Fachbereich Mathematik
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12th Tutorial Sheet Linear Algebra I for MCS
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(T12.1) Inverse of a matrix

Compute the inverse of the matrix $A := \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 2 & 6 & -1 \end{pmatrix} \in \mathbb{R}^{(3,3)}$.

(T12.2) Row and column rank

Let t be any real number. Determine, using Gauß-Jordan elimination, the row and column rank of the following matrix, as a function of t. Give also a basis for its row and column space.

$$\begin{pmatrix} 1 & 0 & -1 & -1 & t-6 \\ -t & 0 & 3 & t & 9 \\ -1 & 0 & t-6 & 1 & 3 \\ 0 & 0 & 0 & t-3 & 0 \end{pmatrix}$$

(T12.3) Affine transformations

Show that the affine transformations of $\mathbb{R}^2$ form a group with respect to composition, and $\mathrm{id}_{\mathbb{R}^2}$ as neutral element. (This is Exercise 3.4.2 on page 112 of the notes.)

Hint: Recall from Section 3.4 of the notes that affine transformations of $\mathbb{R}^2$ are pairs $[\varphi, \mathbf{u}]$ where φ is a linear automorphism of $\mathbb{R}^2$ and $\mathbf{u}$ is any vector in $\mathbb{R}^2$, and that such affine transformations act on a vector $\mathbf{v}$ in $\mathbb{R}^2$ by:

$$[\varphi, \mathbf{u}](\mathbf{v}) = \varphi(\mathbf{v}) + \mathbf{u}.$$

To verify the group axioms, represent the composition $[\varphi_2, \mathbf{u}_2] \circ [\varphi_1, \mathbf{u}_1]$ as $[\varphi, \mathbf{u}]$ for suitable φ and $\mathbf{u}$.