Technische Universität Darmstadt

PhDs in Logic VIII

9th – 11th of May 2016

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Supported by:

DFG International Research Training Group 1529

Deutsche Forschungsgemeinschaft DFG

Technische Universität Darmstadt

Fachbereich Mathematik

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Welcome Address

Welcome to PhDs in Logic VIII! "PhDs in Logic" has been an ongoing, yearly tradition in the community of PhD students and young researchers in Logic. Since its start in 2009 "PhDs in Logic" took place (in chronological order) in Ghent, Tilburg, Brussels, Ghent, Munich, Utrecht and Vienna. We are happy to continue this series here in Darmstadt and we did our best to promote the spirit of interdisciplinarity and internationality.

This year's program involves 5 tutorials by established researchers in different fields and 18 presentations by PhD students on their research work in Mathematical Logic, Philosophical Logic and Logic in Computer Science. In total we can count 43 participants which come from various universities and institutes in Australia, Austria, Denmark, France, Germany, Japan, the Netherlands, Poland, Portugal, Romania, Russia, Sweden, Switzerland, USA and UK.

We hope that PhDs in Logic VIII will provide both a stimulating educational experience for all participants and a chance for cultivating contacts and future collaborations. Furthermore, we hope that the tradition of PhDs in Logic will continue strong in the future.

We wish you a pleasant stay and many fruitful discussions.

The organisers of PhDs in Logic VIII

Julian Bitterlich
Felix Canavoi
Daniel Körnlein
Angeliki Koutsoukou-Argyraki
Florian Steinberg

Acknowledgements

This event was supported by the International Reseach Training Group 1529, the DFG Project KO 1737/5-2, the Department of Mathematics of the Technische Universität Darmstadt and the Association for Symbolic Logic. We would also like to thank Betina Schubotz, the secretary of the Darmstadt Logic Group, as well as Esther Bauer, the secretary of the IRTG 1529, for their kind help. The booklet is based on the template 'A Basic Conference Abstract Booklet' created by LianTze Lim and licensed under Creative Commons CC BY 4.0.

Program

Monday, May 9th

9:00 - 9:30	Registration	
9:30 - 10:30	Nina Gierasimczuk (Tutorial) Topological Modelling of Knowledge Change (A)	p. 1
10:30 - 11:30	Mirna Džamonja (Tutorial) Independence Results at the Successors of Singular Cardinals (A)	p. 1
11:30 - 12:00	Coffee break	
12:00 - 12:30	Eduard Eiben Using Decomposition-Parameters for QBF: Mind the Prefix!	p. 8
12:30 - 13:00	Annemarie Borg Assumptive Hypersequent-Based Argumentation	p. 5
13:00 - 15:00	Lunch break	
15:00 - 15:30	Michael Lettmann Towards Π_2-Cut-Introduction	p. 21
15:30 - 16:00	Holger Thies On Data-Types for Multidimensional Functions in Exact Real Arithmetic	p. 32
16:00 - 17:00	Martin Otto (Tutorial) Bisimulation & Games (A)	p. 3
17:00 - 18:00	Piotr Kowalski (Tutorial) Model Theory of Fields with Operators (A)	p. 2
19:00 - ?	Social activity	

Tuesday, May 10th

9:00 - 10:00	Nina Gierasimczuk (Tutorial) Topological Modelling of Knowledge Change (B)	p. 1
10:00 - 11:00	Ulrich Kohlenbach (Tutorial) Proof Mining: Proof Interpretations and Their Use in Mathematics (A)	p. 2
11:00 - 11:30	Coffee break	
11:30 - 12:00	Dino Rossegger Enumerable Functors	p. 27
12:00 - 12:30	Matthias Hofer Game Semantical Rules for Vague Proportional Quantifiers	p. 17
12:30 - 13:00	Michał Tomasz Godziszewski Cofinal Elementary Cuts in Countable Models of Compositional Arithmetical Truth	p. 12
13:00 - 15:00	Lunch break	
15:00 - 15:30	Martin Sticht Multi-Agent Dialogue Games and Dialogue Sequents	p. 31
15:30 - 16:00	Malvin Gattinger A Model Checker for the Hardest Logic Puzzle Ever	p. 11
16:00 - 17:00	Mirna Džamonja (Tutorial) Independence Results at the Successors of Singular Cardinals (B)	p. 1
17:00 - 17:30	Bartosz Wcisło & Mateusz Łełyk Understanding the Strength of the Compositional Truth	p. 34
17:30 - 18:00	Dariusz Kalociński Bridging Recognition and Coordination of Semantics	p. 18
19:00 - 22:00	Dinner	

Wednesday, May 11th

9:00 - 10:00	Martin Otto (Tutorial) Bisimulation & Games (B)	p. 3
10:00 - 11:00	Ulrich Kohlenbach (Tutorial) Proof Mining: Proof Interpretations and Their Use in Mathematics (B)	p. 2
11:00 - 11:30	Coffee break	
11:30 - 12:00	Andrei Sipoş Proof Mining and Families of Mappings	p. 30
12:00 - 12:30	Antoine Mottet Reducts of finitely bounded homogeneous structures, and lifting tractability from finite-domain constraint satisfaction	p. 23
12:30 - 13:00	Martin Pape A Computable Spectral Theorem	p. 24
13:00 - 15:00	Lunch break	
15:00 - 15:30	Svenja Schalthöfer Definability of Cai-Fürer-Immerman Problems in Choiceless Polynomial Time	p. 28
15:30 - 16:00	Tiago Rezende de Castro Alves Synonymy and Identity of Proofs	p. 7
16:00 - 17:00	Piotr Kowalski (Tutorial) Model Theory of Fields with Operators (B)	p. 2
17:00 - 17:30	Mariusz Popieluch Toward a Similarity Semantics for Counterlogicals	p. 25
17:30 - 18:00	Wilfried Keller Dilution and its Elimination	p. 20

Contents

* Speaker

Abstracts of the Tutorials

Independence Results at the Successors of Singular Cardinals

Mirna Džamonja

University of East Anglia, United Kingdom

The universe of set theory is easy to change by forcing at the successors of regular cardinals, but this is not at all the case at the successors of singulars. This is to do with covering, discovered by Jensen, which says that unless there are large cardinals in the universe, the universe is very close to L and in particular it satisfies the singular cardinal hypothesis. So, even just to have different cardinal arithmetic, we have to assume large cardinals, and if we obtain independence results, the presence of such a large cardinal better be used. In particular, any forcing notion used must necessarily use large cardinals. In the tutorials we shall discuss some known techniques of forcing in these circumstances and will indicate further challenges.

Topological Modelling of Knowledge Change

Nina Gierasimczuk

DTU Compute, Denmark

Learning can be viewed as a process of adjusting one's knowledge on the basis of observations. So far, combining belief revision procedures with learning-theoretic notions led to many interesting observations about reliability and rationality of mind-change policies within various formal frameworks. In this tutorial we will focus on knowledge change modelled with the use of modal logic (dynamic epistemic logic) and inductive inference (finite and limiting identifiability). General topology allows neat characterisations of many concepts relevant for epistemic logic and inductive inference. In particular, we will see how learnability can be characterised with the use of topological separation properties and we will overview various aspects of employing topological semantics for epistemic logic. Finally, we will browse through open questions concerning the relationships between the two paradigms on topological grounds.

Proof Mining: Proof Interpretations and Their Use in Mathematics

Ulrich Kohlenbach

Technische Universität Darmstadt, Germany

This tutorial gives an introduction to an applied form of proof theory that has its roots in G. Kreisel's pioneering ideas on 'unwinding proofs' but has evolved into a systematic activity only during that last 20 years. The general approach is to apply proof theoretic techniques (notably specially designed proof interpretations) to concrete mathematical proofs with the aim of extracting new quantitative results (such as effective bounds) as well as new qualitative uniformity results from (prima facie ineffective) proofs. Logical metatheorems have been developed which guarantee the extractability of highly uniform bounds from large classes of proofs in nonlinear analysis. We will give both (i) an introduction to the underlying proof-theoretic methodology as well as (ii) a survey of some applications to fixed point theory, ergodic theory, optimization and operator theory.

Basic Literature

[1] U. Kohlenbach: "Applied Proof Theory: Proof Interpretations and their Use in Mathematics". Springer Monographs in Mathematics, xx+536pp., 2008.

Model Theory of Fields with Operators

Piotr Kowalski

University of Wrocław, Poland

The aim of my tutorial is to familiarize the audience with the basic notions and ideas of algebraic model theory. I will focus on the model theory of fields with a possible extra structure on a field. In the first hour, I will try to give a friendly introduction to model theory focusing more on concrete examples rather than abstract definitions. I will also show how to prove using model theory some classical results concerning the theory of fields (Ax's theorem about injectivity/surjectivity of polynomials and Hilbert's Nullstellensatz). In the second hour, I will discuss the model theory of fields with operators (derivation/automorphism) and its connections with some problems from diophantine geometry.

Bisimulation & Games: Model-Theoretic Aspects

Martin Otto
Technische Universität Darmstadt, Germany

Bisimulation is a game-based notion of structural equivalence, which embodies the quintessential format of back & forth equivalences in the model-theoretic tradition of Ehrenfeucht-Fraïssé games. As the name suggests, *bi-simulation* captures an essentially dynamic, behavioural equivalence between transition systems in terms of a challenge-response probing of possible transitions. In logical terms it captures a natural condition for two states to satisfy exactly the same formulae of (even infinitary) modal logic, whose characteristic quantification pattern also precisely explores the available transitions. In this tutorial I want to give an introduction to bisimulation equivalence, bisimulation games, and their model-theoretic uses in relation to both the classical picture from the perspective of modal logic and to the wider tradition of model-theoretic games starting from Ehrenfeucht-Fraïssé games for classical first-order logic. Among the classical model-theoretic results to be discussed in some detail are van Benthem's characterisation of modal logic as the bisimulation-invariant fragment of first-order logic, as well as some of its extensions and variations that require other than the classical model-theoretic techniques, and put the focus the combinatorics of bisimulation-respecting model constructions.

Abstracts of the Contributed Talks

Assumptive Hypersequent-Based Argumentation

Annemarie Borg
Ruhr University Bochum

Defeasible reasoning (DR) is a type of inference where the truth of the conclusion is not warranted by the truth of the premises, rather, the conclusion is plausible or likely to hold in view of the premises. One of the major challenges, when formally modeling DR, lies in its dynamic character: defeasible inferences are sometimes retracted in view of new information. One way to model DR is by using argumentation frameworks (AFs), where an argument is only warranted in case it can be defended against counterarguments. AFs, as originally introduced by Dung [9], represent this idea by means of directed graphs where nodes are abstract representations of arguments and arcs represent argumentative attacks.

However, because of its abstract character, Dung's account is often not expressive enough when it comes to applications. Another representation of AFs, building upon Dung's, is logical argumentation, where a structure for arguments is provided in terms of formal languages (see e.g. [8, 12, 14, 15]).

The focus of this talk will be on sequent-based argumentation, as introduced by Arieli [1] and in more detail by Arieli and Straßer [3], which is based on proof-theoretical methods introduced by Gentzen [10]. In their framework, arguments are represented by sequents ($\Gamma \Rightarrow \Delta$, where Γ and Δ are sets of formulas and "$\Rightarrow$" is a reserved symbol), which are derivable using a sequent calculus of a given underlying core logic, and attacks are represented by so-called sequent elimination rules. There are several advantages to representing arguments like this. For instance, different core logics can be used, there is no restriction on the support set Γ (such as consistency or minimality) and arguments can be automatically constructed and identified by means of sequent calculi. The acceptability of arguments is based on Dung's semantics [9] which are applied to the resulting AF. Recently, also a dynamic proof theory has been presented [2], in which elimination rules are used for the retraction of defeasible inferences.

In this talk I will generalize the existing framework, by letting hypersequents, as independently introduced by Avron [4], Mints [11] and Pottinger [13], represent the arguments. Hypersequents are generalizations of Gentzen's sequents. For logics, such as the modal logic $\mathsf{S5}$ and the substructural logics RM and RMI_m, for which no finite cut-free sequent calculus exists, hypersequents provide a system in which a cut-free sequent calculus can be expressed [4]. A hypersequent is a finite sequence of sequents, divided by "$|$" that intuitively represents a choice of sequents, which is true if and only if at least one of its components (a sequent) is true relative to some semantics. By replacing usual sequents with hypersequents, more core logics can be used as a basis for the AFs in the sequent-based system of Arieli and Straßer.

Additionally, the enhanced expressibility of the generalized system is useful for the encoding of defeasible assumptions. For example, a hypersequent of the form $\Gamma_1 \Rightarrow \Delta_1 \mid \Gamma_2 \Rightarrow \Delta_2$ can be interpreted in such a way that the left most component, $\Gamma_1 \Rightarrow \Delta_1$, will be reserved for representing normality assumptions. The framework that is obtained in this way provides a basis for hypersequent-based argumentation with assumptions. I will exemplify its high expressive power by showing that adaptive logics (ALs) [6, 16] can be embedded in it.

I will demonstrate how ALs, based on the paraconsistent logic CLuN [7], can be embedded in the hypersequent system. The cut-free sequent calculus for CLuN as presented in [5] will be generalized into a cut-free hypersequent system. To handle the abnormalities, sequent rules are added.

If time allows, I will present a dynamic proof theory, that allows for defeasible derivations. During a dynamic derivation a derived sequent can be eliminated (in terms of sequent-based argumentation theory). As part of the hypersequent calculus, rules are defined for the elimination of sequents for both strategies. The dynamic proof theory will be useful for automated reasoning.

The resulting hypersequent system, adjusted to encode defeasible assumptions, with rules for the handling of normality assumptions and the elimination of sequents, provides a highly expressive AF. Any AL, based on a logic that has a corresponding (hyper)sequent system, can be embedded in it and hence can be used as the core logic for a logical AF. This way, some of the conflict-management mechanisms of ALs are integrated within argumentation.

References

[1] Ofer Arieli. "A Sequent-Based Representation of Logical Argumentation". In: *Computational Logic in Multi-Agent Systems*. Ed. by Joo Leite et al. Vol. 8143. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2013, pp. 69–85.

[2] Ofer Arieli and Christian Straßer. "Dynamic Derivations for Sequent-Based Logical Argumentation". In: *Proceedings COMMA'14*. Frontiers in Artificial Intelligence and Applications 266. IOS Press, 2014, pp. 89–100.

[3] Ofer Arieli and Christian Straßer. "Sequent-based logical argumentation". In: *Argument & Computation* 6.1 (2015), pp. 73–99.

[4] Arnon Avron. "The method of hypersequents in the proof theory of propositional non-classical logics". In: *Logic: Foundations to Applications*. Oxford Science Publications, 1996, pp. 1–32.

[5] Arnon Avron, Beata Konikowska, and Anna Zamansky. "Cut-free sequent calculi for C-systems with generalized finite-valued semantics". In: *Journal of Logic and Computation* 23.3 (2013), pp. 517–540.

[6] Diderik Batens. "A Universal Logic Approach to Adaptive Logics". English. In: *Logica Universalis* 1.1 (2007), pp. 221–242.

[7] Diderik Batens. "Paraconsistent extensional propositional logics". In: *Logique et Analyse* 90–91 (1980), pp. 195–234.

[8] Philippe Besnard and Anthony Hunter. "A logic-based theory of deductive arguments". In: *Artificial Intelligence* 128.12 (2001), pp. 203–235.

[9] Phan Minh Dung. "On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games". In: *Artificial Intelligence* 77.2 (1995), pp. 321–357.

[10] Gerhard Gentzen. "Untersuchungen über das logische Schließen I, II". In: *Mathematische Zeitschrift* 39 (1934), pp. 176–210, 405–431.

[11] Grigori E. Mints. "Lewis' systems and system T (1965–1973)". In: *R. Feys "Modal Logic" (Russian Translation)*. Nauka, 1974, pp. 422–501.

[12] John L. Pollock. “How to reason defeasibly”. In: *Artificial Intelligence* 57.1 (1992), pp. 1–42.

[13] Garrell Pottinger. “Uniform, Cut-free formulations of T, S4 and S5”. In: *Journal of Symbolic Logic* 48 (1983). Abstract, pp. 900–901.

[14] Henry Prakken. “Two approaches to the formalisation of defeasible deontic reasoning”. In: *Studia Logica* 57.1 (1996), pp. 73–90.

[15] Guillermo R. Simari and Ronald P. Loui. “A mathematical treatment of defeasible reasoning and its implementation”. In: *Artificial Intelligence* 53.23 (1992), pp. 125–157.

[16] Christian Straßer. *Adaptive Logics for Defeasible Reasoning. Applications in Argumentation, Normative Reasoning and Default Reasoning.* Vol. 38. Trends in Logic. Springer, 2014.

Synonymy and Identity of Proofs

Tiago Rezende de Castro Alves

Eberhard-Karls-Universität Tübingen

I intend to present a general outline of a central aspect of my doctoral research, the main subject of which is the relatively well-known problem of identity of proofs. Kosta Dosen argues that, in case all proofs of a certain conclusion from certain assumptions were to be regarded as identical, the whole field of so-called general proof theory would be trivial (*Identity of Proofs Based on Normalization and Generality*, 2004); together with Zoran Petric, he thus claims that identity of proofs is in fact the central question of general proof theory (*Isomorphic Formulae in Classical Propositional Logic*, 2012). Nevertheless, it is — even if considered only after the mathematical formulations given to it by the literature in the seventies — a rather elusive one.

Firstly, I shall argue that: (a) accounting for a notion of identity of proofs, as outlined by authors such as Kreisel (*A survey of proof theory II*, 1971) and Prawitz (*Ideas and results in proof theory*, 1971) in the early 1970s, should not — unlike Kreisel clearly suggests — be understood as a quest for a way of identifying synonymous derivations; and that (b) from a conceptual point of view, characterizing adequately a notion of synonymy of proofs is a necessary condition for any sensible answer to the problem of identity of proofs.

Bearing this in mind, I shall propose a rudimentary notion of synonymy whose aim is accounting for synonymy of natural deduction derivations in propositional logic. The idea behind it is that whenever we derive synonymous conclusions from respectively synonymous sets of premises by means of applications of isomorphic inference rules — in a sense to be specified — , then we have synonymous derivations. On the base cases, viz. atoms and propositional connectives, synonymy is defined respectively in terms of interderivability and in terms of a very strict correspondence relation between the syntactical structures of the inference rules of the connectives.

Logical accounts of synonymy are by no means a novelty. Some semantic puzzles raised as early as 1892 by Frege in his famous late work *ber Sinn und Bedeutung* appear to have served as a trigger to the emergence of some interesting general accounts of synonymy (or akin notions, e.g.*intensional isomorphism*) in the literature. Eminent examples are those by authors such as Carnap (*Meaning and Necessity*, 1947), Montague (*English as a formal language*, 1970) and Moschovakis (*A logical calculus of meaning and synonymy*, 2006). The one to be put forward now, however, is quite different in spirit from these, especially due to its considerably more modest goals and its proof-theoretic rather than model-theoretic characterization. Unlike the proof-theoretic proposal put forward by Dosen and Petric (*Isomorphic Formulae in Classical Propositional Logic*, 2012), it does not depend on the previous acceptance of some criterion of identity of proofs — which is very important, for one of its aims is precisely to contribute as an adequacy criterion for identifying proofs. Furthermore, none of the mentioned authors accounts of synonymy contemplate derivations; the present one, on the other hand, treats derivations in a most natural way.

It shall then be pointed out that both Prawitzs and Lambeks criteria for identifying proofs — respectively, that two derivations are identical (i.e. represent the same proof) if and only if they are $\beta\eta$-equivalent (or, equivalently, if and only if they reduce to the same $\beta\eta$-normal derivation); and that two derivations are identical if and only if they generalise in the same direction — quite obviously fail to identify some synonymous derivations (including some from the same set of assumptions to the same conclusion).

Using Decomposition-Parameters for QBF: Mind the Prefix!

Eduard Eiben*, Robert Ganian and Sebastian Ordyniak

Technische Universität Wien

Many important computational tasks such as verification, planning, and several questions in knowledge representation and automated reasoning can be naturally encoded as the problem of evaluating quantified Boolean formulas [5, 8, 11, 14], a generalization of the propositional satisfiability problem (SAT). In recent years quantified Boolean formulas have become a very active research area. The problem of evaluating quantified Boolean formulas, called QBF, is the archetypical PSPACE-complete problem and is therefore believed to be computationally harder than the NP-complete propositional satisfiability problem [7, 10, 16].

In spite of the close connection between QBF and SAT, many of the tools and techniques which work for SAT are not known to help for QBF, and this is especially true for so-called decomposition-based techniques [1]. Such techniques use various kinds of decompositions to capture the structure of the input, leading to efficient algorithms for computing solutions with runtime guarantees. Decomposition-based techniques are tied to a numerical *parameter* k, which represents the fitness of the decomposition. The goal is then to obtain algorithms whose running time is polynomial in the input size n and exponential only in k, i.e., with a running time of $f(k) \cdot n^{\mathcal{O}(1)}$ where f is some computable function; such algorithms are called *fixed parameter tractable (FPT) algorithms*. Prominent examples of decompositions used in such techniques include decompositions for the structural parameters *treewidth* [12], *pathwidth* [13], *clique-width* [3] and *rank-width* [9]; all of these are known to support FPT algorithms for SAT [17, 6], but the same is not true for QBF [2] under established complexity assumptions.

In this work we introduce and develop *prefix pathwidth*, which is, to the best of our knowledge, the first decomposition-based parameter, with the size of the formula and number of quantifier alternations not bounded in terms of the parameter, that allows an FPT algorithm for QBF. Prefix pathwidth is an extension of pathwidth which takes into account not only the structure of clauses in the formula, but also the structure contained in the quantification of variables. To achieve the latter, we make use of the *dependency schemes* introduced by Samer and Szeider [15]. Dependency schemes capture how the assignment of individual variables in a QBF depends on other variables, and research in this direction has uncovered a large number of distinct dependency schemes.

Prefix pathwidth can be used in conjunction with any dependency scheme that is cumulative [15], which holds for almost all known dependency schemes. In practice, using different dependency schemes may lead to better prefix path-decompositions, in turn resulting in significantly faster algorithms.

In their full generality, our main results on solving QBF using prefix pathwidth can be separated into two steps:

1. using a prefix path-decomposition of small prefix pathwidth to solve the given QBF I, and
2. finding a suitable prefix path-decomposition to be used for step 1.

We resolve the first task by applying advanced dynamic programming techniques on partial existential strategies for the Hintikka game (see, e.g., the work of Grädel et al. ([4])) played on the QBF. Essentially, the game approach allows us to translate the question of whether a QBF is true to the question of whether there exists a winning strategy for one player in the Hintikka game. We show that although the number of such strategies is unbounded, at each point in the prefix path-decomposition there is only a small number of partial strategies on the processed vertices that need to be considered. Thus we obtain:

Theorem 1. QBF *is FPT parameterized by the width of a prefix path-decomposition w.r.t. any cumulative dependency scheme, when such a decomposition is provided as part of the input.*

Resolving step 2 boils down to a graph-algorithmic problem which is related to the problem of computing various established parameters of directed graphs, such as directed pathwidth or directed treewidth. It is an important open problem of whether computing these parameters is FPT or not [18], and the same obstacles seem to also be present for computing our parameter in the general sense. To bypass this barrier, we develop new algorithmic techniques for computing prefix path-decompositions; the efficiency of the obtained algorithm in this case additionally depends on the *poset-width* (i.e., the size of a maximum anti-chain) of the dependency relation.

In combination with the previous Theorem 1, we get our main general contribution, formalized in Theorem 2 below.

Theorem 2. *Let τ be a fixed cumulative dependency scheme. There exists an FPT algorithm which takes as input a* QBF *I and decides whether I is true in time $f(k, w) \cdot |I|^{\mathcal{O}(1)}$, where f is a computable function, k is the prefix pathwidth and w is the poset-width of I w.r.t. τ.*

Our results also have implications for the tractability of QBF with respect to other parameters. For example, we show that QBF is FPT when parameterized by the *vertex cover number* of the matrix (irrespective of the prefix).

References

[1] Markus Aschinger et al. "Structural Decomposition Methods and What They are Good For". In: *28th International Symposium on Theoretical Aspects of Computer Science, STACS 2011, March 10-12, 2011, Dortmund, Germany.* 2011, pp. 12–28.

[2] Albert Atserias and Sergi Oliva. "Bounded-width QBF is PSPACE-complete". In: *J. Comput. Syst. Sci.* 80.7 (2014), pp. 1415–1429.

[3] B. Courcelle and S. Olariu. "Upper bounds to the clique-width of graphs". In: *Discr. Appl. Math.* 101.1-3 (2000), pp. 77–114.

[4] E. Grädel et al. *Finite Model Theory and Its Applications (Texts in Theoretical Computer Science. An EATCS Series).* Springer, 2005. ISBN: 3540004289.

[5] Uwe Egly et al. "Solving Advanced Reasoning Tasks Using Quantified Boolean Formulas". In: *Proceedings of the Seventeenth National Conference on Artificial Intelligence and Twelfth Conference on on Innovative Applications of Artificial Intelligence, July 30 - August 3, 2000, Austin, Texas, USA.* 2000, pp. 417–422.

[6] Robert Ganian, Petr Hliněný, and Jan Obdržálek. "Better Algorithms for Satisfiability Problems for Formulas of Bounded Rank-width". In: *Fundam. Inform.* 123.1 (2013), pp. 59–76.

[7] Hans Kleine Büning and Theodor Lettman. *Propositional logic: deduction and algorithms*. Cambridge University Press, Cambridge, 1999.

[8] Charles Otwell, Anja Remshagen, and Klaus Truemper. "An Effective QBF Solver for Planning Problems". In: *Proceedings of MSV/AMCS*. CSREA Press, 2004, pp. 311–316.

[9] Sang-il Oum and P. Seymour. "Approximating Clique-width and Branch-width". In: *J. Combin. Theory Ser. B* 96.4 (2006), pp. 514–528.

[10] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley, 1994.

[11] J. Rintanen. "Constructing Conditional Plans by a Theorem-Prover". In: *J. Artif. Intell. Res.* 10 (1999), pp. 323–352.

[12] Neil Robertson and P. D. Seymour. "Graph minors. II. Algorithmic aspects of tree-width". In: *J. Algorithms* 7.3 (1986), pp. 309–322.

[13] Neil Robertson and Paul D. Seymour. "Graph minors. I. Excluding a forest". In: *J. Comb. Theory, Ser. B* 35.1 (1983), pp. 39–61.

[14] Ashish Sabharwal et al. "QBF Modeling: Exploiting Player Symmetry for Simplicity and Efficiency". In: *Theory and Applications of Satisfiability Testing - SAT 2006, 9th International Conference, Seattle, WA, USA, August 12-15, 2006, Proceedings*. 2006, pp. 382–395.

[15] Marko Samer and Stefan Szeider. "Backdoor sets of quantified Boolean formulas". In: *Journal of Automated Reasoning* 42.1 (2009), pp. 77–97.

[16] L. J. Stockmeyer and A. R. Meyer. "Word problems requiring exponential time". In: *Proc. Theory of Computing*. ACM, 1973, pp. 1–9.

[17] Stefan Szeider. "On fixed-parameter tractable parameterizations of SAT". In: *Theory and Applications of Satisfiability, 6th International Conference, SAT 2003, Selected and Revised Papers*. Ed. by Enrico Giunchiglia and Armando Tacchella. Vol. 2919. Lecture Notes in Computer Science. Springer Verlag, 2004, pp. 188–202.

[18] Hisao Tamaki. "A Polynomial Time Algorithm for Bounded Directed Pathwidth". In: *Graph-Theoretic Concepts in Computer Science - 37th International Workshop, WG 2011, Teplá Monastery, Czech Republic, June 21-24, 2011. Revised Papers*. 2011, pp. 331–342.

A Model Checker for the Hardest Logic Puzzle Ever

Malvin Gattinger
University of Amsterdam

Puzzles about knights and knaves have been around for a long time, in particular made popular by Raymond Smullyan. The most famous version is the following from George Boolos [1]:

> Three gods A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for "yes" and "no" are "da" and "ja", in some order. *You do not know which word means which.*

This "Hardest Logic Puzzle Ever" and even harder variations recently received attention from philosophical and logical perspectives [3,6,2]. Besides being entertaining, such riddles motivate a critical and formal analysis of knowledge and communication.

In [2] the puzzle inspired a new variant of Dynamic Epistemic Logic which can describe questions, agent types and utterances. Examples of agent types are truth tellers, liars and bluffers: Depending on which type an agent has, what they say can take on a different meaning. If the type of a speaker is unknown, multiple possible meanings of what they said have to be considered. Additionally, the number of possible meanings increases further if the meaning of utterances like "da" in the puzzle above is unknown. Still, using the right kind of questions, it is possible to rule out some possibilities, no matter what the answer will be. For example, we can ask B "If you were asked whether A is a bluffer, would you say *ja*?" as a first step of solving the puzzle.

The formal framework presented in [2] allows us to make this reasoning fully precise, including which questions are asked, who is answering them, of which type they are, which utterance they use to answer and which meanings this utterance has. For example, the nested question just mentioned is given by the formula $?_B([?_B(\mathsf{Bluffer}(A)]\langle !_B\mathsf{ja}\rangle\top)$.

One of the motivations for this formal approach given by the authors of [2] was to save effort by using model checking. However, so far no published implementation exists. We picked up this task and implemented the logic with questions, utterances and agent-types in Haskell. Our implementation is self-contained but uses similar methods as DEMO-S5 [5] and SMCDEL [4].

Alongside a standard recursive model checking algorithm – which takes a model and a formula as input and returns true or false – we provide functions to work with puzzles in a more intuitive manner. The resulting program can verify most claims from [2] in fractions of seconds. In particular this allows us to automatically verify solutions to the Hardest Logic Puzzle Ever.

We make heavy use of Haskell features like infix notation, custom data types and pattern matching to make the code readable and self-documenting. As an example, here is an excerpt from the code which defines puzzles, question strategies and a function which checks whether a strategy is a correct solution:

```
type Puzzle   = (Model,QUForm)
type Question = (AgentName,QUForm)
data Strategy =
  FState
  | QState Question [(Utterance,Strategy)]

solves :: Strategy -> Puzzle -> Bool
solves strategy (model,goal) = all
  (qIsValid model . seq2f goal)
  (sequencesOf strategy)
```

This uses the functions `sequencesOf` and `seq2f` which translate a strategy and a goal to a formula that is true if and only if this strategy ensures that goal. The user can now check a given strategy simply by typing `mystrategy `solves` thispuzzle`.

In the future we are planning to i) use bounded search to find solutions automatically and verify unsolvability results [6], ii) implement the translations to public announcement logic to use them for model checking and compare the performance, iii) extend the framework to cover more complex agent types, for example answering to the previous question, and iv) automate the generation of new puzzles, together with human-readable descriptions and solutions.

The source code is available at `https://github.com/m4lvin/mchlpe`.

References

[1] George Boolos. The hardest logic puzzle ever. *The Harvard Review of Philosophy*, 6(1):62–65, 1996.

[2] Fenrong Liu and Yanjing Wang. Reasoning about agent types and the hardest logic puzzle ever. *Minds and Machines*, 23(1):123–161, 2013.

[3] Gabriel Uzquiano. How to solve the hardest logic puzzle ever in two questions. *Analysis*, 70(1):39–44, 2010.

[4] Johan van Benthem, Jan van Eijck, Malvin Gattinger, and Kaile Su. Symbolic model checking for dynamic epistemic logic. In *Proceedings of The Fifth International Conference on Logic, Rationality and Interaction (LORI-V)*, 2015.

[5] Jan van Eijck. DEMO—a demo of epistemic modelling. In *Interactive Logic. Selected Papers from the 7th Augustus de Morgan Workshop, London*, volume 1, pages 303–362, 2007.

[6] Gregory Wheeler and Pedro Barahona. Why the hardest logic puzzle ever cannot be solved in less than three questions. *Journal of Philosophical Logic*, 41(2):493–503, 2011.

Cofinal Elementary Cuts in Countable Models of Compositional Arithmetical Truth

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We study certain model-theoretic properties of countable recursively saturated models of arithmetic. The study of possible semantics for arithmetized languages in nonstandard models has been a lively research field since the seminal paper of A. Robinson [11]. Our primary inspiration for examining mathematical features of such structures, and recursively saturated in particular, is that every countable recursively saturated model of Peano Arithmetic supports a great variety of nonstandard satisfaction classes that can serve as models for formal theories of truth - those models allow to investigate the role of arithmetic induction in semantic considerations. In the other direction, nonstandard satisfaction classes are used as a tool in model theoretic constructions providing answers to questions in the model theory of formal arithmetic and often allow to solve problems that do not explicitly involve nonstandard semantics.

A satisfaction class is a subset of the model of PA corresponding to the notion of truth in nonstandard models (see S. Krajewski [9], F. Engstrom [3] and H. Kotlarski [8]). We provide the definition via a forma truth theory axiomatizing Tarski's compositional conditions (with an undisturbing abuse of temrinology, substituting truth for satisfiability).

Definition 1. (Stratified Compositional Truth Theory)
Stratified Compositional Truth Theory (CT^{-}) *is an axiomatic theory obtained from* PA *by adjoning to it the following axioms:*
(CT1) $\forall s,t \in Trm[Tr(s=t) \equiv val(s)=val(t)]$,
(CT2) $\forall x[Sent_{\mathcal{L}}(x) \Rightarrow (Tr(\neg x) \equiv \neg Tr(x))]$,
(CT3) $\forall x,y[(Sent_{\mathcal{L}}(x \wedge y)) \Rightarrow (Tr(x \wedge y) \equiv Tr(x) \wedge Tr(y))]$,

(CT4) $\forall x, y[(Sent_{\mathcal{L}}(x \vee y)) \Rightarrow (Tr(x \vee y) \equiv Tr(x) \vee Tr(y))]$,
(CT5) $\forall v, x[Sent_{\mathcal{L}}(\forall vx) \Rightarrow (Tr(\forall vx) \equiv \forall t Tr(x(t/v)))]$,
(CT6) $\forall v, x[Sent_{\mathcal{L}}(\exists vx) \Rightarrow (Tr(\exists vx) \equiv \exists t Tr(x(t/v)))]$,

where by $Sent_{\mathcal{L}}(x)$ we mean that x is the Gödel number af an arithmetical (without any occurence of the truth predicate) sentence of the arithmetical language $\mathcal{L}$. Let us note that there are no instances of the induction axiom scheme for formulae of the extended language $\mathcal{L}_{Tr}$ other then those for formulae of the original language of PA among the axioms of CT^{-}. A sentence $(\varphi(0) \wedge \forall x(\varphi(x) \Rightarrow \varphi(s(x)))) \Rightarrow \forall x \varphi(x)$ is an axiom of CT^{-} only if $\varphi \in Frm_{\mathcal{L}}$, i.e. the truth predicate Tr does not occur in φ.[1]

Definition 2. *Let $\mathcal{M}$ be a model of PA. A set $S \subseteq |\mathcal{M}|$ is a* ***full satisfaction class*** *for $\mathcal{M}$ if and only if $(\mathcal{M}, S) \models CT^{-}$.*

Although we will not prove this fact here, it is worth noting that not each countable model of PA admits a full satisfaction class.

Definition 3. *A set p of the formulae of the language $\mathcal{L}_{\mathcal{M}}$ (i.e. the language $\mathcal{L}$ extended with a constant name for every element of the model $\mathcal{M}$) with exactly one free variable x is* ***finitely satisfied*** *in $\mathcal{M}$ if and only if for any finite $q \subset p$ there exists an $a \in |\mathcal{M}|$ such that for any $\varphi(x) \in q$ $\mathcal{M} \models \varphi(a)$. A* ***type*** *over a model $\mathcal{M}$ is a finitely satisfied set of formulae of the form $\varphi(x, b)$ with exactly one free variable x and at most one parameter $b \in M$. A type p over $\mathcal{M}$ is* ***recursive*** *if and only if the set of codes of formulae $\varphi(x, y)$ such that $\varphi(x, b) \in p$ is recursive. A type p over $\mathcal{M}$ is (globally)* ***realised*** *if and only if there exists an $a \in M$ such that for any $\varphi(x, b) \in p$ $\mathcal{M} \models \varphi(a, b)$. Model $\mathcal{M}$ of PA is* ***recursively saturated*** *if and only if each recursive type over $\mathcal{M}$ is realised.*

Theorem 1. (**Lachlan's theorem**, A. Lachlan [10], also see R. Kaye [4], p. 150, pp. 228-233) *If a nonstandard model $\mathcal{M} \models (PA)$ admits a full satisfaction class, then $\mathcal{M}$ is recursively saturated.*

Theorem 2. (KKL) (H. Kotlarski and S. Krajewski and A. Lachlan [7], J. Barwise and J. Schlipf [1], A. Enayat, and A. Visser [2])
If a countable model $\mathcal{M} \models (PA)$ is recursively saturated, than it admits a full satisfaction class.

Therefore, for a countable model of arithmetic, it is equivalent to admit a full satisfaction class (i.e. satisfy the formal theory of compositional truth) and to be recursively saturated. Therefore the project can be thought of as an investigation into structure of possible interpretations for theory of compositional, arithmetical truth. It needs to be underlined that the purpose of our research is to examine model-theoretic but purely arithmetical properties

[1] The system CT^{-} is called **stratified** since in the axioms $(CT2) - (CT6)$ all the sentences being in the scope of the quantifiers are the sentences of the language $\mathcal{L}$. The axiom $(CT1)$ speaks only of $\mathcal{L}$-sentences because equational theorems are all sentences of $\mathcal{L}$.. A system that is obtained from CT^{-} by adjoining to it all the instances of induction formulated in the ful language $\mathcal{L}_{Tr}$ is called CT.

of models admitting satisfaction classes. In particular, we study various substructures of recursively saturated models of PA.

We study here certain logical properties of cofinal extensions of models of PA. It might be said in doing so we follow the advice of Smorynski expressed in [12]:

> A relatively neglected aspect of the study of nonstandard models of arithmetic is the study of their cofinal extensions. These extensions certainly do not present themselves to the intuition as readily as do their more popular cousins the end extensions; but they are not exactly shrouded in mystery or unnatural objects of study either. They are equal partners with end extensions in the construction of general extensions of models; they offer both special advantages and disadvantages worthy of our interest; and, occasionally, they are useful in understanding the generally more simply behaved end extensions. Cofinal extensions deserve more attention than they have traditionally received.

First-order theories of pairs (N, M), where $N \models PA$ and M is an elementary cofinal submodel of M reveal great diversity and demand systematic study. The case of models admitting satisfaction classes is of particular interest in this respect: all countable recursively saturated models of PA have continuum many nonisomorphic cofinal submodels, and after acknowledging the variety of the abovementioned pairs for N being countable recursively saturated, the next goal is to consider isomorphism types and first-order theories for pairs of models (N, M) for a fixed countable recursively saturated model N and a fixed isomorphism type of M. The method that has already been shown quite effective in this direction is the method of *gaps* (also called *skies*). We present briefly the gap terminology and explain why it is useful.v

Skolem terms, also called simply definable functions[1], are paramter-free definable and PA-provably total functions. Let $\mathcal{M}$ be a nonstandard model of arithmetic and let $\mathcal{F}$ be some family of Skolem terms $f : M \to M$ such that $\forall x, y \in M\ x < y \Rightarrow x \leq f(x) \leq f(y)$. There is a partition of M into sets, which we call $\mathcal{F}$-gaps. For any $a \in M$, $gap_{\mathcal{F}}(a)$ is the smallest set $C \subseteq M$ such that $a \in C$ and:

$$\forall b \in C\, \forall f \in \mathcal{F}\, \forall x \in M\ \ b \leq x \leq f(b)\ \vee\ x \leq b \leq f(x)) \Rightarrow x \in C.$$

This is a natural generalization of an idea of partitioning the universe of a nonstandard model into $\mathbb{Z}$-blocks around each element (then, each such block is $gap_{\mathcal{F}}(a)$ for some a, where $\mathcal{F}$ consists only of the successor function s).

Definition 4. *The gap of $a \in M$, denoted by $gap(a)$, is the $\mathcal{F}$-gap of a, where $\mathcal{F}$ is the family of* ***all*** *such definable functions, i.e.*

$$\mathcal{F} = \{f : M \to M : f \text{ is definable and } \forall x, y \in M\ x < y \Rightarrow x \leq f(x) \leq f(y)\}.$$

[1] with a slight abuse of terminology that is unimportant to our investigations

Definition 5. *For any model $\mathcal{M}$, an* ***initial segment*** *of $\mathcal{M}$, denoted $\mathcal{I} \subseteq_{end} \mathcal{M}$, is such a subset $\mathcal{I} \subseteq \mathcal{M}$ that is closed* downwards*, i.e. $\forall n \in \mathcal{I}$ and $\forall a \in \mathcal{M}$ if $\mathcal{M} \models a < n$, then $a \in \mathcal{I}$. An initial segment $\mathcal{I} \subseteq_{end} \mathcal{M}$ is a* ***cut*** *of $\mathcal{M}$, denoted $\mathcal{I} \subseteq_{cut} \mathcal{M}$ if it is nonempty and closed under successor, i.e. $\forall n \in \mathcal{M}\,(n \in \mathcal{I} \Rightarrow s(n) \in \mathcal{I})$. A cut $\mathcal{I} \subseteq_{cut} \mathcal{M}$ is a* ***proper cut*** *if $\mathcal{I} \neq \mathcal{M}$ (fact: any countable and nonstandard $\mathcal{M} \models PA$ has $2^{\aleph_0}$ proper cuts that are closed under $+, \times$).*

Every model $\mathcal{M}$ has the least gap, the $gap(0)$. Let $A \subseteq M$. Then, we denote $sup(A) = \{x \in M : \exists y \in A\, x \leq y\}$. If for some $a \in M$, $M = sup(gap(a))$, then we call $gap(a)$ the **last gap of $\mathcal{M}$**. A model with a last gap is called **short**. If $\mathcal{M} \preceq_{cut} \mathcal{N}$ (i.e. $\mathcal{M}$ is an elementary cut of $\mathcal{N}$), we say that $\mathcal{M}$ is **short elementary cut** of $\mathcal{N}$ if $\mathcal{M}$ is short - in other words, if by $Scl(a)$ we denote the set $\{t(a) :\ t \text{ is a Skolem term of PA}\}$ $\mathcal{M}$ is short if there is such an element $a \in M$ that its Skolem closure in $\mathcal{M}$ is cofinal in $\mathcal{M}$, i.e. for all $x \in M$ there is $b \in Scl(a)$ such that $x <_{\mathcal{M}} b$. An elementary cut is **coshort** if $\mathcal{N} \setminus \mathcal{M}$ has the least gap, i.e. there is $a \in N \setminus M$ s.t. $M = inf(gap(a))$, where $inf(A) = \{x \in M : \forall y \in A\, x \leq y\}$.

Now, to clarify the gap terminology, if we put:

- $\mathcal{M}(a) = sup(Scl(a))$, and
- $\mathcal{M}[a] = \{b \in M :\ \forall t \in Scl(b)\, t(b) < a\}$,

then the set $[a) = \mathcal{M}(a) \setminus \mathcal{M}[a]$ is exactly the $gap(a)$. It can be shown that $\mathcal{M}(a)$ is the smallest elementary cut of $\mathcal{M}$ containing a, and that $\mathcal{M}[a]$ is empty if and only if everey elementary cut of $\mathcal{M}$ contains a. Gap terminology is particularly useful in the study of recursively saturated models of PA (see e.g. [5] for a reference to many methods and properties).

One of the interesting and natural questions conerning *pairs* for countable recursively saturated models of arithmetic and its cofinal submodels is the following *big* question of our particular interest:

Let $\mathcal{M} \models PA$ be a countable recursively saturated model and let $\mathcal{K}, \mathcal{K}'$ be elementary cuts of $\mathcal{M}$. Suppose that $(\mathcal{M}, \mathcal{K}) \equiv (\mathcal{M}, \mathcal{K}')$. Does it follow that $(\mathcal{M}, \mathcal{K}) \cong (\mathcal{M}, \mathcal{K}')$?

Another way to put it is: under what conditions, does the identity of theories of such pairs imply their isomoprhism? It is known that the answer to the *big* question above is negative, if $\mathcal{K}$ and $\mathcal{K}'$ in question are coshort, as shown by R. Kossak and J. Schmerl in [6]. However, it remains open (and is considered to be difficult) what is the answer for the case in which $\mathcal{K}$ and $\mathcal{K}'$ are short elementary cuts of $\mathcal{M}$, i.e. are of the form $\mathcal{M}(a)$ and $\mathcal{M}(b)$ for some $a, b \in M$.

Since it is not hard to prove the equivalence that there exists an automorphism of such $\mathcal{M}$ if and only if $tp(a) = tp(b)$ (in the purely arithmetical language $\mathcal{L}$), where $tp(a) = \{\varphi(x) : \mathcal{M} \models \varphi(a)\}$ is the set of formulae satisfied in $\mathcal{M}$ by $a \in M$, the natural way to proceed is to consider the definable sets in $(\mathcal{M}, \mathcal{M}(a))$ and complete types realized in the last gap of $\mathcal{M}$. We might then first ask under what circumstances there is an element

$c \in gap(a)$ such that $tp(c) \in Def(\mathcal{M}, \mathcal{M}(a))$ for $\mathcal{M}$ being a countable recursively saturated model of PA.

Using results of Smorynski from [12] and working with gaps and standard systems $SSy(\mathcal{M})$ of $\mathcal{M}$, i.e. the family of all subsets of $\mathbb{N}$ that are coded in $\mathcal{M}$[1] we show that

Theorem 3 (Tin Lok Wong, MTG)**.** *Let $\mathcal{M} \models PA$ be a countable recursively saturated model and let $a, b \in M$. Suppose that $(\mathcal{M}, \mathcal{M}(a)) \equiv (\mathcal{M}, \mathcal{M}(b))$ (recall that $\mathcal{M}(a)$ and $\mathcal{M}(b)$ are short). If $SSy(\mathcal{M}) \subseteq Def(\mathbb{N})$, then $(\mathcal{M}, \mathcal{M}(a)) \cong (\mathcal{M}, \mathcal{M}(b))$.*

As the project is essentially *in progress*, we end with perspective paths for further work.

The conceptual import of the result is that taking a nonstandard model of compositional truth such that all its coded sets are already definable in the standard model, we are able to identify isomorphic cofinal short elementary cuts of the model just by looking on the arithmetical theory of both pairs considered.

References

[1] J. Barwise and J. Schlipf, *An introduction to recursively saturated and resplendent models*, **Journal of Symbolic Logic** 41 (1976), 531-536.

[2] A. Enayat and A. Visser, *New constructions of satisfaction classes*, in *Unifying the Philosophy of Truth*, ed. T. Achourioti, H. Galinon, J. Martinez Fernandez, K. Fujimoto, 321-335.

[3] F. Engström, *Satisfaction classes in nonstandard models of first-order arithmetic*, Chalmers University of Technology and Göteborg University, Göteborg, 2002.

[4] R. Kaye, *Models of Peano Arithmetic*, Oxford University Press, Oxford, 1991.

[5] R. Kossak and J. Schmerl, *The structure of models of Peano Arithmetic*, Clarendon Press, Oxford, 2006.

[6] R. Kossak and J. Schmerl, *On Cofinal Submodels and Elementary Interstices*, **Notre Dame Journal of formal Logic** 53 (2012), 267 - 287.

[7] H. Kotlarski and S. Krajewski and A. Lachlan, *Construction of satisfaction classes for nonsatndard models*, **Canadian Mathematical Bulletin** 24 (1981), 283-293.

[8] H. Kotlarski, *Full satisfaction classes: a survey*, **Notre Dame Journal of Formal Logic** 32 (1991), 573-579.

[9] S. Krajewski, *Nonstadard satisfaction classes*, in *Set Theory and Hierarchy Theory*, ed. W. Marek, M. Srebrny, A. Zarach, Heidelberg, 1976.

[1]It turns out that the standard system tells you a lot about the model; for example, any two countable recursively saturated models of the same completion of PA with the same standard system are isomorphic.

[10] A. Lachlan *Full satisfaction classes and recursive saturation*, **Canadian Mathematical Bulletin** 24 (1981), 295-297.

[11] A. Robinson *On languages based on on-standard arithmetic*, **Nagoya Mathematical Journal** 22 (1963), 83 - 107.

[12] C. Smorynski, *Cofinal extensions and nonstandard models of arithmetic*, **Notre Dame Journal of formal Logic** 2 (1981), 133 - 144.

[13] C. Smorynski, *Elementary extensions of recursively saturated models of arithmetic*, **Notre Dame Journal of formal Logic** 2 (1981), 193 - 203.

[14] C. Smorynski, *Recursively saturated nonstandard models of arithmetic*, textbfJournal of Symbolic Logic 46 (1981), 259 - 286.

Game Semantical Rules for Vague Proportional Quantifiers

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[1]Reasoning with vague quantificational expressions like "*almost all*", "*about half*", or "*a bit more than none*", requires adequate formal models. Developing such constitutes a major scientific challenge that links to linguistics, logic, philosophy as well as to computer science. Game semantics (Hintikka, 1973; Hintikka and Sandu, 2010) can be used to express vagueness (Fermüller and Roschger, 2014). We employ a version where there are two players, called **P** (for proponent) and **O** (for opponent), who either challenge or defend some (interpreted) sentence F of Łukasiewicz logic (Giles, 1974). Since in natural language quantifiers are usually binary, like in, e.g., "*Almost all the students passed the exam*", which can be formalized as *almost all* $_x(S(x), PE(x))$ (S and PE being short for the predicate symbols expressing "being a student" and "passed the exam"), we give as an example the rule for the binary universal quantifier (in this abstract we assume there is a unique constant for every domain element):

($R^2_\forall$) If **P** asserts $all_x(F(x), G(x))$ then **O** has to either quit the game or else pick some constant c and assert $F(c)$, thereby forcing **P** to assert $G(c)$.

For all the quantified natural language statements that start with "*almost x%*", "*about x%*", or "*a bit more than x%*" (and synonymous ones), for $x \in [0, 100]$, we propose a model for classical formulas F and G, which relies on a process that determines plausible tolerances t $(\geq 1)^2$, with respect to which we can evaluate the statements. For such a process we require, that for each size r (≥ 2, cf. footnote 4) of the range, the probabilities for the respective tolerance values $t \in \{1, ..., r-1\}$ sum up to 1, in order to constitute a probability distribution. Also, the values must be decreasing in both arguments, range and tolerance, while the expectation of t has to increase with r increasing. These conditions ensure, that, with a growing r, the values of t increase absolutely, but decrease relatively. To be able to refer to such a process we call it T-process. As an example we consider the rule for the quantifier expression "*almost all*":

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[1]Supported by FWF projects I1897-N25 and W1255-N23

[2]The cases in which the range formula corresponds to only one domain element, or else non at all, are special cases and have to be treated separately.

($R^2_{\tilde{\forall}}$) If **P** asserts *almost all*$_x(F(x), G(x))$, then **O** has to either quit the game, or else t gets chosen through applying a T-Process, and then $(t+1)$ different constants $c_1, ..., c_{t+1}$, all fulfilling the range formula F, get picked randomly, and **P** has to assert $G(c_i)$ for an $i \in \{1, ..., t+1\}$.

Note that for non-atomic formulas F and G, subgames have to played in order to fit with the general idea of game semantics (Benthem, 2014). However, for fixed t, the resulting truth functions correspond to hypergeometric probability distributions (Georgii, 2008). The overall truth value of a corresponding statement can then be interpreted as the expectation of the truth values for a fixed t, weighted with the probabilities of t. Slightly modified[1] game rules can be used to model "*almost x%*" and "*a bit more than x%*". In case of a statement starting with "*about x%*", we take the disjunction of two corresponding statements, starting with "*almost x%*" and "*a bit more than x%*". Finally, we will also show how similar techniques can be used to model vague quantifiers with stronger intensional aspects, like "*many*" or "*few*", still staying in a uniform game semantical framework.

References

[Benthem(2014)] Benthem, J. (2014). *Logic in Games.* MIT Press.

[Fermueller(2014)] Fermüller, C.G. and Roschger, C. (2014). *Randomized Game Semantics for Semi-Fuzzy Quantifiers.* Logic Journal of the IGPL, 223, 413–439.

[Georgii(2008)] Georgii, H.-O. (2008). *Stochastics; Introduction to Probability and Statistics.* de Gruyter.

[Giles(1974)] Giles, R. (1974). *A non-classical logic for physics.* Studia Logica, 33, 397-415.

[Hintikka(1973)] Hintikka, J. (1973). *Logic, language-games and information: Kantian themes in the philosophy of logic.* Clarendon Press Oxford.

[Hinsa(2010)] Hintikka, J. and Sandu, G. (2010). *Game-theoretical semantics.* Handbook of Logic and Language, Elsevier.

Bridging Recognition and Coordination of Semantics

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Traditionally, language learning is conceived as a process of adopting an externally imposed standard. In the case of semantics, this process is usually modelled in terms of a hypothetical learning mechanism which enables a subject to eventually internalize the correct meanings by mere observation of language use (see e.g. [1, 5]). We shall refer to such type of learning as to learning by recognizing. Crucial characteristics of this approach include: the stability of the semantic standard and the full adaptability of the learner.

In contrast, coordinative approach to learning does not assume any external standard. It is concerned with modelling a population of communicating agents who may differ in their semantic interpretations. Each individual is equipped with a coordination mechanism. This mechanism allows an agent to adapt his hypotheses in face of information he gathers while engaging with others in communication games. After playing a sufficient amount of games, a common standard may emerge (similar ideas may be traced back to [3], see [6] for a recent survey).

[1] We have to mind little subtleties regarding the range size, but basically "*almost*" and "*a bit more than*" get treated inversely.

In this work, we link the two approaches: learning by recognizing and learning by coordination. To connect them, we use a modified version of the framework introduced in [2]. We model the scenario of an explorer who joins a perfect language community (i.e., a community which interprets language expressions in the same way) with an eye towards successful communication (Quine [4] considers a similar scenario). For simplicity, we are concerned with learning of a single expression. Our agents are allowed to choose from a (possibly infinite) set of semantic hypotheses which are understood as algorithms (for details on the algorithmic theory of meaning see [7]). Learners prefer simpler hypotheses (according to specific, computationally justifiable notion of simplicity). In a single interaction, the speaker and the hearer are confronted with a shared context. The speaker checks the output of his current algorithmic hypothesis on the shared context and conveys this information to the hearer. A communication game is a sequence of such realized single interactions. At each stage agents play a communication game and then each of them coordinates by picking up the simplest hypothesis which guarantees maximal communicative success (weighted by the authorities of the agents) in face of the communication games he has played so far.

Our main results are as follows. We provide sufficient conditions for semantic stability of the community and for the explorer to converge to the rest of the population. Briefly, these conditions say that the community should receive appropriate feedback from its members (roughly speaking, a provides feedback to b on a given interaction in which b is the hearer and c is the context if there is another interaction in the game where a speaks to b about c) and the explorer should observe sufficiently rich sample of language use. Under these conditions, the explorer scenario collapses to learning by recognizing, where the externally imposed standard is represented by the community. However, if these conditions are not satisfied the linguistic stability may be broken, the explorer may not converge or in the worst case, the community may adopt the semantics of the explorer. The proof of the theorem provides a more detailed view about how authorities and the structure of communication games contribute to those effects. The interplay between these properties is easily seen in the limit case, when the ratio between the authority of the explorer and the lower bound of the authorities of the community is sufficiently small.

Our analysis of the explorer scenario may indicate that recognition and coordination are not two different cognitive mechanisms. Instead, learning by recognizing may be seen as a specific manifestation of a more generic mechanism of coordination.

References

[1] Nina Gierasimczuk. "The Problem of Learning the Semantics of Quantifiers". In: *TBiLLC'05: 6th International Tbilisi Symposium on Logic, Language, and Computation. Revised Selected Papers.* Vol. 4363. Lecture Notes in Artificial Intelligence. Springer, 2007, pp. 117–126.

[2] Dariusz Kalociński, Nina Gierasimczuk, and Marcin Mostowski. "Quantifier Learning: An Agent-based Coordination Model". In: *Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems.* AAMAS '15. Richland, SC: IFAAMAS, 2015, pp. 1853–1854. ISBN: 978-1-4503-3413-6. URL: `http://dl.acm.org/citation.cfm?id=2772879.2773470`.

[3] David Lewis. *Convention.* Cambridge: Harvard University Press, 1969.

[4] Willard von Orman Quine. *Word and object.* Harvard University Press, 1960.

[5] Steven T. Piantadosi, Joshua B. Tenenbaum, and Noah D. Goodman. "Bootstrapping in a language of thought: A formal model of numerical concept learning". In: *Cognition* 123.2 (2012), pp. 199–217. ISSN: 0010-0277.

[6] Luc Steels. “Modeling the cultural evolution of language”. In: *Physics of Life Reviews* 8.4 (2011), pp. 339–356. ISSN: 1571-0645. DOI: http://dx.doi.org/10.1016/j.plrev.2011.10.014. URL: http://www.sciencedirect.com/science/article/pii/S1571064511001060.

[7] Jakub Szymanik. *Quantifiers and Cognition: Logical and Computational Perspectives.* en. Studies in Linguistics and Philosophy. Springer International Publishing, 2016, pp. 3–8. ISBN: 978-3-319-28747-8 978-3-319-28749-2. URL: http://link.springer.com/chapter/10.1007/978-3-319-28749-2_1.

Dilution and its Elimination

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In his classical essay “*What is logic?*” [3] Ian Hacking turns on three structural criteria to demarcate the realm of logic: He calls these cut-, reflexivity- and dilution-elimination. The first two are familiar for example as Nuel Belnap’s criteria for proof-theoretic harmony.[1] Hacking adds to these a further requirement: What he calls dilution-elimination is the requirement to reduce all applications of the thinning (aka dilution aka monotonicity) rule to atomic formulae.

Hacking then goes on to argue, that modal logics fail on that requirement, and hence are not logic proper – cf. [3, pp. 297f] and [5, pp. 10ff]. While this is true for usual sequent systems which require side conditions on the application of rules (f.i. systems of **S4** which require for $\Box$-right-introduction that the entire antecedent should be fully modalized), the situation is different with respect to the display versions of modal logic. Heinrich Wansing, building on Belnap’s [1], has shown the modal display calculus to be a satisfactory general proof-theoretic framework, that satisfies a systematic cut- and reflexivity-elimination procedure. I will argue, that the situation with regard to monotonicity is a bit more subtle, and its elimination (in Hacking’s sense) cannot be carried out by means as general as that for cut and reflexivity – i.e. independently of *all* the other structural rules.

Interestingly, with regard to display systems the reduction of dilutions of boxed formulae to the atomic case *on the right* requires just the structural equivalent of *necessitation*, while elimination of boxed formulae *on the left* depends on the structural counterpart of the **(T)** schema. However, the situation here is not unprecedented, since even in the case of (display calculi of) classical extensional logics dilution elimination with regard to binary connectives depends on the structural rule of *contraction*![2]

However, nowadays all these criteria are somewhat controversial as criteria for logicality, in part because of the rise of substructural logics. In particular dilution can be seen as a source of *irrelevance*, and proponents of a relevance requirement on logical consequence have opted for eliminating it *altogether* – for

[1] Although Belnap’s terminology is different, cf. [2] or [1]; in the latter paper he speaks of the Elimination and Identity theorems. The first criterion states that the structural rule Cut (or transitivity) is an admissable rule, which leads in the usual Gentzen or display systems to (normalized, i.e. cut-free) proofs having a subformula-property. This in turn shows not only the concistency or better non-triviality of the system, but the conservativity of each connective over the others. The second criterion states that as initial sequents it suffices to use those of the form ‘$p \to p$’, where ‘p’ is atomic, which can be seen as proof-theoretic uniqueness. In some sense these criteria grant the well-definedness of the new connective.

[2] Of course, here the observation appears less important, especially if one endorses a view of consequence as obtaining between sets: This gives contraction ‘for free’, while reflexivity, transitivity, and monotonicity are not entailed by the nature of the arguments alone.

complex formulae as well as for atoms. A paradigmatic approach on these lines is that of Neil Tennant, cf. f.i. [4].

In this talk I will discuss some technical and philosophical issues that pertain to dilution and its elimination – in one form or the other, and in their comparison.

References

[1] Nuel Belnap. "Display Logic". In: *Journal of Philosophical Logic* 11.4 (1982), pp. 375–417.

[2] Nuel Belnap. "Tonk, Plonk and Plink". In: *Analysis* 22.6 (1962), pp. 130–134.

[3] Ian Hacking. "What is Logic?" In: *Journal of Philosophy* 76.6 (1979), pp. 285–319.

[4] Neil Tennant. "Relevance in Reasoning". In: *The Oxford Handbook of Philosophy of Mathematics and Logic.* Ed. by Stewart Shapiro. Oxford UP, 2004. Chap. 23, pp. 696–726. DOI: http://dx.doi.org/10.1093/0195148770.003.0023.

[5] Heinrich Wansing. "The Idea of a Proof-Theoretic Semantics and the Meaning of the Logical Operations". In: *Studia Logica* 64.1 (2000), pp. 3–20.

Towards Π_2-Cut-Introduction

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Lemmata play an important role in proofs in mathematics and computer science. They can structure a proof and reduce its complexity, and they are often non-trivial abstractions [3]. Lemmata of the latter case usually occur in proofs with induction, where a formula could be only provable by a more abstract induction-formula. Hence, not only in the context of automated theorem proving there is an enormous demand for automated methods to find non-trivial lemmata.

In more precise terms, in sequent calculus the cut-rule simulates the use of a lemma. Let a sequent $A_1, \ldots, A_n \vdash B_1, \ldots, B_m$ represent a consequence relation among formulae [4] [9]. The interpretation of a sequent is the implication of the disjunction over all B_j by the conjunction over all A_i. Then the cut-rule can be described as follows: If it is possible to prove a formula A in a context $\Delta \vdash \Gamma$, i.e. the sequent $\Delta \vdash \Gamma, A$ is a tautology, and with the assumption that A is true it is possible to prove that the context is true, i.e. $A, \Delta \vdash \Gamma$ is a tautology, then also the sequent $\Delta \vdash \Gamma$ is a tautology.

In certain circumstances it is useful to embed in the corresponding proof the information that is stored in a cut, i.e. to simulate the cut-rule by other rules. This process is called cut-elimination. Stefan Hetzl, Alexander Leitsch, Giselle Reis, and Daniel Weller investigated the cut-elimination of an interesting class of cut-formulae and used these results successfully to introduce non-trivial cut-formulae [7] [6] [3]. To describe this in more detail we look at the arithmetical hierarchy [8]. It classifies formulae in prenex normal form according to the first quantifier and the number of quantifier alternations. If the first quantifier is universal and the quantifiers alternate n times between existential and universal quantifiers the formula is in the class Π_n. Stefan Hetzl found out that the elimination of cut-rules with a Π_1-cut-formula corresponds to the computation of so-called totally rigid acyclic tree grammars [5]. Based on this connection Stefan Hetzl, Alexander Leitsch, Giselle Reis, and Daniel Weller showed that it is possible to invert cut-elimination for cuts with a Π_1-cut-formula: Assume a provable sequent and a totally rigid acyclic tree grammar, whose

language describes instantiations of the sequent such that it is a tautology. Then there is a proof with a non-trivial Π_1-cut-formula.
Here, we extend this method for a subclass of Π_2-formulae. Again we use the connection between the cut-elimination of a Π_2-cut and the corresponding grammar [1]. We define schematic Π_2-grammars in order to characterise the Π_2-cut-elimination. Given a provable sequent and a schematic Π_2-grammar, whose language describes instantiations of the sequent such that it is a tautology, we would like to produce a proof with a non-trivial Π_2-cut. By formulating the property of G^*-unifiability we provide an algorithm to generate non-trivial Π_2-cut-formulae for a big class of problems. G^*-unifiability is a unification method based on a type-0 grammar. Type-0 grammars are the most general grammars in the Chomsky hierarchy [2].
In a proof with Π_2-cut of a given sequent it is possible that some axioms only depend on parts of the cut-formula. A cut-formula of this kind interacts with itself. We say that a cut is balanced if it does not interact with itself, i.e. it interacts only with the context. For this class of cuts our method applies.
We intend to introduce this form of cut-introduction on the basis of Π_1-cut-introduction and to present our main results for Π_2-cut-introduction.

References

[1] Bahareh Afshari, Stefan Hetzl, and Graham E. Leigh. Herbrand disjunctions, cut elimination and context-free tree grammars. *to appear at Typed Lambda Calculi and Applications*, 2015.

[2] Noam Chomsky. Three models for the description of language. *Information Theory, IRE Transactions on*, 2(3):113–124, 1956.

[3] Sebastian Eberhard and Stefan Hetzl. Inductive theorem proving based on tree grammars. *Annals of Pure and Applied Logic*, 166(6):665–700, 2015.

[4] Gerhard Gentzen. Untersuchungen über das logische Schließen. I. *Mathematische Zeitschrift*, 39(1):176–210, 1934.

[5] Stefan Hetzl. Applying tree languages in proof theory. In Adrian-Horia Dediu and Carlos Martìn-Vide, editors, *Language and Automata Theory and Applications*, pages 301–312. Springer, 2012.

[6] Stefan Hetzl, Alexander Leitsch, Giselle Reis, Janos Tapolczai, and Daniel Weller. Introducing quantified cuts in logic with equality. In S. Demri, D. Kapur, and C. Weidenbach, editors, *Automated Reasoning*, volume 8562 of *LNAI*, pages 240–254. Springer, 2014.

[7] Stefan Hetzl, Alexander Leitsch, Giselle Reis, and Daniel Weller. Algorithmic introduction of quantified cuts. *Theoretical Computer Science*, 549:1–16, 2014.

[8] Hartley Rogers. *Theory of recursive functions and effective computability.* McGraw-Hill New York, 1967.

[9] Anne S. Troelstra and Helmut Schwichtenberg. *Basic Proof Theory*, volume 43 of *Cambridge Tracts in Theoretical Computer Science.* Cambridge University Press, The Pitt Building, Trumpington Street, Cambridge, United Kingdom, second edition, 1996.

Reducts of Finitely Bounded Homogeneous Structures, and Lifting Tractability from Finite-Domain Constraint Satisfaction

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Many computational problems in various areas of theoretical computer science can be formulated as *constraint satisfaction problems (CSPs)*. Constraint satisfaction problems where the variables take values from a finite domain are reasonably well understood with respect to their computational complexity. The Feder-Vardi dichotomy conjecture for finite domain CSPs, which states that every finite-domain CSP is either in P or NP-complete, is still open, but there is a stronger *tractability conjecture* [8] which provides an effective characterisation of those finite-domain CSPs that are NP-complete, and those that are conjectured to be in P. The tractability conjecture has been confirmed in many special cases, such as for finite structures on domains of size two [12] and three [7], or for finite undirected graphs [9] and finite directed graphs without sources and sinks [1]. The strongest complexity classification results have been obtained using concepts and results from universal algebra. The universal-algebraic approach can also be applied to classify the complexity of some classes of CSPs over infinite domains. This approach works particularly well if the constraints can be defined over a homogeneous structure with finite relational signature. The class of CSPs that can be formulated in this way is a considerable extension of the class of CSPs over finite domains, and captures many computational problems that have been studied in various research areas. For example, almost all CSPs studied in qualitative temporal and spatial reasoning belong to this class.

The class of CSPs where the constraints can be defined over $(\mathbb{Q}; <)$ has been classified [3]. Also constraint languages definable over the random graph [4] or over homogeneous tree-like structures [2] have been classified. These results were obtained by using a generalisation of the universal-algebraic approach from finite-domain CSPs, and structural Ramsey theory. However, it would be desirable to go further and to reduce complexity classification tasks for CSPs over infinite domains to the rather more advanced classification results that are known (or have been conjectured) for finite-domain CSPs.

In this paper, we present a first result in this direction. We study structures that have been called *first-order definable structures with atoms* [10] which recently attracted attention in automata theory as a natural class of infinite structures with finite descriptions; see [6, 11, 5], and references therein. In the context of constraint satisfaction problems they were first studied in [10], where the authors focussed on structures that are additionally *locally finite*, a very strong restriction that is not needed for our approach. More precisely, our result is as follows. Let $\mathbb{A}$ be a τ-structure, where τ is a finite relational signature. The *constraint satisfaction problem of* $\mathbb{A}$ is the decision problem which takes as input a τ-sentence ϕ of the form

$$\exists x_1, \ldots, x_n. \bigwedge R_i(x_{i1}, \ldots, x_{ir_i})$$

where $R_i \in \tau$, and whose question is "Is ϕ true in $\mathbb{A}$?". Let $\mathcal{C}$ be the class of all those structures $\mathbb{A}$ with finite relational signature whose domain is countably infinite, and whose relations are all first-order definable using equality and constants from the domain.

Theorem 1. *The tractability conjecture for finite-domain CSPs is true iff for every $\mathbb{A} \in \mathcal{C}$, if $\mathbb{A}$ has a so-called* Siggers polymorphism modulo endomorphisms*, then the constraint satisfaction problem of $\mathbb{A}$ is tractable in polynomial-time.*

References

[1] Barto, L., Kozik, M., and Niven, T. The CSP dichotomy holds for digraphs with no sources and no sinks (a positive answer to a conjecture of Bang-Jensen and Hell). *SIAM Journal on Computing 38*, 5 (2009).

[2] Bodirsky, M., Jonsson, P., and Pham, T. V. The complexity of phylogeny constraint satisfaction. In *Proceedings of the Symposium on Theoretical Aspects of Computer Science (STACS)* (2016). Preprint arXiv:1503.07310.

[3] Bodirsky, M., and Kára, J. The complexity of temporal constraint satisfaction problems. *Journal of the ACM 57*, 2 (2009), 1–41. An extended abstract appeared in the Proceedings of the Symposium on Theory of Computing (STOC).

[4] Bodirsky, M., and Pinsker, M. Schaefer's theorem for graphs. *Journal of the ACM 62*, 3 (2015), Article No. 19, 52 pages. A conference version appeared in the Proceedings of STOC 2011, pages 655–664.

[5] Bojańczyk, M., Klin, B., and Lasota, S. Automata theory in nominal sets. *Logical Methods in Computer Science 10*, 3 (2014).

[6] Bojańczyk, M., Klin, B., Lasota, S., and Toruńczyk, S. Turing machines with atoms. In *28th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2013, New Orleans, LA, USA.* (2013), pp. 183–192.

[7] Bulatov, A. A. A dichotomy theorem for constraint satisfaction problems on a 3-element set. *Journal of the ACM 53*, 1 (2006), 66–120.

[8] Bulatov, A. A., Krokhin, A. A., and Jeavons, P. G. Classifying the complexity of constraints using finite algebras. *SIAM Journal on Computing 34* (2005), 720–742.

[9] Hell, P., and Nešetřil, J. On the complexity of H-coloring. *Journal of Combinatorial Theory, Series B 48* (1990), 92–110.

[10] Klin, B., Kopczynski, E., Ochremiak, J., and Toruńczyk, S. Locally finite constraint satisfaction problems. In *30th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2015, Kyoto, Japan.* (2015), pp. 475–486.

[11] Klin, B., Lasota, S., Ochremiak, J., and Toruńczyk, S. Turing machines with atoms, constraint satisfaction problems, and descriptive complexity. In *Conference on Computer Science Logic (CSL) and Symposium on Logic in Computer Science (LICS), CSL-LICS'14, Vienna, Austria.* (2014), pp. 58:1–58:10.

[12] Schaefer, T. J. The complexity of satisfiability problems. In *Proceedings of the Symposium on Theory of Computing (STOC)* (1978), pp. 216–226.

A Computable Spectral Theorem

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The spectral theorem [2], a central result in quantum mechanics, provides a bijection between self-adjoint operators and projection-valued measures. We prove a computable version, constructing the necessary data types in the category $\mathbf{QCB}_0$ of *T_0-quotients of countably based spaces and continuous maps* [1], which is equivalent to the category of admissible representations and continuously realizable maps, used in Weihrauch's computable analysis [3].

The category $\mathbf{QCB}_0$ is cartesian closed and closed under regular subobjects. It contains the complex separable infinite dimensional Hilbert space $\mathcal{H}$, the unit interval with Euclidean topology $[0, 1]$, the unit interval with lower topology $\mathcal{I}$, and the Sierpiński space $\Sigma = \{0, 1\}$, whose only non-trivial open set is $\{0\}$.

Closed subsets of a space X in $\mathbf{QCB}_0$ correspond to elements of Σ^X by identifying each closed set A with the map $\chi_A : X \to \Sigma$ defined by $\chi_A(x) = 1$ if $x \in A$ and 0 otherwise.

Let $\mathcal{E}$ be the regular subobject of $\mathcal{H}^{\mathcal{H}}$ consisting of the *effects*, i.e. the positive linear maps, bounded by one. It carries the strong operator topology. The *Hilbert lattice* $\mathcal{L}$ of closed linear subspaces of $\mathcal{H}$ is the regular subobject of linear maps in $\Sigma^{\mathcal{H}}$. Instead of projection-valued measures, we consider a constructively better behaved object: Let $\mathcal{SV}$ be the regular subobject of $\mathcal{L}^{\Sigma^{[0,1]}}$ consisting of the *spectral valuations*, which are the maps $\nu : \Sigma^{[0,1]} \to \mathcal{L}$ such that for every unit vector $x \in \mathcal{H}$ the map $\nu(x) : \Sigma^{[0,1]} \to \mathcal{I} : A \mapsto \langle \nu(A)x|x\rangle$ is a probability valuation. A sequence ν_n converges to ν in $\mathcal{SV}$ iff for every unit vector $x \in \mathcal{H}$, the sequence of probability valuations $\nu_n(x)$ converges weakly to $\nu(x)$, i.e. $\limsup \nu_n(x)(A) \leq \nu(x)(A)$ for every closed set A in $[0,1]$.

Theorem 1. *The map*

$$\mathcal{E} \to \mathcal{SV} : E \mapsto \nu(x)(A) = \inf\{\langle f(E)x|x\rangle \mid \chi_A \leq f, f \in \mathbb{R}^{[0,1]}\}^1$$

is a computable isomorphism with regard to the effective standard representations for $\mathcal{H}$*,* $[0,1]$*,* $\mathcal{I}$ *and* Σ*.*

References

[1] I. Battenfeld, M. Schröder, and A. Simpson. A convenient category of domains. *Electronic Notes in Theoretical Computer Science*, 172:69–99, 2007.

[2] J. von Neumann. *Mathematische Grundlagen der Quantenmechanik.* Springer Verlag, Berlin, Germany, 1932.

[3] K. Weihrauch. *Computable Analysis.* Springer, Berlin, 2000.

Toward a Similarity Semantics for Counterlogicals

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An approach to developing a non-vacuous analysis of counterpossibles (subjunctive conditionals whose antecedents expresses an impossibility) that has gained a fair amount of interest in the last couple of decades is to take as the starting point one of the best accounts of the counterfactual—one due to David K. Lewis [5]—and extend the range of the world accessibility relation to include impossible worlds.

I follow the key suggestion of a number of authors [10, 8, 2] and interpret closeness as similarity in *relevant respects* or just *relevant* similarity—an interpretation that Lewis [5, 6, 7] consistently endorses, albeit only informally. The key modification proposed here that—I will argue—can free Lewis' analysis from the burden of number of families of objections [9 ,3,4], is one that makes the role of *relevance*—in terms of context dependence—*explicit* in the truth conditions. Building on Nolans [8] suggestion—indirectly endorsed by Berto [1]—and Brogaard & Salernos [2] insights along the same lines, I propose a model of *relevant comparative similarity* of worlds—a method that generalizes nicely to the extended analysis, i.e. of some counterpossibles.

Following Lewis' [6] *ordering semantics* for counterfactuals, with frame properties (except for *centering*) as those given by him on the *comparative similarity* formulation in [5], an ordering frame $\mathbf{F}$ is a function

[1] where $f(E)$ is defined by the usual functional calculus

$\mathbf{F} : W \to \wp\left(W^2\right)$ that assigns a *total preorder* $\lesssim_w$ on the set of possible worlds W, to each possible world $w \in W$, such that w is minimal in $(W, \lesssim_w)$. That is, frames of interest here, are just *Weakly Centered* and *Universal* Lewis frames. The structures that play a key technical role in the proposal are *frame refinements.* Given the restricted class of frames of interest at this stage, a frame $\mathbf{F}^*$ is a *proper refinement* of a frame[1] $\mathbf{F}$ iff $\mathbf{F}^*$ preserves the orderings $\lesssim_w$ of $\mathbf{F}$, for each world w, and for some $u \in W$ and symmetric pair $j, k \in W$, i.e. $\{(j,k), (k,j)\} \subseteq \lesssim_u$, that pair's symmetry is broken on $\lesssim_u^*$, i.e. either $(j,k) \notin \lesssim_u^*$ or $(k,j) \notin \lesssim_u^*$. In short, $\mathbf{F}^*$ is a *refinement* of $\mathbf{F}$ iff $\lesssim_w^* \subseteq \lesssim_w$ for all $w \in W$. Intuitively, refinements *partially resolve* (up to *totally*) similarity *ties* between worlds that are present according to similarity assignments of the original frames.

Building on an equivalence result between frames and their refinements, given by Lewis [6], I model the intended relationship between the structures that are proposed to model context indices and their corresponding contexts. Context indices are modelled by ordering frames.[2] The idea here is that each ordering frame $\mathbf{F}$ is to be thought of as capturing the *essentially relevant similarity* of the context that it determines, i.e. the $\mathbf{F}$-context. Finally, contexts are modelled by (identified with) sets of all refinements of some context-index, i.e. some ordering frame. In support of the proposed modification, I give a general theorem regarding logical inference over the expanded language, which—I will argue—exhibits properties that a contextualized account is expected to have.

The proposed, modified account of Lewis' counterfactual analysis not only avoids a number of serious objections, but also offers surprisingly simple truth conditions despite an increase in the complexity of comparative similarity assignments (the expanded object language). Finally, an elegant model of *logical similarity* and *logical contexts* falls out of the offered modified account of counterfactuals, paving the way for developing comparative logical similarity.

KEYWORDS: Philosophical Logic, Modal Logic, Conditional Logic, Counterfactuals, Counterpossibles, Counterlogicals, Similarity Semantics, Ordering Semantics, Relevant Similarity. MSC: 03A05, 03B45, 03B60.

References

[1] Berto, F. (2014), VOn Conceiving the Inconsistent. Proceedings of the Aristotelian Society (Hardback), 114: 103121.

[2] Brogaard B. & Salerno J., 2013, Remarks on Counterpossibles, Synthese March 2013, Vol. 190, Issue 4, pp 639-660.

[3] Fine, Kit. 1975. Review of Counterfactuals. Mind 84: 451-8.

[4] Hjek, Alan. *Most Counterfactuals are false* draft.

[5] Lewis, D. *Counterfactuals*, Oxford: Blackwell.

[6] Lewis, D. Ordering Semantics and Premise Semantics for Counterfactuals, Journal of Philosophical Logic, 10: 217234.

[7] Lewis, D. 1986, *On the Plurality of Worlds*, Oxford: Blackwell.

[8] Nolan D. (1997): Impossible Worlds: A modest Approach. Notre Dame Journal of Formal Logic, 38 (4), 535-572.

[1] Each ordering frame, whose similarity assignments are total, is a refinement of itself.

[2] This is a lot like what Lewis [5 ,6] intended—the exception being an expansion of the object language that explicitly accounts for context dependence *via* indexed counterfactuals.

[9] Quine, W. V. O., 1960, textitWord & Object. The MIT Press.

[10] Read, S., *Thinking About Logic*, Oxford University Press, Oxford, 1995.

Enumerable Functors

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In computable structure theory we study algorithmic properties of structures. We measure the algorithmic complexity of a structure $\mathcal{A}$ by its Turing degree, which is defined as the Turing degree of its atomic diagram $D(\mathcal{A})$, the set of quantifier free sentences of $\mathcal{A}$ expanded by a constant symbol for each variable in the universe of $\mathcal{A}$. We study various algorithmic properties of structures, one very widely studied notion are degree spectra under equivalence relations, usually isomorphism. The degree spectrum of a structure under isomorphism is the set of Turing degrees of all structures isomorphic to it. When a new property is investigated one often wants to know whether this property can be found in an intuitive class of structures such as graphs or fields. To show this, one can prove that this property exists for this class from scratch or use transformations of structures into structures in that class. The advantage of using transformations is that they often preserve many properties and are therefore reusable, and that they let us compare the algorithmic complexity of classes of structures. As an example, the most widely used transformation is a transformation of arbitrary structures to structures in the class of asymmetric graphs. This transformation shows that a structure possessing any known algorithmic property can be found in the class of asymmetric graphs.

Many notions of transformations between structures, such as Muchnik -, Medvedev reducibility, computable embeddings and Σ-definability, are known. Recently computable functors, an effective version of the categoricity theoretic notion of functors have been proposed as a new method to reduce one structure into another [2] [5]. Computable functors, based on Turing reducibility, preserve most computability theoretic properties of structures such as degree spectra under isomorphism, computable dimension or Scott rank. They are equivalent to effective interpretability, an effective version of the model theoretic concept of interpretability equivalent to Σ-definability. Computable functors might be the strongest notion of reducibility investigated until now and the structure induced by it is well understood, mainly because all the interpretations between classes of structures given in [3] induce computable functors between these classes.

We propose an even stronger notion of reducibility which is also an effectivization of functors, enumerable functors, based on enumeration reducibility. We say that a structure $\mathcal{B}$ is enumerable from another structure $\mathcal{A}$ if there is an enumerable operator Ψ containing pairs (α, φ) such that every atomic sentence φ of $D(\mathcal{B})$ is defined by a finite subset α of $D(\mathcal{A})$ in Ψ. We require our structures in the image of the functor to be enumerable from the structures in its domain, whereas computable functors require them to be computable. We use results from [4] to show that given an enumerable functor one can construct a computable functor. One of the main results of our work is that enumerable functors preserve degree spectra under Σ_n equivalence [1]. Two structures are Σ_n equivalent iff they model the same sentences up to n quantifier alternations. Enumerable functors are therefore a strictly stronger notion of reducibility than computable functors which in general do not preserve this property. By analyzing the results in [3] we furthermore investigate the structure induced by enumerable functors and its usefulness as a notion of reduction between classes of structures. We establish a connection between enumerable functors and an effective version of interpretability, similar to the result in [2] and discuss other effectivizations of functors which we neglected in favor of our definition.

References

[1] Ekaterina Fokina, Pavel Semukhin, and Daniel Turetsky. “Degree spectra of structures under equivalence relations”. In: (in preparation).

[2] Matthew Harrison-Trainor et al. “Computable functors and effective interpretability”. In: *arXiv preprint arXiv:1506.02737* (2015). URL: `http://arxiv.org/abs/1506.02737` (visited on 06/28/2015).

[3] Denis R. Hirschfeldt et al. “Degree spectra and computable dimensions in algebraic structures”. In: *Ann. Pure Appl. Logic* 115.1 (2002), pp. 71–113. DOI: `10.1016/S0168-0072(01)00087-2`.

[4] Julia F. Knight, Sara Miller, and M. Vanden Boom. “Turing Computable Embeddings”. In: *The Journal of Symbolic Logic* 72.3 (Sept. 1, 2007), pp. 901–918. ISSN: 0022-4812. URL: `http://www.jstor.org/stable/27588576` (visited on 05/02/2015).

[5] Russell Miller et al. “A computable functor from graphs to fields”. In: *arXiv preprint arXiv:1510.07322* (2015). URL: `http://arxiv.org/abs/1510.07322` (visited on 02/01/2016).

Definability of Cai-Fürer-Immerman Problems in Choiceless Polynomial Time

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One of the most important questions in descriptive complexity theory is whether there is a logic capturing PTIME. Choiceless Polynomial Time (CPT) is currently one of few promising candidates for such a logic. This formalism, which was introduced by Blass, Gurevich and Shelah[5], is based on the creation and manipulation of hereditarily finite sets over the input structure, and can thus also be considered as an isomorphism-invariant model of computation on structures. The intuition behind studying that kind of logic is that it seems to be as close to PTIME computations as possible without sacrificing isomorphism-invariance: CPT is choiceless in the sense that common algorithmic instructions like ”choose an arbitrary vertex“ are not possible. To date, CPT has not been separated from PTIME, and it can express many queries proposed for that purpose:

- Any query definable in fixed-point logic with counting (FP+C) is also definable in CPT.
- On structures with sufficiently large padding, CPT captures PTIME [6, 3]. This can be used to show that CPT is a strict extension of FP+C.
- The Cai-Fürer-Immerman query is definable in CPT (without counting) if the underlying graphs are linearly ordered [7].
- More generally, CPT captures PTIME on structures with bounded colour class size if the colour classes have Abelian automorphism groups. In particular, the solvability of *cyclic linear equation systems* over finite rings is definable in CPT [1].
- The isomorphism problem for multipedes is definable in Choiceless Polynomial Time [1, 6].

- Perfect Matching [2] and determinants of matrices over finite fields [4] are CPT-definable[1].

These results illustrate the surprising expressive power of Choiceless Polynomial Time. However, there are some properties for which we do not know whether they can be expressed in CPT, and which might witness a separation from polynomial time. Most importantly, it is open whether the Cai-Fürer-Immerman problem over all graphs can be expressed in Choiceless Polynomial Time.

The CFI query has been introduced to separate fixed-point logic with counting from PTIME, and has since remained the main benchmark for the expressibility of logics within PTIME. It is based on a graph construction that associates with each connected graph a set of CFI-graphs, that can be partitioned into exactly two isomorphism classes called odd and even CFI-graphs. The problem is to decide, given a CFI-graph, whether it is odd or even. In the original version, the underlying graphs are linearly ordered, and for this case, Dawar, Richerby and Rossman[7] provided an elegant construction proving expressibility in CPT. Essentially, their approach is to succinctly represent the complete isomorphism class of a given CFI-graph (which is a class of exponential size) as a hereditarily finite set over the input structure. To this end they invent a very clever data structure which requires the use of highly nested sets. They further prove that this nesting cannot be avoided: with sets of constant rank it is not possible to define the CFI query over ordered graphs in CPT. However, the CFI query over general, i.e. unordered, graphs remains one of the few known examples for which CPT-definability is open. We identify new classes of graphs over which the CFI query is CPT-definable. First, we generalise the result by Dawar, Richerby and Rossman to the variant of the CFI query where the underlying graphs have colour classes of logarithmic size. (Note that the case of ordered graphs corresponds to colour class size one.) Secondly, we consider the CFI query over graph classes where the maximal degree is linear in the size of the graphs. For these classes, we establish CPT-definability using only sets of small, constant rank, which is known to be impossible for the general case.

For our CFI-procedures we strongly make use of the mechanisms of CPT to create sets, rather than tuples only, and we further prove that the use of sets is indeed unavoidable. We introduce a notion of "sequence-like objects" based on the structure of their symmetry groups, and we show that no CPT-program which only uses sequence-like objects can decide the CFI-query over complete graphs, which have linear maximal degree. From a more general perspective, this generalises a result by Blass, Gurevich, and van den Bussche about the power of isomorphism-invariant machine models (for polynomial time) to a setting with counting.

References

[1] F. Abu Zaid et al. "Choiceless Polynomial Time on structures with small Abelian colour classes". In: *MFCS 2014*. Vol. 8634. LNCS. Springer, 2014, pp. 50–62.

[2] M. Anderson, A. Dawar, and B. Holm. "Maximum matching and linear programming in fixed-point logic with counting". In: *LICS, 2013*. IEEE. 2013, pp. 173–182.

[3] B. Laubner. "The Structure of Graphs and New Logics for the Characterization of Polynomial Time". PhD thesis. HU Berlin, 2011.

[4] A. Blass and Y. Gurevich. "A quick update on open problems in Blass-Gurevich-Shelahs article On polynomial time computations over unordered structures". In: *Online at http://research. microsoft. com/gurevich/annotated. html* (2005).

[5] A. Blass, Y. Gurevich, and S. Shelah. "Choiceless polynomial time". In: *Annals of Pure and Applied Logic* 100.1 (1999), pp. 141–187.

[1] in fact, both problems turned out to be definable in FP+C

[6] A. Blass, Y. Gurevich, and S. Shelah. "On polynomial time computation over unordered structures". In: *J. Symb. Logic* 67.3 (2002), pp. 1093–1125.

[7] A. Dawar, D. Richerby, and B. Rossman. "Choiceless polynomial time, counting and the Cai–Fürer–Immerman graphs". In: *Ann. Pure Appl. Logic* 152.1 (2008).

Proof Mining and Families of Mappings

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Proof mining is a research program introduced by U. Kohlenbach in the 1990s ([2] is a comprehensive reference), which aims to obtain explicit quantitative information (realizers and bounds) from proofs of an apparently ineffective nature. This paradigm in applied logic has successfully led so far to obtaining some previously unknown effective bounds, primarily in nonlinear analysis and ergodic theory. A large number of these are guaranteed to exist by a series of logical metatheorems which cover general classes of bounded or unbounded metric structures.

We contribute to this program by working the case of families of mappings. More specifically, there exist several algorithms and iteration schemas devoted to computing common fixed points of families of self-mappings $(T_i : C \to C)_i$, where C is a nonempty convex subset of a metric space endowed with a notion of convexity. This framework can be, for example, put to work in problems of convex optimization. Frequently, Banach or Hilbert spaces are used, but in recent years, there has been a steady growth in the usage of nonlinear structures like (uniformly convex) W-hyperbolic spaces, CAT(0) spaces or other classes of geodesic spaces. A typical algorithm starts from an arbitrary point of C and repeatedly computes approximations to the desired fixed point by applying convex combinations of our mappings and possibly some fixed pivot points to the previous approximation. The kind of information that proof mining can extract in this context is a rate of convergence, of metastability (in the sense of Tao [4]) or various forms of asymptotic regularity. Some examples of previous work in this area, studying the Kuhfittig and Halpern iterative processes, are [1], [3]. Our aims are to extract the corresponding quantitative results for these cases where proof mining has only been done in the case of single mappings and to extend this kind of results in two directions - firstly, to nonlinear spaces having the least necessary structure imposed of them in order for the algorithms to give the required answer, and secondly, to more general classes of mappings than the (quasi-)nonexpansive ones which have largely been at the focus of research in nonlinear analysis – one such example is the class of strict pseudo-contractions.

References

[1] Muhammad Aqeel Ahmad Khan, Ulrich Kohlenbach, *Bounds on Kuhfittig's iteration schema in uniformly convex hyperbolic spaces*, ***J. Math. Anal. Appl.***, vol. 403, pp. 633–642 (2013).

[2] Ulrich Kohlenbach, *Applied proof theory: Proof interpretations and their use in mathematics*, Springer Monographs in Mathematics, ***Springer-Verlag***, Berlin-Heidelberg (2008).

[3] Laurenţiu Leuştean, Adriana Nicolae, *Effective results on compositions of nonexpansive mappings*, ***J. Math. Anal. Appl.***, vol. 410, pp. 902–907 (2014).

[4] Terence Tao, *Soft analysis, hard analysis, and the finite convergence principle*, Essay posted May 23, 2007, appeared in: 'T. Tao, *Structure and Randomness: Pages from Year One of a Mathematical Blog.* ***AMS*** (2008)'.

Multi-Agent Dialogue Games and Dialogue Sequents

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We extend Lorenzen's and Lorenz's two-player approach of Dialogical Logic [2] to a multi-player version which is inspired by the multi-conclusion sequent calculus LJmc for intuitionistic logic by Maehara [3]. In this dialogue game we have several proponents (*agents*) fighting one opponent (*environment*). Whenever an agent has a choice of how to react to an opponent's move, a new proponent is introduced who performs the alternative reaction, while the opponent has to react to each of these moves.

Focusing on the proof-theoretic features of dialogues, we introduce a sequent calculus that implements the informal dialogical rules in a concrete way. Its sequents have the form $\Phi \vdash_p \Psi$ where Φ and Ψ are sequences of *labeled formulas* and p indicates the *party* whose turn it is in the current state of the game (agents or environment). The formulas of Φ and Ψ are *signed* with an *announcer label*: 'e' for the environment (opponent) and 'a' for the agents (proponents). Formulas labeled with 'e' also refer to an *addressee* written as subscript of the label. Every label can be marked with lines indicating that a formula has been *attacked* and not yet defended.

For example, the sequent "$\overline{e_{a1}} : A \vee B \ \vdash_E \ a0 : A \supset B,\, a1 : A$" represents a game state with two agents ($a0$, $a1$) and one attacked opponent e_{a_1}. It is currently the opponent's turn ($\vdash_E$) The horizontal bars over announcer labels indicate that the corresponding formula has been attacked by the other party, i.e., e's formula $A \vee B$ has been attacked by agent $a1$. The sequent style of dialogues make the rules concrete and simplify the soundness and completeness proof for the system.

Every multi-agent game consists of several *rounds* which again consist of three phases. The first one (*decide phase*) is necessary in intuitionistic logic, where the agents have to decide about their strategy. It is the only phase in which there are choices with consequences for them, i.e., their decisions in this phase are decisive concerning winning or losing. In the following phase, the agents perform their other moves. The order of moves in this phase is irrelevant, as the result is always the same. The last phase is the *e-phase*, in which e reacts on all of the agents' moves of the round. Whenever she has a choice, the sequent tree branches as usual.

An important advantage of this approach is that we have a kind of *normalization* of sequent proofs as in focused *single-conclusion* sequent calculi by Liang, Miller [1] and recently Simmons [4] for intuitionistic logic. In this way, the number of different sequent trees leading to the same result can be reduced as the critical non-deterministic process for the proponents happens in the decide phase in which the agents choose whether they defend against an attack of an implication or not. In case of such a *critical* defence, the other agents are *deactivated*, i.e., excluded for the rest of the game — a feature that corresponds to the $\supset r$-rule of sequent calculus LJmc. This issue allows us to focus on this phase when performing further analyses of the agents' strategies to increase the efficiency of proof searches.

To operate in modal logic, following an idea of Van Dun [5], we introduce several players on the opponent side as well, where each of these players corresponds to a Kripke world connected by a *coalition relation*, e.g, when an environment $e0$ states "$\Box\varphi$", this can be interpreted as "*All of my friends (coalition partners) can show you that φ is true.*" The agent who then attacks this assertion may choose the coalition partner of $e0$ who then has to defend A.

References

[1] Chuck Liang and Dale Miller. Focusing and polarization in linear, intuitionistic, and classical logics. *Theoretical Computer Science*, 410(46):4747 – 4768, 2009. Abstract Interpretation and Logic Programming: In honor of professor Giorgio Levi.

[2] Paul Lorenzen and Kuno Lorenz. *Dialogische Logik*. Wissenschaftliche Buchgesellschaft, 1978.

[3] Shôji Maehara. Eine Darstellung der intuitionistischen Logik in der klassischen. *Nagoya mathematical journal*, 7:45–64, 1954.

[4] Robert J. Simmons. Structural focalization. *ACM Transactions on Computational Logic (TOCL)*, 15 (3):21, 2014.

[5] Frank Van Dun. On the modes of opposition in the formal dialogues of P. Lorenzen. *Logique et analyse*, 15:103–136, 1972.

On Data-Types for Multidimensional Functions in Exact Real Arithmetic

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Exact Real Arithmetic deals with error free computations on real numbers in the sense of arbitrarily precise approximations. A theoretical foundation for such computations is given by real computability and complexity theory (see e.g. [2, 4, 9]). In contrast to the usual approach in Numerical Engineering where correctness and efficiency is mostly demonstrated empirically, algorithms in Exact Real Arithmetic can be verified to be correct, have sound semantics and are closed under composition.

Of great practical importance are not only functions over real numbers, but also operators on real functions, i.e., functions mapping real functions to real functions. Examples include Differentiation, Integration or solving Initial Value Problems for Differential Equations. Such problems commonly occur in applications in science and engineering and are heavily studied in numerical analysis.

However, results from real complexity suggest that many basic operators are computationally hard. For example, parametric maximization relates to the $\mathcal{P}$ vs. $\mathcal{NP}$ problem [5] and integration to the stronger $\mathcal{FP}$ vs. $\#\mathcal{P}$ problem [1] in the sense that the complexity classes are equal if those operators map polynomial time computable functions to polynomial time computable functions. This remains true even if one restricts the input to smooth functions. However, for many of those operators, the result when applied only to polynomial time computable analytic functions will again be a polynomial time computable analytic function. This can often even be turned into uniform algorithms for those operators by choosing the right representation.

One example is the problem of finding the solution of Initial Value Problems for Systems of Ordinary Differential Equations of the form

$$\dot{y}_i = F_i(t, y_1, \ldots, y_d),\ y_i(0) = 0,\ d \in \mathbb{N}$$

where the functions $F_i : \mathbb{R}^{d+1} \to \mathbb{R}$ are analytic. It is well known that the solution of such a system will again be analytic and polynomial-time computable if the right-hand side functions are polynomial time computable, while in the general case the problem has been shown to be $\mathcal{PSPACE}$-hard [3]. An implementation for polynomial right-hand side functions has recently been presented by Müller and Korovina [8]. To extend this solver to the analytic case, one first needs to define how to represent analytic functions in a programming language like `C++`.

Analytic functions can be locally represented by a power series around some point. For uniform computability, however, knowledge of the power series alone is not sufficient. The data type has to be enriched by some additional natural information about the function [6]. Such enrichment has been studied in theory and parameterized complexity bounds are known. However, previous work considered only one-dimensional analytic functions, while for the aforementioned problem of solving Initial Value Problems at least two-dimensional functions are necessary. There are several ways to generalize the theory to the multidimensional case. One possibility is to view a d-dimensional function as a one-dimensional function where the coefficients of the power series are given by evaluating a $(d-1)$-dimensional analytic function. For this it is necessary to find the right parameters for the recursive evaluation.

An implementation of Exact Real Arithmetic in `C++` can be found in Norbert Mller's library `iRRAM` [7]. `iRRAM` extends `C++` by data-types to compute (seemingly) exactly with real numbers and already provides implementations for many standard functions over the reals.

Based on this library, we give a prototypical implementation of data-types for functions $f : \mathbb{R}^d \to \mathbb{R}$ for arbitrary $d \in \mathbb{N}$ where f is at least (complex) analytic on some compact domain $D \subseteq \mathbb{R}^d$. Operators for Addition, Subtraction, Multiplication, Division, Composition, Partial Differentiation and Analytic Continuation as well as a solver for Initial Value Problems on such functions have been implemented.

While all those operations have been studied in real complexity theory, for an efficient implementation many details that are usually neglected have to be considered and various design choices have to be made. To this end, different algorithms for the above operators are evaluated and empirical results are compared with complexity bounds known from theory.

References

[1] Harvey Friedman. "The computational complexity of maximization and integration". In: *Adv. in Math.* 53.1 (1984), pp. 80–98. ISSN: 0001-8708. DOI: `10.1016/0001-8708(84)90019-7`. URL: `http://dx.doi.org/10.1016/0001-8708(84)90019-7`.

[2] A. Grzegorczyk. "On the definitions of computable real continuous functions". In: *Fund. Math.* 44 (1957), pp. 61–71. ISSN: 0016-2736.

[3] Akitoshi Kawamura. "Lipschitz Continuous Ordinary Differential Equations are Polynomial-Space Complete". In: *Computational Complexity* 19.2 (2010), pp. 305–332.

[4] Ker-I Ko. *Complexity theory of real functions.* Progress in Theoretical Computer Science. Boston, MA: Birkhäuser Boston Inc., 1991, pp. x+309. ISBN: 0-8176-3586-6. DOI: `10.1007/978-1-4684-6802-1`. URL: `http://dx.doi.org/10.1007/978-1-4684-6802-1`.

[5] Ker-I Ko and Harvey Friedman. "Computational complexity of real functions". In: *Theoret. Comput. Sci.* 20.3 (1982), pp. 323–352. ISSN: 0304-3975. DOI: `10.1016/S0304-3975(82)80003-0`. URL: `http://dx.doi.org/10.1016/S0304-3975(82)80003-0`.

[6] Norbert Th. Müller. "Constructive Aspects of Analytic Functions". In: *Proc. Workshop on Computability and Complexity in Analysis.* Vol. 190. InformatikBerichte. FernUniversität Hagen. 1995, pp. 105–114.

[7] Norbert Th. Müller. "The iRRAM: Exact Arithmetic in C++". In: *CCA*. Ed. by Jens Blanck, Vasco Brattka, and Peter Hertling. Vol. 2064. Lecture Notes in Computer Science. Springer, 2000, pp. 222–252. ISBN: 3-540-42197-1.

[8] Norbert Th. Müller and Margarita V. Korovina. "Making big steps in trajectories". In: *Proceedings Seventh International Conference on Computability and Complexity in Analysis, CCA 2010, Zhenjiang, China, 21-25th June 2010.* 2010, pp. 106–119. DOI: `10.4204/EPTCS.24.15`. URL: `http://dx.doi.org/10.4204/EPTCS.24.15`.

[9] Klaus Weihrauch. *Computable Analysis.* An Introduction. Berlin/Heidelberg: Springer, 2000, p. 285. ISBN: 3-540-66817-9.

Understanding the Strength of the Compositional Truth

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Our talk concerns axiomatic theories of truth. We define these theories as extensions of Peano Arithmetic PA with axioms governing a unary predicate $T(x)$ not in the arithmetical signature with the intended reading "x is (a Gödel code of) a true sentence". By choosing axioms governing $T(x)$ we specify a concrete theory of truth.

One of the most natural theories one can formulate is the theory CT^- (or $\mathrm{CT}\!\upharpoonright$) of the classical compositional stratified truth predicate. Its axioms for truth predicate comprise Tarski–like clauses for the arithmetical sentences, but no induction for the sentences containing the truth predicate.

By a theorem of Enayat and Visser (preceded by a related result of Kotlarski–Krajewski–Lachlan and proved independently by Leigh using different methods), CT^- is conservative over PA, i.e. it does not prove any arithmetical theorems unprovable in PA itself. On the other hand, if we add the full induction for the formulae containing the truth predicate, the resulting theory CT can prove the consistency of PA by a formalized soundness argument. Therefore a natural question arises of how to isolate the axioms actually needed to derive new arithmetical consequences. Our general strategy is to define possibly natural axioms or axiom schemes which are clearly provable in CT (or actually in its fragment CT_1 with the induction for the truth predicate restricted to Σ_1-formulae) and to compare their arithmetical strength. We try to establish where the borderline is located between conservative and non-conservative theories of compositional truth.

It turns out that the principles analysed so far which are strictly weaker than CT_1 are either conservative over PA or share arithmetical consequences with the theory CT_0 which is CT^- with induction scheme for Δ_0-formulae of the extended language. The arithmetical content of this theory happens to enjoy a number of independent characterisations and to be surprisingly robust. Namely, CT^- with (a formalised version of) any of the following principles added has the same arithmetical consequences:

1. Δ_0-induction scheme for the formulae containing the truth predicate.
2. Axioms of PA are true and true sentences are closed under derivations in the first-order logic.
3. True sentences are closed under derivations in the propositional logic.
4. Sentences provable in pure first-order logic are true.
5. Axioms of PA are true and a disjunction of arbitrary length is true iff some of its disjuncts is.

Our original import is that (5) proves all the arithmetical consequences of (2). Along with a simple observation that (1) implies (5), this shows that CT_0 is not conservative over PA (and is indeed a surprisingly strong theory), thus solving (a variant of) a problem posed independently by Visser and Heck. The equivalence of (3) and (1) is a previously known result due to Cieliski (in [2]). The fact that (4) implies (5) follows from Theorem 1 in [1] and an unpublished theorem, both of the same author.

As a consequence, our findings show that the borderline between conservative and non-conservative theories should be sought for among still weaker theories and pose a natural problem of formulating good candidates for such intermediate principles. If time allows, we will try to list a few such natural principles, which are conceivably nonconservative, but still weaker than CT_0.

References

[1] C. Cieliski, "Truth, Conservativeness, and Provability, *Mind 119*, pp. 409-422

[2] C. Cieliski, "Deflationary Truth and Pathologies", *Journal of Philosophical Logic 39*, pp. 325-337.

[3] A. Enayat, A. Visser, "New Constructions of Satisfaction Classes", *Logic Group Preprint Series 303* .

[4] V. Halbach, *Axiomatic Theories of Truth*, Cambridge University Press 2011.

[5] H. Kotlarski, "Bounded Induction and Satisfaction Classes", *Zeitschrift für matematische Logik und Grundlagen der Mathematik 32*, pp. 531-544.

[6] G. Leigh, "Conservativity for theories of compositional truth via cut elimination", *The Journal of Symbolic Logic 80*, pp. 825-865.

Author Index