

Research Résumé

My research interest is model theory for monadic second-order logic. In particular, I consider the following topics:

- (i) algorithmic model theory;
- (ii) model constructions and decompositions;
- (iii) the expressive power of monadic second-order logic;
- (iv) automata theory and algebraic language theory;
- (v) algorithmic issues.

(i) Algorithmic model theory

Algorithmic model theory studies algorithmic properties of infinite structures. To this end, one considers finite presentations of *infinite* structures and operations on them. The central questions regarding a given encoding are:

- (a) Which structures have a representation of this kind?
- (b) Which operations on such representations are computable?

Frequently, the algorithmic properties of a given type of representation can be determined easily. But characterising those structures that possess such a representation is usually highly non-trivial.

Previous work. My own contributions to this field concentrate on characterisation results. The following classes of finitely presented structures were investigated:

- (a) automatic structures [T3,C5,C2,J10],
- (b) tree-interpretable structures [C3,J11],
- (c) the Caucal hierarchy [J6].

The focus of these papers was on methods to show that a given structure does *not* belong to the class under consideration. By now, the formal framework proposed in my work has become a standard technique in the field.

(ii) Model constructions and decompositions

An invaluable tool in algorithmic model theory are operations that are compatible with a given logic in the sense that, when applying the operation, one can compute the theory of the result from the theories of the arguments. Classical examples of such operations are interpretations, the products of Feferman and Vaught (for first-order logic) and the generalised sums of Shelah (for monadic second-order logic). The survey [H1] gives an overview with emphasis on operations that are compatible with first-order logic or with monadic second-order logic. Such operations have many applications.

(i) They yield a notion of reduction between classes of structures. For instance, every structure that one can interpret a structure with undecidable theory in also has an undecidable theory.

(ii) In algorithmic model theory one can use operations that are compatible with logical theories to represent (infinite) structures by finite terms. Such term representations provide a uniform formalism to describe classes of finitely presentable structures. It encompasses nearly all classes considered in the literature yielding a unification of their definitions, which originally were based on ad-hoc methods like automata, term rewriting systems, grammars, etc.

(iii) Besides algorithmic applications of term representations one can also use terms to obtain decompositions of representable structures. In that way, depending on the choice of operations, it is possible to develop a structure theory for the given class.

Previous work. (a) [H1] is a survey presenting common operations that are compatible with first-order logic or monadic second-order logic and some of their applications.

(b) In [J8,T2] we study the class of structures that can be interpreted in a tree. These structures can be characterised in several ways:

- (1) They can be interpreted by monadic second-order logic in a tree.
- (2) They have a hierarchical decomposition where a certain complexity measure (the partition width) is bounded.
- (3) They can be built up from finite structures using (i) disjoint unions and (ii) quantifier-free interpretations.

Besides these characterisations we also show that one can use tools from first-order model theory to study such structures.

(c) [H2] is a survey on an iteration operation, introduced by Muchnik, that is compatible with monadic second-order logic. The article [J9] extends these results by showing that the Muchnik iteration is also compatible with several extensions of monadic second-order logic.

(d) The article [J7] studies various algebras of finite structures and the corresponding notions of recognisable and equational classes.

(iii) Expressive power

In the last two decades the beginnings of a model theory for monadic second-order logics were developed. Of particular interest in this context are questions of definability and interpretability in given structures.

On the one hand, there are structures whose monadic theory is simple enough such that we can develop a structure theory for them. All known examples of such structures have the property of being interpretable in a tree.

On the other hand, there are structures whose monadic theory is extremely complicated. A prominent example are structures containing large definable grids. According to a conjecture of Seese these to extremes form a dichotomy: either a structure can be interpreted in a tree, or it contains a large grid.

Previous work. (a) One emphasis of my work is on different kinds of interpretations. For transductions, a strong form of interpretation, and classes of finite structures we obtained a complete

description of the resulting hierarchy in [J4]. In particular, we developed concrete combinatorial criteria for the existence of a transduction between two given classes of finite structures.

When considering the conjecture of Seese interpretations in trees are of particular importance. These are the subject of [J8,T2], where structures interpretable in trees are characterised in various ways. In particular, it is shown that these are exactly those structures whose partition width is bounded.

At the other extreme there are structures containing definable grids or pairing functions. Such structures were studied in [J3,J1]. The main result is a proof of a weak variant of Seese's conjecture.

(b) A further topic of my work concerns variants of monadic second-order logic. An important extension of this logic is the so-called guarded second-order logic. In general, it is strictly more expressive than monadic second-order logic. But, according to a result of Courcelle, on countable sparse structures the expressive power of guarded second-order logic collapses to that of monadic second-order logic. The article [J5] contains, among other results on the expressive power of guarded second-order logic, a generalisation of Courcelle's result to sparse structures of arbitrary cardinality.

(c) A large class of structures with a decidable monadic second-order theory is the Caucal hierarchy. Each of these structures has a finite partition width. In [J6] we study methods to prove that certain structures do not belong to a given level of the hierarchy.

(iv) Automata theory and algebraic language theory

There is a tight connection between automata and the monadic second-order theories of certain structures, like the order of the natural numbers or the infinite binary tree. In particular, Büchi and Rabin have shown that one can obtain decision procedures for these theories by translating formulae into automata. Automata theory has therefore become an essential tool in the investigation of monadic second-order logic.

Besides using monadic second-order logic one can also characterise regular languages algebraically via homomorphisms into finite algebras. This point of view is particularly suited to classify fragments of monadic second-order logic and to develop corresponding decision procedures. There are well-developed algebraic theories for languages of finite and infinite words. For languages of finite trees a preliminary theory has also been developed, but for infinite trees only partial results exist. The main obstacle in the development of an algebraic language theory in this context are missing combinatorial tools, like Ramseyan factorisation theorems for trees.

Previous work. (a) The paper [U4] makes a first contribution to the development of an algebraic language theory for languages of infinite trees. In particular, it contains a characterisation of the regular languages via homomorphisms in certain algebras.

(b) In [C1] we study fixed-point inductions of monadic second-order formulae on finite words. The main result is a proof that it is decidable whether the length of these inductions is uniformly bounded.

(c) Besides considering automata as recognisers of languages, we can also use them to present infinite structures. For instance, the graphs in the Caucal hierarchy coincide with the configuration graphs of higher-order pushdown automata. In [J6] we use higher-order pushdown automata to study the classes in the Caucal hierarchy. In particular, we develop methods to prove

that certain structures do not belong to a given level of the hierarchy. The main technical result is a pumping lemma for these automata.

(d) Both, monadic second-order logic and the algebraic approach can be used to generalise formal language theory to arbitrary structures. For certain kinds of graph algebras this was done in [J7]. This paper extends formal language theory to various algebras of finite structures. We study the corresponding algebraic notions of recognisable and equational classes, and we relate them to the notion of definability in monadic second-order logic.

(v) Finite model theory, descriptive complexity theory, and algorithmic issues

For many applications, one needs logics with the right balance of expressive power and algorithmic manageability. In many cases, in particular in verification and database theory, one can obtain such logics by extending some weak logic by fixed-point operators. This has led to a wide range of fixed-point logics.

Previous work. (a) [H3] is a survey on guarded fixed-point logic. We present automata-based algorithms for model checking and satisfiability testing for this logic, and we study the complexity of these problems.

(b) In [C1] we study fixed-point inductions of a monadic second-order formulae on finite words. The main result is a proof that it is decidable whether the length of these inductions is uniformly bounded.

(c) Descriptive complexity theory studies the correspondence between the computational complexity of classes of finite structures and the logics these classes can be axiomatised in. In [C4] we introduce a different setting where we consider the complexity and definability of sets definable in a fixed structure. Several complexity classes are characterised in this way.

(d) Ehrenfeucht-Fraïssé Games are one of the main model-theoretic tools in finite model theory. Unfortunately, on nontrivial structures the combinatorics involved in playing these games quickly become unmanageable. In [J2] we study several ways to simplify games and to decompose them into simpler subgames. While in the literature one mostly considers games on sparse structures, in this article we place the emphasis on structures that are not sparse.

Publications

Unpublished papers

- [U1] *Logic and Algebra*, book in preparation. A draft is available at www.mathematik.tu-darmstadt.de/~blumensath
- [U2] (with Martin Otto and Mark Weyer) *Boundedness of Monadic Second-Order Formulae Over Trees*, in preparation.
- [U3] *An Algebraic Proof of Rabin's Tree Theorem*, in preparation.
- [U4] *Recognisability for Algebras of Infinite Trees*, submitted.

Journal articles

- [J1] *Simple Monadic Theories and Partition Width*, *Mathematical Logic Quarterly*, to appear.
- [J2] *Locality and Modular Ehrenfeucht-Fraïssé Games*, *Journal of Applied Logic*, to appear.
- [J3] *Simple Monadic Theories and Indiscernibles*, *Mathematical Logic Quarterly*, to appear.
- [J4] (with Bruno Courcelle) *The Monadic Second-Order Transduction Hierarchy*, *Logical Methods in Computer Science*, 6 (2010).
- [J5] *Guarded Second-Order Logic, Spanning Trees, and Network Flows*, *Logical Methods in Computer Science*, 6 (2010).
- [J6] *On the Structure of Graphs in the Caucal Hierarchy*, *Theoretical Computer Science*, 400 (2008), pp. 19–45.
- [J7] (with Bruno Courcelle) *Recognizability, Hypergraph Operations, and Logical Types*, *Information and Computation*, 204 (2006), pp. 853–919.
- [J8] *A Model Theoretic Characterisation of Clique-Width*, *Annals of Pure and Applied Logic*, 142 (2006), pp. 321–350.
- [J9] (with Stephan Kreutzer) *An Extension to Muchnik's Theorem*, *Journal of Logic and Computation*, 15 (2005), pp. 59–74.
- [J10] (with Erich Grädel) *Finite Presentations of Infinite Structures: Automata and Interpretations*, *Theory of Computing Systems*, 37 (2004), pp. 641–674.
- [J11] *Axiomatising tree-interpretable structures*, *Theory of Computing Systems*, 37 (2004), pp. 3–27.

Handbook chapters

- [H1] (with Thomas Colcombet and Christof Löding) *Logical theories and compatible operations*, in *Logic and Automata* (J. Flum, E. Grädel, T. Wilke, eds.), Amsterdam University Press, 2007, pp. 72–106.
- [H2] (with Dietmar Berwanger) *The Monadic Theory of Tree-like Structures*, in *Automata, Logic, and Infinite Games* (E. Grädel, W. Thomas, T. Wilke, eds.), LNCS 2500 (2002), pp. 285–301.

- [H3] (with Dietmar Berwanger) *Automata for Guarded Fixed Point Logics*, in Automata, Logic, and Infinite Games (E. Grädel, W. Thomas, T. Wilke, eds.), LNCS 2500 (2002), pp. 343–355.

Papers in refereed conferences

- [C1] (with Martin Otto and Mark Weyer) *Boundedness of Monadic Second-Order Formulae Over Finite Words*, ICALP, LNCS 5556 (2009), pp. 67–78.
- [C2] (with Erich Grädel) *Finite Presentations of Infinite Structures: Automata and Interpretations*, Proc. 2nd Int. Workshop on Complexity in Automated Deduction, CiAD 2002.
- [C3] *Axiomatising tree-interpretable structures*, Proc. 19th Int. Symp. on Theoretical Aspects of Computer Science, LNCS 2285 (2002), pp. 596–607.
- [C4] *Bounded Arithmetic and Descriptive Complexity*, Proc. 14th Ann. Conference of the European Association for Computer Science Logic, LNCS 1862 (2000), pp. 232–246.
- [C5] (with Erich Grädel) *Automatic Structures*, Proc. 15th IEEE Symp. on Logic in Computer Science, 2000, pp. 51–62.

Theses

- [T1] *Simple Monadic Theories*, Habilitation Thesis, TU Darmstadt, 2008.
- [T2] *Structures of Bounded Partition Width*, Ph.D. Thesis, RWTH Aachen, 2003.
- [T3] *Automatic Structures*, Diploma Thesis, RWTH Aachen, 1999.

Preprints of all my papers are available from:

<http://www.mathematik.tu-darmstadt.de/~blumensath/Publications.html>