

December 5, 2006

8th Tutorial Sheet Linear Algebra I for MCS Winter Term 2006/2007

(T8.1) Dimension and dimension formula

- (i) Let W be a subspace of an n -dimensional \mathbb{F} -vector space V . Prove that $\dim(W) \leq n$ and that, if $\dim(W) = n$, then $W = V$.
- (ii) Suppose U and W are distinct 4-dimensional subspaces of an \mathbb{F} -vector space V of dimension 6. Find all possible dimensions of $U \cap W$. Give examples in each case.
- (iii) Suppose U and W are 2-dimensional subspaces of \mathbb{F}^3 . Show that $U \cap W \neq \{0\}$.

(T8.2) Morphisms

Let V be a finite dimensional \mathbb{F} -vector space and $\varphi : V \rightarrow V$ a linear transformation. Show the equivalence of the following:

- a) φ is injective.
- b) φ is surjective.
- c) φ is bijective.

(T8.3) Images of lines and planes

Consider a linear transformation $\varphi : \mathbb{F}^n \rightarrow \mathbb{F}^n$. Show that the image under φ of a line (i.e., a 1-dimensional affine subspace) is either a point or again a line. Discuss the same problem for the image of a plane (i.e., a 2-dimensional affine subspace).

Does it make a difference if φ is assumed to have an inverse?