A Conservative Multi-Tracer Transport Scheme for Spectral-Element Spherical Grids (SPELT for CAM-SE)

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Community Atmosphere Model – Spectral Elements (HOMME)

- Cubed-sphere grids resulting from equi-angular gnomonic projection.
- Domain decomposition (horizontal), that means parallel strategy over elements, highly scalable - \( \mathcal{O}(100k) \) cores.
- Supports unstructured grids.
- For \( \mathcal{O}(100k) \) tracers, SE advection is inefficient.

Spectral Element Lagrangian Transport

Finite volume flux form of continuity equation

\[
\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{v}) = \frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{F} = 0. 
\]

Integrating over the cell \( A_k \) and temporally \([t^n, t^{n+1}]\) results in

\[
\bar{\psi}_{k}^{n+1} | A_k = \bar{\psi}_{k}^{n} | A_k - \left( \int_{t^n}^{t^{n+1}} \mathbf{F} \cdot n \, dt \right) \] with \( \Gamma_k = \partial A_k \).

\( \mathbf{F} \) time-integrated flux, \( \bar{\psi}_{k}^{n+1} \) and \( \bar{\psi}_{k}^{n} \) are the cell average (prognostic variable) at old and new time-level, respectively. Conservation of mass is guaranteed, no matter how the fluxes are approximated.

The trajectories joining upstream points \((t = t^n)\) and arrival points \((t = t^{n+1})\) are shown as smooth curves. The fluxes \( \mathbf{F} \) on the filled black circles are computed by integrating flux values along the characteristics. We need the reconstruction values (see below) only at \( t^n, \ t^{n+1/2}, \ t^{n+1} \).

The flux \( F_{AB} \) for an edge \( AB \) uses three point fluxes (entering perpendicular with respect to the edge). Then, the four fluxes \( F_{AB}, F_{BC}, F_{CD}, F_{DA} \) define \( \bar{\psi}_{k}^{n+1} \) by the standard flux form.

SPaRAL Transport Scheme (SPELT) in CAM-SE

SPELT works on a mesh based on the SE elements, uses the cubed-sphere structure of CAM-SE but has its own data structure. Communication differs significant because of a different halo zone.

- No need for complex upstream area searching: pointwise search.
- Multi-moment based scheme: local reconstruction.
- Easy to implement on unstructured quad grids.
- Search is only done once \( \rightarrow \) multi tracer efficient.
- Flux-based method, positivity (monotonic) option possible by FCT.

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SPELT on unstructured grids

Figure: Left: unstructured (perturbed) grid with \( 20 \times 20 \) finite volume cells per cube face. Right: convergence of normalized errors on unstructured grids: Gaussian hill advection with solid-body rotation over a cube corner (almost third order).

Figure: Time traces of the normalized errors for solid-body (cosine bell) rotation problem for one complete revolution (12 days). Left: on uniform grid. Right: on the perturbed grid.

Figure: Orthographic projection of the numerical solution (slotted cylinder) at different stages during a complete revolution along the north-east direction (solid-body) on a uniform grid with CFL = 0.625. Left: initial condition. Middle: centered over a corner. Right: initial position. The final monotone solution appears to be slightly smoothed, but the cylinder is symmetric.

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