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Introduction to Mathematical Software

2nd exercise sheet

Exercise 1 (Representation of integers)

For Computers the least logical entity is a *bit*, which can have two states – typically written as 0 and 1. Thus in most programming languages the natural numbers are represented binary; e.g. the decimal number $13_{(10)}$ would have the binary representation $1101_{(2)}$.

(a) Convert the following decimals to binaries: $4_{(10)}$, $5_{(10)}$, $78_{(10)}$, $127_{(10)}$

(b) Convert the following binaries to decimals: $111_{(2)}$, $10101_{(2)}$, $10111_{(2)}$, $1110111_{(2)}$

To also represent negative "natural" numbers, i.e. integers, somehow the "−"-sign has to be represented. This is achieved by first fixing the number of bits the representation of an integer may use (e.g. 8 bits) and then using the leftmost bit for the sign. (Of course this limits the integers that may be represented at all.)

That's not the whole story. In fact negative integers are typically represented by its "Two's Complements". The Two's Complements of a negative integer n is built by inverting all bits of the representation of $|n|$ and then adding 1 to it. To represent e.g. $-13_{(10)}$ in an 8 bits wide representation, you first represent $13_{(10)} = 00001101_{(2)}$, then invert all its bits to $11110010_{(2)}$ and then add 1 to it, thus $-13_{(10)} = 11110010_{(2)} + 1 = 11110011_{(2)}$.

(c) Find the 8 bits wide representations for $-27_{(10)}$ and $-78_{(10)}$.

(d) Which integers may be represented at all with a

  (i) 8 bits wide representation? (ii) 32 bits wide representation?

(e) How could you build the Two's Complement using decimal calculations?

With the Two's Complement, you can simply use binary addition to add two integers. Do the following calculations like a computer using 8 bits:

(f) $13 + 8$ (g) $27 - 78$ (h) $-27 - 78$ (i) $-78 - 78$

What happens in exercise (i)?

Exercise 2 (Lists and sets in Maple)

(a) Explain the difference of lists and sets in Maple.

(b) Use Maple to find the common divisors of 23545800, 25491186 and 229420674.

(c) Let Maple evaluate the function sin for all the solutions of the equation

$$x^4 - 4x^3\pi + \frac{26}{9}x^2\pi^2 + \frac{4}{9}x\pi^3 - \frac{1}{3}\pi^4 = 0.$$

(d) Program a Maple procedure that returns for a given set M and a given number k all the subsets of M with exactly k elements.

(e) Program a Maple procedure which builds the cartesian product of a set of functions. Let the input be a list of function-names, e.g. $[\sin, \cos]$. Then the output should be the cartesian product of this list of functions with itself, given in the form of a list: $[(x, y) \to (\sin(x), \sin(y)),\ (x, y) \to (\sin(x), \cos(y)),\ (x, y) \to (\cos(x), \sin(y)),\ (x, y) \to (\cos(x), \cos(y))]$.