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Introduction to Mathematical Software
3rd exercise sheet

Exercise 1 (plot and plot3d)

1. Let $f(x) := e^{-e^2} sin(\pi x^3)$. Plot this function from 0 to infinity. Then try to plot f together with e^{-x^2} and $-e^{-x^2}$ with $x \in [-2, 2]$, all into one picture.

2. How can you plot the function $cos(xy), x \in [-3, 3], y \in [-3, 3]$? Take a look into Maple's Help, in order to find out more about plotting.

Exercise 2 (Limits)

For the following problems, try to find out whether they can be solved algorithmically. How do solutions look like in Maple, and how can Maple help solving the problems?

1. Input: a, b integer numbers, $b \neq 0$. Output: the rational number a/b in decimal notation. The output might be finite or periodic. If you do not recall from your calculus class, you can get some help from google. Enter 'decimal notation' into the search field, and the first entry should lead you to http://en.wikipedia.org/wiki/Decimal.

2. Each bounded sequence of real numbers contains a convergent sub-sequence, concerning the Bolzano-Weierstrass theorem. The problem here is:
   Input: bounded sequence of real numbers. Output: a convergent sub-sequence.

3. Input: convergent sequence f of real numbers with $\lim_{n\to\infty} f(n) = x$ and $\epsilon > 0$. Output: natural number N such that $|f(N) - x| < \epsilon$.

   If this has a solution apply the procedure to the series representation of the sine function which can be obtained from the definition:

   $$\sin x := \frac{1}{2i}(e^{ix} - e^{-ix}).$$

   Hint: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

4. Input: convergent sequence f of real numbers with $\lim_{n\to\infty} f(n) = x$ and $\epsilon > 0$. Output: natural number N, such that $|f(n) - x| < \epsilon$ for all $n > N$.

Exercise 3 (Cauchy-Product of series)   Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be absolutely convergent series. Find an algorithm for the computation of c_n with $(\sum_{n=0}^{\infty} c_n) = (\sum_{n=0}^{\infty} a_n)(\sum_{n=0}^{\infty} b_n)$.

Exercise 4 (Learning with the help of Maple)

Use the menu of your Maple and click on 'Tools/Tutors/Calculus - Single Variable/Limit Methods'. With the help of that tutor, you can learn about various rules which you can apply to limits. When you click on 'Next Step', Maple will show you what it proposes as the next transformation. You can also try to apply the different available rules, ask 'Help' what to do next.