INVARIANT CONDITIONALS AND INDEPENDENCE IN ŁUKASIEWICZ LOGIC

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Abstract

By the Kroupa-Panti theorem, states of an MV-algebra $A$ are (in canonical one-one correspondence with) regular probability Borel measures on the maximal spectral space of $A$. Thus the finite additivity of states completely captures the sigma-additivity of their corresponding probability measures.

Further, states are coherent probability assessments, in the sense of De Finetti, on events described by Łukasiewicz (infinite-valued propositional) logic. Pursuing this line of research, to every consistent finite set $\Theta$ of conditions, expressed by formulas (equivalently, by one formula) in Łukasiewicz logic, we attach a map $P_\Theta$ assigning to each formula $\psi$ a rational number $P_\Theta(\psi) \in [0, 1]$ that represents "the conditional probability of $\psi$ given $\Theta$". Algebraically speaking, $P_\Theta$ yields a state of the Lindenbaum algebra of $\Theta$. The map $\Theta \mapsto P_\Theta$ has the following properties:

(i) (Effectiveness): The value $P_\Theta(\psi)$ is effectively computable from $\Theta$ and $\psi$.

(ii) (Faithfulness): $P_\Theta(\psi) = 1$ iff $\psi$ is a syntactic consequence of $\Theta$, iff $\psi$ is a semantic consequence of $\Theta$.

(iii) (Additivity): For any two formulas $\phi$ and $\psi$ whose $\&$-conjunction is falsified by $\Theta$, letting $\chi$ be their $\lor$-disjunction we have $P_\Theta(\chi) = P_\Theta(\phi) + P_\Theta(\psi)$.

(iv) (Invariance): Whenever $\Theta'$ is a finitely axiomatizable theory and $i$ is an isomorphism between the Lindenbaum algebras of $\Theta$ and of $\Theta'$, then for any two formulas $\psi$ and $\psi'$ that correspond via $i$ we have $P_\Theta(\psi) = P_{\Theta'}(\psi')$. Invariance generalizes the natural requirement that the probability of $\psi$ given $\Theta$ should be the same as the probability of a new variable $X$ given ($\Theta$ and $(\psi \leftrightarrow X)$).

Using our conditional $\Theta \mapsto P_\Theta$, we can simply say that event $\psi$ is independent of $\Theta$ if the conditional probability of $\psi$ given $\Theta$ is the same as the unconditional probability of $\psi$, i.e., the probability of $\psi$ given the tautology $\psi \leftrightarrow \psi$.

We will survey the wealth of recent results on MV-algebraic states and their applications.

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