Herbrand centenary lecture

Georg Kreisel, F.R.S. (Salzburg), *Aspects of Herbrand's thèse: combinations with contemporary ideas and relations to foundational perennials*

Invited lectures

Hans Adler (Leeds), *The role of matroids in model theory*

Abstract. Matroids (also known as pregeometries) are combinatorial structures with an excellent dimension theory. They were introduced into model theory with Morley's categoricity theorem. A part of classification theory can be understood by regarding the lattice of algebraically closed sets as a "matroid that need not have elements": an (almost) M-symmetric algebraic lattice. The connection illuminates some recent research.

Sergei Goncharov (Novosibirsk), *On two problems of Turing complexity for strongly minimal theories*

Abstract. It is proved that all countable models of strongly minimal theories with computable model are computable with respect to $\mathbf{0}''$. All countable models of uncountably categorical theories with computable model are computable with respect to $\mathbf{0}^n$ for an appropriate n. We build a series of examples of strongly minimal theories with computable models with unbounded complexity in arithmetical hierarchy. The results disprove two known hypotheses of S. Lempp.

Joel David Hamkins (New York), *Set-theoretic geology* (joint work with Gunter Fuchs and Jonas Reitz)

Abstract. The technique of forcing in set theory is customarily thought of as a method for constructing outer as opposed to inner models of set theory. A set theorist typically has a model of set theory V and constructs a larger model $V[G]$, the forcing extension, by adjoining a V-generic filter G over some partial order P in V. A switch in perspective, however, allows us to view forcing as a method of describing inner models as well. The idea is simply to search inwardly for how the model V itself might have arisen by forcing. Given a set theoretic universe V, we consider the classes W over which V can be realized as a forcing extension $V = W[G]$ by some W-generic filter $G \subset P$ in W. This change in viewpoint is the basis for a collection of questions constituting the topic we refer to as set-theoretic geology. In this talk, I will present some of the most interesting initial results in the topic, along with an abundance of open questions, many of which concern fundamental issues.

A ground model of the universe V is a class W such that V is obtained by set forcing over W, so that $V = W[G]$ for some W-generic filter $G \subset P$ in W. The model V satisfies the Ground Axiom if there are no such W properly contained in V. The model W is a bedrock of V if W is a ground of V and satisfies the Ground Axiom. The mantle of V is the intersection of all grounds

of V. The generic mantle of V is the intersection of all grounds of all set forcing extensions of V. Our main initial result is that every model of **ZFC** is the mantle and generic mantle of another model of **ZFC**. We prove this theorem while also controlling the HOD of the final model, as well as the generic HOD, the intersection of the HODs of all forcing extensions. Iteratively taking the mantle penetrates down through the inner mantles to what we call the outer core, what remains when all outer layers of forcing have been stripped away. Many fundamental questions remain open.

This is joint work with Gunter Fuchs (Münster) and Jonas Reitz (NY City Tech).

Robert Lubarsky (Boca Raton), *Topological Semantics with Settling*
Abstract. Under the standard topological semantics, truth is determined by open sets only. In the variant to be presented, points also play a role. The original motivation for this change was to prove independence results for constructive set theories weaker than **IZF**, even weaker than **CZF** (for which the standard semantics is inadequate, as it is self-realizing for **IZF**). It will be shown that this settling semantics forces only Bounded (not Full) Separation to hold, as well as only a weakened form of Power Set. Natural conditions on the topological space will be discussed that yield Full Separation and Exponentiation.

Nicole Schweikardt (Frankfurt), *Gaifman's locality theorem revisited*
Abstract. Gaifman's locality theorem, proven in the early 1980s, provides a normal form for first-order formulas. It states that every first-order sentence is equivalent to a local sentence, i.e., a sentence where quantification is basically restricted to local neighbourhoods of elements.

Apart from proving limitations in the expressive power of first-order logic, Gaifman's theorem has found various applications in algorithms and complexity. For example, it has been used as a tool for proving so-called "algorithmic meta theorems" stating that first-oder definable problems can be solved efficiently on various classes of structures.

In this talk I want to give an overview of different versions of Gaifman's theorem and of their various applications. Particular emphasis will be on recent results concerning the succinctness of formulas in Gaifman's normal form and the complexity of the translation of arbitrary first-order formulas into equivalent formulas in normal form.

Michiel van Lambalgen (Amsterdam), *Logic in a neuroscience lab*
Abstract. A major problem of psycholinguistics is to understand the nature of the cognitive processes that enable language comprehension and production. Various methods are used to further our understanding, including brain imaging and the study of developmental language disorders. The purpose of this talk is to show that logic can play a pivotal role in such investigations. I explain how use of logic may lead to refined models of what goes on during language processing, and also leads to precise testable hypotheses, in particular concerning

the linguistic encoding of temporal notions. This encoding leaves a trace as an E(vent-)R(elated) P(otential), a form of electro-encephalogram, especially useful for studying the time-course of linguistic processing. Baggio, van Lambalgen and Hagoort (2008) derived predictions about ERPs from the computational theory of tense and aspect (the "event calculus"), proposed in van Lambalgen and Hamm (2004), concerning temporal propostions, tense violations and the progressive. A common feature of these predictions, dictated by the underlying logic of the event calculus, is that linguistic processing in these domains is non-monotonic, and indeed the experiments yielded ERPs which give evidence of non-monotonic recomputations.

References

G. Baggio and M. van Lambalgen, The processing consequences of the imperfective paradox, *Journal of Semantics* (2007), doi: 10.1093/jos/ffm005.

G. Baggio, M. van Lambalgen and P. Hagoort, Computing and recomputing disourse models: an ERP study, *Journal of Memory and Language* (2008).

M. van Lambalgen and F. Hamm, *The proper treatment of events*, Blackwell 2004.

Contributed talks

Achim Blumensath (Darmstadt), *The transduction hierarchy for guarded second-order logic* (joint work with Bruno Courcelle)

Abstract. We study guarded second-order interpretations between classes of finite structures. The main result is a complete description of the resulting hierarchy. It is linear of order type $\omega + 3$. Each level can be characterised in terms of a suitable variant of tree-width. Canonical representatives of the various levels are: the class of (i) all trees of height n, for $n < \omega$; (ii) all paths; (iii) all trees; and (iv) all grids.

Eyvind Briseid (Darmstadt), *On some applications of proof mining in metric fixed point theory*

Abstract. This talk will deal with some aspects of the use of methods from proof theory in finding new information from given proofs of established theorems in metric fixed point theory. Specifically, we give general conditions under which one can extract explicit and highly uniform rates of convergence for the Picard iteration sequences for selfmaps of bounded metric spaces from ineffective proofs of convergence to a unique fixed point. In case studies we have found such explicit rates of convergence in two concrete cases. Our method involves an application of a logical metatheorem due to U. Kohlenbach combined with an argument which, loosely speaking, amounts to general conditions which allow us to treat a $\forall\exists\forall$-sentence as a $\forall\exists$-sentence in this specific setting. The resulting reduction in logical complexity allows us to use the existing machinery to extract quantitative bounds of the sort we need.

Merlin Carl (Bonn), *Naproche – Logic and Linguistics of Natural Mathematical Language*

Abstract. It is a desirable task to combine the security of fully formalized proofs with the readability of proofs written in natural language, as they appear, e.g., in textbooks. The Naproche project (Natural Language Proof Checking) aims at approximating textbook proofs by mirroring the use of language and higher argumentation figures we find in such proofs in a rich, but controlled fragment of natural language, allowing mathematical formulae, that is accessible to a formalization routine and hence to automatic proof checking. For this purpose, we use a new intermediate linguistical format: the Proof Representation Structures (PRS).

I will indicate several logical and linguistical phenomena the Naproche system will have to deal with like the use of existential statements for introducing and naming discourse objects or the problem of finding suitable mechanisms for recognizing the introduction and retraction of assumptions. I shall demonstrate a preliminary implementation of the Naproche system.

Ioanna Dimitriou (Bonn), *Successors of singular cardinals in choiceless Chang conjectures*

Abstract. We are interested in the consistency with ZF of higher Chang conjectures of the form $(\kappa, \mu) \twoheadrightarrow (\lambda, \nu)$, when κ is the successor of a singular cardinal. This statement means that whenever we have a structure $\mathcal{A} = \langle A, R, \dots \rangle$ in a countable language and such that $|A| = \kappa$, R is a unary predicate, and $|R| = \mu$ (we say $\mathcal{A}$ is of type (κ, μ)) then we can always find an elementary substructure of type (λ, ν).

This model theoretic property of cardinals is well studied under the axiom of choice. The consistency strength of several versions of this statement with ZFC range from $0^\dagger$ to inconsistency. We'll see that if we drop the requirement of choice the consistency strength of all these statements drops to one strongly compact cardinal with an Erdős on top. More precisely, we show that the consistency of ZFC + "there exist cardinals $\kappa > \theta \geq \lambda$ such that θ is κ-strongly compact and κ is λ-Erdős" implies the consistency of ZF + "for every $\nu < \mu < \kappa$, $(\kappa, \mu) \twoheadrightarrow (\lambda, \nu)$ and κ is the successor of a singular cardinal". This can be done with either some symmetric version of strongly compact Prikry forcing or some symmetric version of a certain iteration of Prikry forcings with θ-complete filters (Gitik-Prikry forcing).

If time permits we'll look briefly at a construction inspired by Moti Gitik's "all uncountable cardinals can be singular". We'll see that if we start from α-many strongly compact cardinals with a λ-Erdős on top, we can get a model of ZF where we have α-many singulars in a row and immediately successing them a cardinal κ for which $(\kappa, \mu) \twoheadrightarrow (\lambda, \nu)$ holds for any $\nu < \mu$ below κ.

Fernando Ferreira (Lissabon), *On commuting conversions* (joint work with Gilda Ferreira)
Abstract. Commuting conversions were introduced in the natural deduction calculus as ad hoc devices for the purpose of guaranteeing the subformula property in normal proofs. In a well known book, Jean-Yves Girard commented harshly on these conversions by saying that "one tends to think that natural deduction should be modified to correct such atrocities". We present an embedding of the intuitionistic predicate calculus into a second-order predicative system for which there is no need for commuting conversions. Furthermore, we show that the redex and the contractum of a commuting conversion of the original calculus translate into second-order derivations which are, in a certain standard sense, equivalent.

Bernhard Irrgang (Bonn), *On ω_3-chains in $P(\omega_1)$ mod finite*
Abstract. It is shown to be consistent that there exists a sequence $\langle X_\alpha \mid \alpha < \omega_3 \rangle$ of subsets $X_\alpha \subseteq \omega_1$ such that $X_\beta - X_\alpha$ is finite and $X_\alpha - X_\beta$ is uncountable for all $\beta < \alpha < \omega_3$. Such a sequence is added by a ccc forcing which is constructed along a simplified $(\omega_1, 2)$-morass. The idea of the proof is to use a finite support iteration of countable forcings which is not linear but three-dimensional. In the same way it is possible to construct along a simplified $(\omega_1, 2)$-morass a ccc forcing which adds ω_3 many distinct functions $f_\alpha : \omega_1 \to \omega$ such that $\{\xi < \omega_1 \mid f_\alpha(\xi) = f_\beta(\xi)\}$ is finite for all $\alpha < \beta < \omega_3$.

Javier Legris (Buenos Aires), *Formality and Universal Language in 19th Century Symbolic Logic: The Cases of Frege and Schröder*
Abstract. In a paper from 1885, "On Formal Theories of Arithmetic", Frege maintained a particular conception of formal theories in Arithmetic. According to it, a theory for Arithmetic is formal if every law of the theory is logically derived only from logical notions via definitions. Frege connected this characterization explicitly with his idea of universality: The concept of number is universally applicable. It can be applied to entities of every kind. He refers in this case to a "general applicability" of this concept. Some years later, in Grundgesetze, Frege characterized his conception of formal theories as contentual. For the meaning of the symbols are the real objects of the theory, the symbols being only a medium to express this meaning. This is the sense according to which Frege constructed arithmetic in his conceptual script. Frege opposed this conception of formal theory to the idea of symbolic systems whose elements had no meaning at all. Now, Ernst Schröder with his program of an abstract algebra had just advocated for the latter idea of formal theory: Formal algebra could be applied to different domains, and the operations received diverse interpretations depending on the domain considered. This general applicability of formal algebra suggested a new idea of universality. Schröder expanded later this idea, when he developed the algebra of relatives, with his pasigraphy: The symbolic system of the algebra of relatives turns into a language, which had the pragmatic aim of expressing every mathematical theory – and prima facie every

scientific theory. The two cases of Frege and Schröder suggest that to different conceptions of formality correspond different conceptions of universality. It will be argued that these different conceptions can be understood properly as alternative ideas of formal entities.

Laurentiu Leustean (Darmstadt), *Recent applications of proof mining in ergodic theory* (joint work with Ulrich Kohlenbach)
Abstract. In this talk, we present recent work in collaboration with Ulrich Kohlenbach. Applying methods of proof mining, we provide an explicit uniform bound on the local stability of ergodic averages in uniformly convex Banach spaces. Our result can also be viewed as a finitary version in the sense of Terence Tao of the Mean Ergodic Theorem for such spaces and so generalizes similar results obtained for Hilbert spaces by Avigad, Gerhardy and Towsner and T. Tao.

Johann Makowsky (Haifa), *Intriguing graph polynomials*
Abstract. We show how model theoretic methods can help in discussing various properties of graph polynomials.

Philipp Schlicht (Münster), *Thin projective equivalence relations*
Abstract. We consider thin equivalence relations on the real numbers in the context of projective determinacy. Thin equivalence relations are those with no perfect set of pairwise inequivalent reals. Projective ordinals, the suprema of the lengths of prewellorders in a fixed projective pointclass, provide optimal bounds for the number of equivalence classes of thin projective equivalence relations. A natural question is how to construct an inner model which has a representative in every equivalence class of every thin $\mathbf{\Pi}^1_n$ equivalence relation defined from a parameter in the inner model. We present an extension of a theorem of Hjorth which describes these inner models for all even n.

Peter Schuster (München), *Calibrating Baire's theorem for closed sets* (joint work with Hajime Ishihara)
Abstract. In the spirit of constructive reverse mathematics we relate a certain version BT′ of Baire's theorem for closed sets to a restricted form WC-N′ of the weak continuity principle for numbers and to the boundedness principle BD-N. More precisely, WC-N′, BT′ + ¬LPO, and BD-N + ¬LPO are equivalent in a constructive system, where LPO stands for the limited principle of omniscience.

Lutz Strüngmann (Duisburg-Essen), *The structure of* $\mathrm{Ext}(G, H)$ *in models of ZFC* (joint work with Saharon Shelah)
Abstract. Since the solution of the Whitehead problem by Saharon Shelah many tools and techniques have been developed in order to determine the structure of $\mathrm{Ext}(G, H)$ for abelian groups or more generally modules G and H over commutative domains. We will present an overview of recent results and show how certain prediction principles in various models of ZFC can be used for characterizing $\mathrm{Ext}(G, H)$.

Benno van den Berg (Darmstadt), *Realizability in algebraic set theory* (joint work with Ieke Moerdijk)
Abstract. The aim of algebraic set theory is to provide a uniform categorical semantics for set theories of different kinds (classical or constructive, predicative or impredicative, well-founded or non-well-founded, etc.). In this talk we show how realizability methods for constructive set theories like **IZF** and **CZF** fit into the framework of algebraic set theory. This allows us to simultaneously recover known realizability interpretations of Friedman, McCarty and Rathjen, and introduce some unfamiliar ones. (The contents of this talk are based a paper written together with Ieke Moerdijk.)

Jip Veldman (Bonn), *Symmetric Submodels and Mutual Stationarity*
Abstract. It is notoriously hard to obtain models of Zermelo-Fraenkel set theory with the axiom of Choice in which $\aleph_\omega$ fulfils certain combinatorial properties. For instance, it is not known whether it is consistent relative to large cardinal axioms that $\aleph_\omega$ can be Jónsson. The same is true for a large class of interesting mutual stationarity properties below $\aleph_\omega$. This situation changes drastically as soon as one is willing to drop the axiom of choice.

I constructed various symmetric submodels in which the axiom of choice fails, $\aleph_\omega$ is Jónsson, and a mutual stationarity property holds below $\aleph_\omega$. In this talk I will present various aspects of these constructions which are part of joint work with Arthur Apter and Peter Koepke.