INVARIANT CONDITIONALS AND INDEPENDENCE IN ŁUKASIEWICZ LOGIC

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Abstract

By the Kroupa-Panti theorem, states of an MV-algebra A are (in canonical one-one correspondence with) regular probability Borel measures on the maximal spectral space of A. Thus the finite additivity of states completely captures the sigma-additivity of their corresponding probability measures.

Further, states are coherent probability assessments, in the sense of De Finetti, on events described by Lukasiewicz (infinite-valued propositional) logic. Pursuing this line of research, to every consistent finite set Θ of conditions, expressed by formulas (equivalently, by one formula) in Lukasiewicz logic, we attach a map $\mathcal{P}_\Theta$ assigning to each formula ψ a rational number $\mathcal{P}_\Theta(\psi) \in [0,1]$ that represents *"the conditional probability of ψ given Θ"*. Algebraically speaking, $\mathcal{P}_\Theta$ yields a state of the Lindenbaum algebra of Θ. The map $\Theta \mapsto \mathcal{P}_\Theta$ has the following properties:

(i) (Effectiveness): The value $\mathcal{P}_\Theta(\psi)$ is effectively computable from Θ and ψ.

(i) (Faithfulness): $\mathcal{P}_\Theta(\psi) = 1$ iff ψ is a syntactic consequence of Θ, iff ψ is a semantic consequence of Θ.

(ii) (Additivity): For any two formulas ϕ and ψ whose $\odot$-conjunction is falsified by Θ, letting χ be their $\oplus$-disjunction we have $\mathcal{P}_\Theta(\chi) = \mathcal{P}_\Theta(\phi) + \mathcal{P}_\Theta(\psi)$.

(iii) (Invariance): Whenever Θ' is a finitely axiomatizable theory and ι is an isomorphism between the Lindenbaum algebras of Θ and of Θ', then for any two formulas ψ and ψ' that correspond via ι we have $\mathcal{P}_\Theta(\psi) = \mathcal{P}_{\Theta'}(\psi')$. Invariance generalizes the natural requirement that the probability of ψ given Θ should be the same as the probability of a new variable X given (Θ and $(\psi \leftrightarrow X)$).

Using our conditional $\Theta \mapsto \mathcal{P}_\Theta$, we can simply say that event ψ is *independent of* Θ if the conditional probability of ψ given Θ is the same as the unconditional probability of ψ, i.e., the probability of ψ given the tautology $\psi \leftrightarrow \psi$.

We will survey the wealth of recent results on MV-algebraic states and their applications.

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Date: January 29, 2009.