On MV-algebras with internal state

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Abstract

We will concern with some subvarieties of the variety $\mathcal{SMV}$ of all State MV-algebras *State MV-algebras* (A, σ). These subvarieties are defined by systems of equations, which define the subvarieties of $\mathcal{MV}$ (the variety of all MV-algebras).

Actually, let

$$\{q_i(x) = p_i(x)\}_{i=1,\ldots,n}$$

be a system of equations defining the subvariety W of $\mathcal{MV}$, then it is clear that the systems of equations

$$\{\sigma(q_i(x)) = \sigma(p_i(x))\}_{i=1,\ldots,n}$$

and

$$\{q_i(\sigma(x)) = p_i(\sigma(x))\}_{i=1,\ldots,n} \quad (\sigma(A) \in W)$$

define two subvarieties of $\mathcal{SMV}$ here denoted by $\Sigma(W)$ and $W(\Sigma)$, respectively.

Since σ fixes the elements of $\sigma(A)$, then

$$\Sigma(W) \subseteq W(\Sigma).$$

Denoting by S_i the MV-chain with $i+1$ elements, below we list the results we are going to display:

1. If $W = V(S_1, \ldots, S_n)$, then

$$\Sigma(W) = W(\Sigma);$$

both coinciding with the subvariety defined by $\sigma((nx)^* \wedge x) = 0$.

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2. If $\sigma(A) \in W = V(C)$, where C is the so called *Chang's* algebra and $\sigma(x) = 0$, then for any $x \in RadA$, σ is a state morphism.
3. Any State MV-algebra, with $\sigma(A) \in V(S_p)$, with p prime number is a Morphism State MV-algebra.