Fachbereich Mathematik
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13th Tutorial Sheet Linear Algebra I for MCS
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(T13.1) Permutations

(i) Let $n \geqslant 1$. Show that the set of even permutations,

$$A_n = \{\sigma \in S_n : \sigma \text{ even}\} \subseteq S_n,$$

forms a group with composition. It is called the *alternating group* (this is Exercise 4.1.1 on page 120 of the notes).

(ii) Let σ be the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$.
Determine pictorially the number $\nu(\sigma)$ (see page 109 of the notes) and represent σ as a composition of nnts. (Hint: Use the proof of Proposition 4.1.6.)

(T13.2) Determinants

Show that for any $A \in \mathbb{F}^{(3,3)}$:

$$\begin{aligned} |A| &= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \\ &= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{31}(a_{12}a_{23} - a_{22}a_{13}) \end{aligned}$$

(this is Exercise 4.1.2 on page 123 of the notes).

(T13.3) Determinants

One can give an alternative proof of Proposition 4.1.16 in the notes, in case $|A| \neq 0$. Show that for any such A, the function

$$f : \mathbb{F}^{(n,n)} \to \mathbb{F}, \quad B \mapsto \frac{|AB|}{|A|}$$

satisfies conditions (D1-3) for a determinant function (see page 118 of the notes).
Hence it must be *the* determinant function with value $|B|$ on B.
(This is Exercise 4.1.4 on page 126 of the notes.)