**Fachbereich Mathematik**
Martin Otto
Benno van den Berg, Sven Herrmann

TECHNISCHE UNIVERSITÄT DARMSTADT

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# 10th Tutorial Sheet Linear Algebra I for MCS
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### (T10.1) Coordinates with respect to different bases

Let $B = (\mathbf{e}_1, \mathbf{e}_2)$ be the standard basis of $\mathbb{R}^2$, and let $\hat{B} = (\mathbf{b}_1, \mathbf{b}_2)$ be the basis consisting of $\mathbf{b}_1 = (1, 1)$ and $\mathbf{b}_2 = (1, -1)$.

(i) Determine the vector $\mathbf{x}$ represented by $[\![\mathbf{x}]\!]_{\hat{B}} = (2, -1)$, and $[\![\mathbf{y}]\!]_{\hat{B}}$ for $\mathbf{x} = (5, -1)$.

(ii) Determine the matrix $A' = [\![\varphi]\!]_{\hat{B}}^{\hat{B}}$ representing the endomorphism $\varphi = \varphi_A$ whose representation w.r.t. the standard basis is $A = \begin{pmatrix} 1 & -2 \\ 4 & 1 \end{pmatrix}$.

### (T10.2) Maps from matrices

What is the geometric interpretation of the linear operations on $\mathbb{R}^2$ represented by the following matrices w.r.t. some given labelled basis $B = (\mathbf{b}_1, \mathbf{b}_2)$? ($\lambda_1, \lambda_2 \in \mathbb{R} \setminus \{0\}$)

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

### (T10.3) Matrices from maps

Find the matrices representing the following linear maps w.r.t. the standard basis of $\mathbb{R}^3$:

(i) reflection of $\mathbb{R}^3$ in the $\mathbf{e}_1$-$\mathbf{e}_2$-plane;

(ii) rotation of $\mathbb{R}^3$ through 60 degrees ($\pi/3$) around the $\mathbf{e}_3$-axis;
(for definiteness: clock-wise, looking in the direction of $\mathbf{e}_3$.)

(iii) rotation of $\mathbb{R}^3$ through 60 degrees ($\pi/3$) around the $\mathbf{e}_2$-axis;
(for definiteness: clock-wise, looking in the direction of $\mathbf{e}_2$.)

(iv) the map $\varphi : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\varphi(\mathbf{x}) = (x_1 a_1 + x_2 a_2 + x_3 a_3)\, \mathbf{b}$,
where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.