

Some Subsidiary Relations from the ECE Lemma

1) Standard Derivation (Appendix J, volume 1)

$$\square \nabla_{\lambda}^a := R \nabla_{\lambda}^a \quad - (1)$$

$$R := \nabla_{\lambda}^{\lambda} \nabla^{\mu} \left(\Gamma_{\mu\lambda}^{\nu} \nabla_{\nu}^a - \omega_{\mu b}^a \nabla_{\lambda}^b \right). \quad - (2)$$

The ECE Lemma is a subsidiary mathematical proposition (1). The scalar curvature is defined as in eq. (2), in terms of the tetrad, gamma connection and spin connection. This means that provided R is defined as in eq. (2), the identity (1) is true. The identity (1) is sought because it has the structure of the main wave equations of physics, and so unifies quantum mechanics and general relativity.

2) Derivation in Paper 37

$$\square \nabla_{\lambda}^a := R \nabla_{\lambda}^a \quad - (3)$$

$$R := \nabla_{\lambda}^{\lambda} \left(\left(D^{\mu} \Gamma_{\mu\lambda}^{\nu} \right) \nabla_{\nu}^a - \left(D^{\mu} \omega_{\mu b}^a \right) \nabla_{\lambda}^b + \Gamma_{\nu\mu}^{\nu} \omega_{\lambda}^{\mu a} - \Gamma_{\nu\mu}^{\nu} \Gamma^{\mu\nu}_{\lambda} \right). \quad - (4)$$

This is a more complicated version of R, so eq. (2) is preferred by Occam's Razor. Eq. (3) is an identity provided that eq. (4) is true.

2)

3) Development of Eq. (2)

It is shown in paper 37 (volume 2), that eq. (2) may be written as:

$$R := -g^{\mu\nu} (\Gamma_{\mu\alpha}^{\nu} \omega^{\alpha}{}_{\sigma\nu} + \omega_{\mu\alpha}^{\nu} \Gamma_{\sigma\nu}^{\alpha}) \quad - (5)$$

This expresses R in terms of the metric, gamma and spin connection.

4) Self Check (Paper 38)

Start with the tetrad postulate:

$$D_{\mu} v^{\lambda a} = d_{\mu} v^{\lambda a} + \omega_{\mu b}^a v^{\lambda b} - \Gamma_{\mu\lambda}^{\nu} v^{\lambda a} = 0 \quad - (6)$$

The Lemma is the identity:

$$D^{\mu} (D_{\mu} v^{\lambda a}) := 0 \quad - (7)$$

i.e.

$$D^{\mu} (d_{\mu} v^{\lambda a} + \omega_{\mu b}^a v^{\lambda b} - \Gamma_{\mu\lambda}^{\nu} v^{\lambda a}) := 0 \quad - (8)$$

Using the Leibnitz theorem and the tetrad postulate again, eq. (8) is:

$$(D^{\mu} d_{\mu}) v^{\lambda a} + (D^{\mu} \omega_{\mu b}^a) v^{\lambda b} - (D^{\mu} \Gamma_{\mu\lambda}^{\nu}) v^{\lambda a} = 0 \quad - (9)$$

3) Now construct a self-check or cross-check by expressing eq. (7) as:

$$d^\mu (D_\mu q_\lambda^a) = 0 \quad \text{--- (10)}$$

So we know that eq. (10) and eq. (9) must give the same result. This is cross-checked as follows.

First use the results:

$$D_\mu d^\mu = \square + \Gamma_{\mu\lambda}^\mu d^\lambda \quad \text{--- (11)}$$

$$D^\mu = g^{\mu\nu} D_\nu \quad \text{--- (12)}$$

$$d_\mu = g_{\mu\nu} d^\nu \quad \text{--- (13)}$$

this means:

$$D^\mu d_\mu = g^{\mu\nu} D_\nu g_{\mu\sigma} d^\sigma = 4 D_\mu d^\mu \quad \text{--- (14)}$$

Now use eq. (14) in eq. (9):

$$4 (D_\mu d^\mu) q_\lambda^a + (D^\mu \omega_{\mu b}^a) q_\lambda^b - (D^\mu \Gamma_{\mu\lambda}^\nu) q_\nu^a = 0 \quad \text{--- (15)}$$

i.e.:

$$4 (D_\mu (d^\mu q_\lambda^a) + d^\mu (D_\mu q_\lambda^a)) + (D^\mu \omega_{\mu b}^a) q_\lambda^b - (D^\mu \Gamma_{\mu\lambda}^\nu) q_\nu^a = 0 \quad \text{--- (16)}$$

4) By comparison of eq. (10) and (16) we know that the following must be true:

$$4) D_{\mu} (d^{\mu} q^a_{\lambda}) + D^{\mu} \omega_{\mu b}^a - (D^{\mu} \Gamma_{\mu\lambda}^{\nu}) q^a_{\nu} = 0 \quad (17)$$

i.e.

$$D^{\mu} (d_{\mu} q^a_{\lambda} + \omega_{\mu b}^a q^b_{\lambda} - \Gamma_{\mu\lambda}^{\nu} q^a_{\nu}) = 0 \quad (18)$$

which is

$$D^{\mu} (D_{\mu} q^a_{\lambda}) = 0 \quad (19)$$

quod erat demonstrandum. (Q.E.D.)

So this self-checks eq. (2). It

checks that Cartan geometry is correct, and that this use of Cartan geometry is correct.

5) Baker's Version of the ECE Lemma.

Baker starts with the tetrad postulate:

$$d_{\mu} q^a_{\nu} + \omega_{\mu b}^a q^b_{\nu} - \Gamma_{\mu\nu}^{\sigma} q^a_{\sigma} = 0 \quad (20)$$

Presumably, he now accepts the tetrad postulate after spending a long time asserting that it is somehow wrong.

5) He ~~then~~ operates a eq. (20) w/ ∂^μ .
 So Bohr now accepts eqn. (10), diametrically
 contradicting himself, but finally getting it
 right. So:

$$\partial^\mu \left(\partial_\mu q^a + \omega_{\mu b}^a q^b - \Gamma_{\mu\sigma}^\sigma q^a \right) = 0 \quad - (21)$$

Elementary differentiation gives:

$$\begin{aligned} \square q^a + (\partial^\mu \omega_{\mu b}^a) q^b + \omega_{\mu b}^a (\partial^\mu q^b) \\ - (\partial^\mu \Gamma_{\mu\sigma}^\sigma) q^a - \Gamma_{\mu\sigma}^\sigma (\partial^\mu q^a) \\ = 0 \end{aligned} \quad - (22)$$

Bohr now uses the tetrad postulate twice
 more, thus contradicting himself twice more,
 because the tetrad postulate is now accepted
 by him but also simultaneously rejected by
 him. So he uses:

$$\partial^\mu q^b = \Gamma^{\mu\rho}_\sigma q^b - \omega^{\mu b}_c q^c \quad - (23)$$

$$\partial^\mu q^a = \Gamma^{\mu\rho}_\sigma q^a - \omega^{\mu a}_c q^c \quad - (24)$$

So he just uses eqns. (23) and (24)

in eq. (22) to obtain:

6)

$$\square q_{\nu}^a + (\partial^{\mu} \omega_{\mu b}^a) q_{\nu}^b - (\partial^{\mu} \Gamma_{\mu \nu}^{\sigma}) q_{\sigma}^a + \omega_{\mu b}^a (\Gamma^{\mu \rho}_{\nu} q_{\rho}^b - \omega^{\mu b c} q_{\nu}^c) - \Gamma_{\mu \nu}^{\sigma} (\Gamma^{\mu \rho}_{\sigma} q_{\rho}^a - \omega^{\mu a c} q_{\nu}^c) = 0 \quad (25)$$

He then re-arranges dummy indices to obtain:

$$\square q_{\nu}^a + (\partial^{\mu} \omega_{\mu c}^a - \omega_{\mu b}^a \omega^{\mu b c}) q_{\nu}^c - (\partial^{\mu} \Gamma_{\mu \nu}^{\rho} + \Gamma_{\mu \nu}^{\sigma} \Gamma^{\mu \rho}_{\sigma}) q_{\nu}^a + 2 \omega_{\mu c}^a \Gamma^{\mu \sigma}_{\nu} q_{\sigma}^c = 0 \quad (26)$$

Finally he tries to create a false impression that eq. (2) is somehow wrong by deliberately omitting the final step, which is:

$$R := q_{\nu}^a \left((\partial^{\mu} \Gamma_{\mu \nu}^{\rho} + \Gamma_{\mu \nu}^{\sigma} \Gamma^{\mu \rho}_{\sigma}) q_{\nu}^a - (\partial^{\mu} \omega_{\mu c}^a - \omega_{\mu b}^a \omega^{\mu b c}) q_{\nu}^c - 2 \omega_{\mu c}^a \Gamma^{\mu \sigma}_{\nu} q_{\sigma}^c \right) \quad (27)$$

$$\square q_{\nu}^a := R q_{\nu}^a \quad (28)$$