

January 9, 2007

## 11th Tutorial Sheet Linear Algebra I for MCS Winter Term 2006/2007

### (T11.1) Matrices from maps

Let  $A_1 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$  and  $A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ .

Consider the linear maps  $\varphi_i$  described by  $A_i$  with respect to some labelled basis  $B$ . Determine the matrices that represent  $\varphi_1 \circ \varphi_1$ ,  $\varphi_2 \circ \varphi_2$ ,  $\varphi_1 \circ \varphi_2$  and  $\varphi_2 \circ \varphi_1$  with respect to  $B$ .

### (T11.2) Regular matrices

Show that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{F}^{(2,2)}$$

is regular iff  $ad - bc \neq 0$ . Determine  $A^{-1}$  if  $A$  is regular.  
(This is Exercise 3.3.7 on page 102 of the notes.)

### (T11.3) Some matrix arithmetic

Let  $A = \begin{pmatrix} a & b & 0 \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$  for some  $a, b \in \mathbb{F}$ .

- (i) Determine  $A, A^2, A^3, \dots$ .
- (ii) Show that  $E_3, A, A^2, A^3$  are linearly dependent.
- (iii) (\*) In case  $A$  is regular determine  $A^{-1}$ .