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1. Exercise Sheet Linear Algebra I for MCS
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(E1.1) [Geometric interpretation of vectors.]

(i) Consider a (non-regular) hexagon $ABCDEF$. The following vertices and sides are given as vectors (points are given as vectors starting from the origin):

$$A = \binom{0}{-2} \quad D = \binom{1}{4} \quad \overrightarrow{AB} = \binom{-2}{2} \quad \overrightarrow{BC} = \binom{1}{3} \quad \overrightarrow{ED} = \binom{-3}{3} \quad \overrightarrow{FA} = \binom{-3}{-1}$$

Here we write vectors as columns instead of rows, $\binom{x_1}{x_2}$ *rather than* (x_1, x_2).

Can you draw the hexagon? Calculate $\overrightarrow{AE}$ and compare with your drawing.

(ii) Let $ABCD$ be an arbitrary quadrangle, where the vertices are given as vectors from the origin. Can you write down a condition that is true if and only if the quadrangle is a parallelogram?

(E1.2) [Parametric representation of a line.]

Consider the linear equation $E : ax_1 + bx_2 = c$ over $\mathbb{R}^2$.

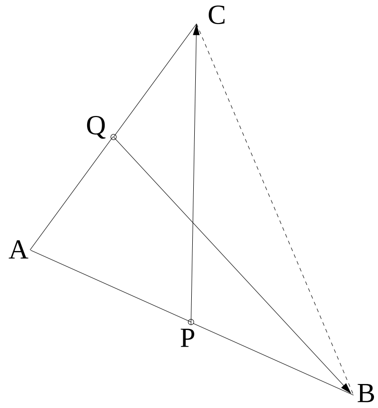
(i) Let $a = 2$, $b = 3$ and $c = 6$. Find in that case the parametric representation for $S(E)$.

(ii) Now in the general case, let $a \neq 0$. Find the parametric representation of $S(E)$.

(iii) Consider lines in $\mathbb{R}^3$.
Find a parametric representation of the line t through the points $(3, 0, 5)$ and $(1, 1, 1)$.

(E1.3) [Parametric representation.]

Draw a triangle ABC and consider the angle BAC with apex [*Scheitelpunkt*] A (we assume that the angle is strictly between 0 and π). Let P be the midpoint of AB and Q the midpoint of AC



(i) Describe the lines through CP and through BQ using a parametric representation involving only the vectors $\mathbf{b} = \overrightarrow{AC}$ and $\mathbf{c} = \overrightarrow{AB}$ and the point A.

(ii) Determine the intersection of the two lines from part(a).

(iii) If you take B or C as apex instead of A, you can argue in a similar way. Which well-known theorem about triangles can you now prove?

(E1.4) [Intersections of planes.]

Consider three arbitrary planes P_1, P_2 and P_3 in $\mathbb{R}^3$.

(i) What are the possible intersections of P_1 and P_2?

(ii) Now consider the intersection of P_1, P_2 and P_3 geometrically. Identify all possibilities and draw a picture for each case.

(iii) In the case of an empty intersection of all three planes: which intersection types between pairs of planes can we have.