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8th Tutorial Sheet Linear Algebra I for MCS
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(T8.1) Dimension and dimension formula

(i) Let W be a subspace of an n-dimensional $\mathbb{F}$-vector space V. Prove that $\dim(W) \leq n$ and that, if $\dim(W) = n$, then $W = V$.

(ii) Suppose U and W are distinct 4-dimensional subspaces of an $\mathbb{F}$-vector space V of dimension 6. Find all possible dimensions of $U \cap W$. Give examples in each case.

(iii) Suppose U and W are 2-dimensional subspaces of $\mathbb{F}^3$. Show that $U \cap W \neq \{\mathbf{0}\}$.

(T8.2) Morphisms

Let V be a finite dimensional $\mathbb{F}$-vector space and $\varphi : V \to V$ a linear transformation. Show the equivalence of the following:

a) φ is injective.

b) φ is surjective.

c) φ is bijective.

(T8.3) Images of lines and planes

Consider a linear transformation $\varphi : \mathbb{F}^n \to \mathbb{F}^n$. Show that the image under φ of a line (i.e., a 1-dimensional affine subspace) is either a point or again a line. Discuss the same problem for the image of a plane (i.e., a 2-dimensional affine subspace).
Does it make a difference if φ is assumed to have an inverse?