

# STATES ON EFFECT ALGEBRAS AND MV-ALGEBRAS THEIR APPLICATIONS

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In our talk we present, the notion of a state on effect algebras and MV-algebras. We recall that an effect algebra is an algebra  $(M, +, 0, 1)$  with a partially defined addition,  $+$  that is (i) associative, (ii) commutative, (iii) for any element  $a \in M$  there is a unique element  $a' \in M$  such that  $a + a' = 1$ , and (iv) if  $a + 1 \in M$ , then  $a = 0$ .

In particular, if  $(M; \oplus, \odot, *, 0, 1)$  is an MV-algebra, then the operation  $+$  defined by  $a + b$  is defined iff  $a \leq b^*$  and in this case,  $a + b = a \oplus b$ . Then  $(M; +, 0, 1)$  is an effect algebra with the Riesz decomposition property.

A state on an effect algebra (an MV-algebra)  $M$  is a mapping  $s : M \rightarrow [0, 1]$  such that  $s(1) = 1$  and  $s(a + b) = s(a) + s(b)$  whenever  $a + b$  is defined on  $M$ .

We describe the state space (that in some situations could be empty), we show state spaces in particular case when it is nonvoid. We apply this notion to represent monotone  $\sigma$ -complete effect algebras or  $\sigma$ -complete MV-algebras as a homomorphic image of an algebra of functions defined on the state space or on the set of extremal states. This is a variation of Loomis–Sikorski type theorems.

In addition, we show state-morphism MV-algebras.

## REFERENCES

- [1] A. Dvurečenskij, *Loomis–Sikorski theorem for  $\sigma$ -complete MV-algebras and  $\ell$ -groups*, J. Austral. Math. Soc. Ser. A **68** (2000), 261–277.
- [2] A. Dvurečenskij, *Loomis–Sikorski theorem for monotone  $\sigma$ -complete effect algebras*, J. Austral. Math. Soc. **79** (2005), 305–318.
- [3] A. Di Nola, A. Dvurečenskij, *State-morphism MV-algebras*, J. Pure Appl. Algebra In: Festschrift for Franco Montagna on the occasion of his 60th birthday. To appear in JPAL. Eds. Luca Spada et al. pp. 63–85.
- [4] A. Di Nola, A. Dvurečenskij, *On some classes of state-morphism MV-algebras*, Math. Slovaca, to appear.
- [5] J. Foulis, M.K. Bennett, *Effect algebras and unsharp quantum logics*. Found. Phys. **24** (1994), 1325–1346.

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