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4th Tutorial Sheet Linear Algebra I for MCS
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(T4.1) Function spaces

Let A be any set, $\mathbb{F}$ a field. Let $\mathcal{F}(A, \mathbb{F})$ be the set of all functions $f : A \to \mathbb{F}$. Show that $\mathcal{F}(A, \mathbb{F})$ with point-wise addition and point-wise multiplication is an $\mathbb{F}$-vector space.
(A borderline case: is it ok to admit $A = \emptyset$?)

(T4.2) Vector spaces of polynomial functions

Let V be the set $\mathrm{Pol}_7(\mathbb{R})$, the space of polynomial functions of degree 7 over $\mathbb{R}$.
Show that V is an $\mathbb{R}$-vector space with point-wise addition and point-wise multiplication.
Determine whether or not W is a subspace of V where

(i) W consists of the degree 7 polynomials with integral coefficients.

(ii) W consists of the degree 7 polynomials with only even powers of x of the form $b_0 + b_1x^2 + b_2x^4 + b_3x^6$.

(T4.3) Polynomials vs polynomial functions

Consider a finite field $\mathbb{F}_p$. Two distinct polynomials over $\mathbb{F}_p$, $q(x) = \sum_{i=0}^{n} a_i x^i$ and $r(x) = \sum_{i=0}^{n} b_i x^i$ with $(a_0, \ldots, a_n) \neq (b_0, \ldots, b_n)$ may induce the same polynomial function.

(i) Give concrete examples of this phenomenon, e.g., over $\mathbb{F}_2$.

(ii) Give a combinatorial counting argument why this must occur for sufficiently large degrees n.

(iii) How is this different for the field $\mathbb{R}$?