\Phi}_\varepsilon = (\Phi_\varepsilon^1(x), \Phi_\varepsilon^2(x), \Phi_\varepsilon^3(x))$ and $q = q(x)$ is unknown velocity fields and pressure at $x \in \Omega_\varepsilon$, respectively. Moreover, κ is a positive friction constant, and $e(\boldsymbol{u}) = \{e_{ij}(\boldsymbol{u})\}_{1\leq i,j\leq 3}$ is the strain tensor defined by

$$e_{ij}(\boldsymbol{u}) := \frac{1}{2}\left(\frac{\partial u^i}{\partial y^j} + \frac{\partial u^j}{\partial y^i}\right), \quad i, j = 1, 2, 3. \tag{2.3}$$

We konw that the spectral set of (2.2) consists of the descrete sequence of positive real numbers. Then, if we arrange them with counting multiplicity, we have that

$$0 < \lambda_1(\varepsilon) \leq \lambda_2(\varepsilon) \leq \cdots \leq \lambda_k(\varepsilon) \leq \cdots \to \infty,$$

where the k-th eigenvalue is denoted by $\lambda_k(\varepsilon)$. In this talk, we consider the ε dependency of the eigenvalue $\lambda(\varepsilon)$. More precisely, we shall establish Hadamard variational formula for the multiple eigenbalue of (2.2). This is a joint work with Professor Shuichi Jimbo in Hokkaido University.

Local unique solvability for compressible-incompressible two-phase flows with phase transitions

Keiichi Watanabe

Waseda University

keiichi-watanabe@akane.waseda.jp

We consider the free boundary problem for compressible-incompressible two-phase flows with phase transitions in a general domain (namely, the boundedness of boundaries are not required). Two fluids are separated by a sharp interface and a surface tension is taken into account. The free surface is parameterized over the sphere by means of a height function. We use the Navier-Stokes-Korteweg equation for the compressible fluid and the Navier-Stokes equation for the incompressible fluid, respectively, whose model is consistent with thermodynamics. We show that for given $T > 0$ the problem admits a unique strong solution on $(0, T)$ provided the initial data are suitably small in their natural norms.

References:

[1] K. Watanabe, Compressible-incompressible two-phase flows with phase transition: model problem, to appear in *J. Math. Fluid Mech*.

[2] K. Watanabe, Strong solutions for compressible-incompressible two-phase flows with phase transitions, submitted.

Some interesting behavior of the Stokes operator in L^1 and L^∞ spaces

Laura Westermann

Heinrich-Heine-Universität Düsseldorf

Laura.Westermann@uni-duesseldorf.de

The L^q-Theory, $1 < q < \infty$, for the Stokes operator is well-established, starting from the pioneering works of Solonnikov and Giga in the seventies and eighties. In the limit values $p = 1$ and $p = \infty$, however, the situation is much more delicate and can even be pretty surprising. In fact, in contrast to the situation for $1 < q < \infty$,

the existence of a Stokes semigroup in L^1 and L^∞, and hence well-posedness of the Stokes equations, substantially depends on the geometry of the domain under consideration, on the imposed boundary conditions, and on the space dimension. In his PhD thesis L. von Below proves the positive or negative result of the generation of a semigroup for the Stokes operator in L^1 and L^∞ in layer domains subject to Dirichlet boundary conditions. The purpose of my talk is to present some new results in this direction.

Renormalisation of active scalar equations

Emil Wiedemann

Leibniz Universität Hannover

wiedemann@ifam.uni-hannover.de

In analogy to the famous (now resolved) conjecture of Onsager for the Euler equations, one may ask what regularity a weak solution of an active scalar equation needs to have in order to guarantee the conservation of L^p norms. While the case $p = 2$ can be handled by the classical commutator method of Constantin-E-Titi, other exponents require novel ideas. Joint work with I. Akramov.

Rayleigh-Taylor instability for the Verigin problem

Mathias Wilke

University of Regensburg

mathias.wilke@ur.de

In this talk, we consider isothermal incompressible two-phase flows in a capillary modeled with and without phase transition in the presence of gravity, employing Darcy's law for the velocity field. The problems are well-posed in an L_p-setting and generate local semiflows in the proper state manifolds. The main result concerns the Rayleigh-Taylor instability of equilibria with flat interface.
(joint work with J. Prüss and G. Simonett)

Global strong convergence of the scaled Navier Stokes equations to the primitive equations

Marc Wrona

Technische Universtität Darmstadt

wrona@mathematik.tu-darmstadt.de

The primitive equations for the ocean and atmosphere are derived from the scaled Navier Stokes equations by taking the aspect ratio formally to zero, also known as *the hydrostatic approximation*. In this talk, a mathematical justification for this approximation is given, namely the strong convergence of the above mentioned limit. More precisely for initial data in $B^{2-2/p}_{q,p}$, $1/q + 1/p \lesssim 1$, global convergence of the horizontal velocity of the scaled Navier Stokes equations to the horizontal velocity of the primitive equations with respect to the maximal regularity norm is shown, generalizing Li and Titi's strong convergence result in [1] where $p = q = 2$ and stronger assumptions upon the initial data are imposed. To this end a *maximal regularity* estimate for the difference equations and the *boundedness of the vertical velocity* of the primitive equations with respect to the maximal regularity norm are established. This is a joint work with Ken Furukawa, Yoshikazu Giga, Matthias Hieber, Amru Hussein and Takahito Kashiwabara.

[1] J. Li and E.S. Titi, *The primitive equations as the small aspect ratio limit of the Navier Stokes equations: rigorous justification of the hydrostatic approximation*, Journal de Mathématiques Pures et Appliquées, (2018), (to appear). arXiv:1706.08885[math.AP]

On Incompressible Inviscid Resistive MHD Surface Waves in 2D

Zhouping Xin

The Institute of Mathematical Sciences
The Chinese University of Hong Kong

zpxin@ims.cuhk.edu.hk

In this talk I will present some studies on the effects of the magnetic diffusions on the motions of an ideal free Surface wave for 2-dimensional incompressible electrically conducting fluids. Such a problem is governed by the inviscid and resistive MHD system, and the effects of the surface tension on the free boundary are considered. We find a strong dissipation for the fluid vorticity due to the transversal magnetic field and thus are able to establish the global well-posedness of the free surface wave motion around an equilibrium. Some of the key ideas of the analysis will be presented. This is a joint work with Professor Yanjing Wang.

Inverse problems for compressible viscous fluids

Masahiro Yamamoto

The University of Tokyo

myama@ms.u-tokyo.ac.jp

In this talk, we mainly consider a compressible viscous fluid in the isothemral case.

$$\begin{cases} \partial_t \rho(x,t) + \mathrm{div}(\rho(x,t)v(x,t)) = 0, \\ \rho(x,t)\partial_t v = \mu(x)\Delta v + \lambda(x)\nabla(\mathrm{div} v) - \rho(v\cdot\nabla)v + g(x,t)\nabla\rho + F(x,t) \quad \text{in } Q. \end{cases}$$

Here $n = 1,2,3$, $\Omega \subset \mathbf{R}^n$ is a bounded domain and $Q = \Omega \times (0,T)$. Let $\Gamma \subset \partial\Omega$ be a suitable subboundary and let $t_0 \in (0,T)$ be a fixed moment.

We consider the following inverse problems.

Inverse source problem. We assume that μ, λ, g are known and $F(x,t) = R(x,t)f(x)$, where an $n \times n$-matrix valued function $R(x,t) \not\equiv 0$ is given. Determine an $\mathbf{R}^n$-valued $f(x)$, $x \in \Omega$ by lateral Cauchy data ρ, v on $\Gamma \times (0,T)$ and $v(\cdot, t_0)$ in Ω.

Inverse coefficient problem. We assume that $F = 0$ and some of μ, λ, g are unknown. Determine these coefficients by lateral Cauchy data ρ, v on $\Gamma \times (0,T)$ and $v(\cdot, t_0)$ in Ω.

We establish the conditional stability as well as the uniqueness for the inverse problems. The main method is on the basis of Carleman estimates (e.g., Bellassoued and Yamamoto [1]). This work is with Professor Oleg Imanuvilov (Colorado State University).

[1] Bellassoued, Mourad and Yamamoto, Masahiro, *Carleman Estimates and Applications to Inverse Problems for Hyperbolic Systems*, Springer-Japan, Tokyo, 2017, 260pp.

Regularity structures for the primitive equations

Sebastian Zaigler

TU Darmstadt

zaigl@mathematik.tu-darmstadt.de

The new technique of regularity structures introduced by Martin Hairer is a method to find local solutions for subcritical SPDEs with rough data, which were formerly inaccessible. Regularity structures are spaces of abstract "Taylor expansions", in which a solution of a SPDE can be found by solving fixed point equations. The sequence of these fixed points, which are renormalised solution of the SPDE, converges to a solution of the SPDE. We will give a short introduction how to modify them to solve the primitive equations with white noise.

Dispersive effect and global well-posedness of the compressible viscoelastic fluids

Ruizhao Zi

Central China Normal University

rzz@mail.ccnu.edu.cn

This talk is concerned with the global well-posedness issue of the compressible viscoelastic fluids in the whole space $\mathbb{R}^N, N \geq 2$. The proof relies on the dispersive estimates to the linearized hyperbolic system combined with energy estimates to the hyperbolic-parabolic system. By exploiting the intrinsic structure of the system under some physical restrictions, we find that the compressible viscoelastic fluids

admits a unique global solution with a class of large initial data in the sense of critical L^2 framework.

Computer Science of Numerics

Martin Ziegler

KAIST, Republic of Korea

ziegler@kaist.ac.kr

Numerics employs digital computers for operating, by means of approximations, on continuous data: real numbers, converging sequences, continuous/smooth functions, and more generally elements of some compact metric space. Its rigorous foundation builds on and combines mathematical analysis, logic, and computer science.

3 Participants

Abels Helmut, Universität Regensburg

Amann Herbert, Universität Zürich

Barker Tobias, University of Oxford

Betz Volker, Technische Universität Darmstadt

Bothe Dieter, Technische Universität Darmstadt

Celik Aday, Technische Universität Darmstadt

Collier Timothy, University of Sidney

Constantin Peter, Princeton University

Danchin Raphaël, Université Paris-Est Créteil

De Anna Francesco, Pennsylvania State University

Denk Robert, University of Konstanz

Deuring Paul, Université du Littoral "Côte d'Opale"

Disser Karoline, Technische Universität Darmstadt and WIAS Berlin

Egger Herbert, Technische Universität Darmstadt

Eiter Thomas, Technische Universität Darmstadt

Farwig Reinhard, Technische Universität Darmstadt

Feireisl Eduard, Institute of Mathematics CAS

Flandoli Franco, Scuola Normale Superiore of Pisa

Giga Yoshikazu, University of Tokyo

Giorgini Andrea, Università di Pavia

Glass Olivier, Université Paris-Dauphine

Gries Mathis, Technische Universität Darmstadt

Heck Horst, Bern University of Applied Sciences

Hieber Matthias, Technische Universität Darmstadt

Hobus Pascal, Heinrich-Heine-Universität Düsseldorf

Hsu Pen-Yuan, University of Tokyo

Hussein Amru, Technische Universität Darmstadt

Ikezaki Natsuko, Waseda University

Iwabuchi Tsukasa, Tohoku University

Kagei Yoshiyuki, Kyushu University

Kajiwara Naoto, University of Tokyo

Kashiwabara Takahito, University of Tokyo

Koba Hajime, Osaka University

Kobayashi Takayuki, Osaka University

Koch Herbert, Bonn University

Köhne Matthias, Heinrich-Heine-Universität Düsseldorf

Korn Peter, Max Planck Institute for Meteorology

Kozono Hideo, Waseda University

Kress Klaus, Technische Universität Darmstadt

Kunstmann Peer, Karlsruhe Institute of Technology

Kyed Mads, Technische Universität Darmstadt

Li Jinkai, The Chinese University of Hong Kong

Lukáčová-Medvidová Mária, Johannes-Gutenberg University of Mainz

Maremonti Paolo, Università degli Studi della Campania "L. Vanvitelli"

Mazzucato Anna, Penn State University

Mizerová Hana, Comenius University in Bratislava

Monniaux Sylvie, Université Aix-Marseille

Mucha Piotr Bogusław, University of Warsaw

Murata Miho, Kanagawa University

Nečasová Šárka, Institute of Mathematics of Sciences of the Czech

Nguyen Thieu Huy, Hanoi University of Science and Technology

Notsu Hirofumi, Kanazawa University

Ogawa Takayoshi, Tohoku University

Okabe Takahiro, Hirosaki University

Otto Felix, Max-Planck-Institut, Leipzig

Ożański Wojciech, University of Warwick

Pal Choudhury Anupam, Johann Radon Institutes, Linz

Racke Reinhard, University of Konstanz

Reichwein Elisabeth, Heinrich-Heine-Universität Düsseldorf

Renardy Michael, Virginia Tech

Rivera Jaime E. Muñoz, National Laboratory for Scientific Computation

Rocca Elisabetta, University of Pavia

Růžička Michael, Albert-Ludwigs-University Freiburg

Saal Jürgen, Heinrich-Heine-University Düsseldorf

Saal Martin, Technische Universität Darmstadt

Saito Hirokazu, Waseda University

Sauer Jonas, Max-Planck-Institut, Leipzig

Sawada Okihiro, Gifu University

Schmidt Andreas, Technische Universität Darmstadt

Seregin Gregory, University of Oxford

Seyfert Anton, Technische Universität Darmstadt

She Bangwei, Institute of Mathematics, Czech Academy of Sciences

Shibata Yoshihiro, Waseda University

Simonett Gieri, Vanderbilt University

Soga Kohei, Keio University

Soga Verena, Technische Universität Darmstadt

Steinberg Florian, INRIA Sophia Antipolis

Takada Ryo, Kyushu University

Titi Edriss S., Texas A&M University

Tolksdorf Patrick, Technische Universität Darmstadt

Tolle Tobias, Technische Universität Darmstadt

Tomoyuki Nakatsuka, Technische Universität Darmstadt

Ulbrich Stefan, Technische Universität Darmstadt

Ushikoshi Erika, Yokohama National University

Watanabe Keiichi, Waseda University

Wegmann David, Technische Universität Darmstadt

Westermann Laura, Heinrich-Heine-Universität Düsseldorf

Wiedemann Emil, Leibniz Universität Hannover

Wilke Mathias, University of Regensburg

Wrona Marc, Technische Universität Darmstadt

Xin Zhouping, The Chinese University of Hong Kong

Yamamoto Masahiro, University of Tokyo

Zaigler Sebastian, Technische Universität Darmstadt

Zi Ruizhao, Central China Normal University

Ziegler Martin, Kaist, Republic of Korea