Bisimulation and Logics for Knowledge and Information

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bisimulation structures information

bisimulation — the quintessential back&forth

model theory, not just in classical settings

logics accessing information — in structural representations

with relevant semantics "up to what?"

two main parts

(I) basics: bisimulation and back&forth games

- bisimulation as modal Ehrenfeucht–Fraïssé
- bisimulation as the mother of back&forth
- model theory of modal logics

(II) survey: variations, generalisations & challenges

- bisimilar coverings for graphs and hypergraphs
- classically beyond FO to MSO
- essentially modal variations within FO
- non-classical modal steps beyond FO:
  team semantic & inquisitive scenarios,
  modal common knowledge

part I: bisimulation as quintessential back&forth

on graph-like structures

Kripke structures (possible worlds/accessibility),
transition systems (states/transitions),
game graphs (positions/moves)

capture informational/behavioural/positional equivalence
that may not be respected in concrete structural representation (!)

core idea: dynamic back&forth probing of possibilities

$\longrightarrow$ **exploration of what is meant to be represented in these structures & eliminating overhead in concrete structural representations (!)**

games in logic: the bigger picture

two distinct model-theoretic traditions:

(1) semantic evaluation games (model checking games)**:**

game protocol to test satisfaction relation: given structure $\mathcal{A}$ and formula $\varphi \in \mathsf{L}$ determine whether $\mathcal{A} \models \varphi$

(2) comparison, equivalence games (back&forth games)**:**

game protocol to test L-equivalence/similarity: given structures $\mathcal{A}$ and $\mathcal{B}$ determine to which extent $\mathcal{A} \equiv_{\mathsf{L}} \mathcal{B}$

with bisimulation notions we focus on the second kind (2)
but key results link it to the first kind (1)
and there is a systematic connection !

bisimulation game & bisimulation relations

the game: two players: **I** (challenger), **II** (defender)

play over two Kripke structures $\begin{cases} \mathcal{A} = (A, \boldsymbol{R}^{\mathcal{A}}, \boldsymbol{P}^{\mathcal{A}}) \\ \mathcal{B} = (B, \boldsymbol{R}^{\mathcal{B}}, \boldsymbol{P}^{\mathcal{B}}) \end{cases}$
or transition systems

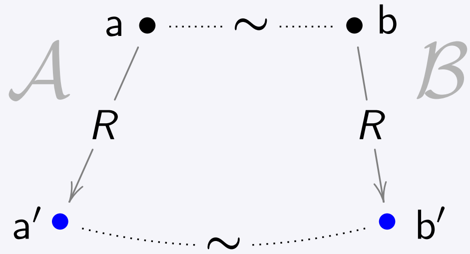
positions: pairs (a, b), correspondences between pebbled worlds

single round, challenge/response:

I shifts pebble in $\mathcal{A}$ or $\mathcal{B}$ along R-edge

II must do likewise on opposite side

effect: $(\boldsymbol{a}, \boldsymbol{b}) \rightsquigarrow (\boldsymbol{a}', \boldsymbol{b}')$



II loses in position (a, b) unless $\boldsymbol{P}^{\mathcal{A}}{\restriction}a \simeq \boldsymbol{P}^{\mathcal{B}}{\restriction}b$ (atom equivalence)
either player loses when stuck

bisimulation game & bisimulation relations

winning regions for II define **bisimulation equivalences:**

$\mathcal{A}, a \sim^\ell \mathcal{B}, b$	**II** has a winning strategy for ℓ rounds from (a, b)
$\mathcal{A}, a \sim \mathcal{B}, b$	**II** has a winning strategy for infinite game from (a, b)

intermediate limit $\sim^\omega := (\sim^\ell \text{ for all } \ell \in \mathbb{N})$

winning strategies in relational formalisation:

$\sim^\ell :\ (Z_m \subseteq A \times B)_{m \leqslant \ell}$

$\sim^\omega :\ (Z_m \subseteq A \times B)_{m \in \mathbb{N}}$ **stratified b&f systems, or**

$\sim :\ Z \subseteq A \times B$ **single bisimulation relation**

bisimulation game & bisimulation relations

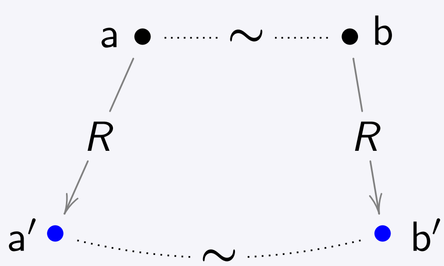
a single bisimulation relation $Z \subseteq A \times B$ for $\sim$

with characteristic b&f requirements

(*back*) for $(a, b) \in Z$ and $(b, b') \in R^{\mathcal{B}}$ there is $a' \in A$ s.t. $(a, a') \in R^{\mathcal{A}}$ and $(a', b') \in Z$

(*forth*) for $(a, b) \in Z$ and $(a, a') \in R^{\mathcal{A}}$ there is $b' \in B$ s.t. $(b, b') \in R^{\mathcal{B}}$ and $(a', b') \in Z$

witnesses winning strategy for **II** in infinite game from any $(a, b) \in Z$



classical motif: Ehrenfeucht–Fraïssé

pebble games for FO and FO_∞

I and **II** over relational structures $\mathcal{A} = (A, \boldsymbol{R}^{\mathcal{A}})$ and $\mathcal{B} = (B, \boldsymbol{R}^{\mathcal{B}})$

positions: local isomorphisms $p\colon \boldsymbol{a} \mapsto \boldsymbol{b}$, $p\colon \mathcal{A}{\restriction}\boldsymbol{a} \simeq \mathcal{B}{\restriction}\boldsymbol{b}$

single round: challenge/response for
extension by one new pebble pair
$(p\colon \boldsymbol{a} \mapsto \boldsymbol{b}) \;\rightsquigarrow\; (p'\colon \boldsymbol{a}a' \mapsto \boldsymbol{b}b')$

winning regions:
b&f equivalences
$$\begin{cases} \mathcal{A}, \boldsymbol{a} \simeq^{\ell} \mathcal{B}, \boldsymbol{b} & \ell \text{ rounds} \\ \mathcal{A}, \boldsymbol{a} \simeq^{\infty} \mathcal{B}, \boldsymbol{b} & \text{infinite game} \end{cases}$$

$\simeq^\infty$ classically known as partial isomorphy,
intermediate level $\simeq^\omega$ as finite isomorphy

Ehrenfeucht–Fraïssé

linking game equivalence to equivalence w.r.t. FO and FO_∞

Ehrenfeucht–Fraïssé/Karp thms

$\mathcal{A}, \boldsymbol{a} \simeq^{\ell} \mathcal{B}, \boldsymbol{b} \;\Leftrightarrow\; \mathcal{A}, \boldsymbol{a} \equiv^{\ell}_{\mathrm{FO}} \mathcal{B}, \boldsymbol{b}\ ^*$ FO-equiv. to qfr-depth ℓ

$\mathcal{A}, \boldsymbol{a} \simeq^{\infty} \mathcal{B}, \boldsymbol{b} \;\Leftrightarrow\; \mathcal{A}, \boldsymbol{a} \equiv^{\infty}_{\mathrm{FO}} \mathcal{B}, \boldsymbol{b}$ FO_∞-equiv.

$*$ for finite relational vocabularies
where $\simeq^\ell$ has finite index

proof ingredients:

- $\big(Z_m := \{(p\colon \boldsymbol{a} \mapsto \boldsymbol{b}) : \mathcal{A}, \boldsymbol{a} \equiv^{m}_{\mathrm{FO}} \mathcal{B}, \boldsymbol{b}\}\big)_{m \in \mathbb{N}}$
  satisfies stratified b&f conditions
- **I** wins according to $\mathcal{A}, \boldsymbol{a} \not\equiv^{m}_{\mathrm{FO}} \mathcal{B}, \boldsymbol{b} \;\rightsquigarrow\; \mathcal{A}, \boldsymbol{a}a' \not\equiv^{m-1}_{\mathrm{FO}} \mathcal{B}, \boldsymbol{b}b'$
- equivalence classes $[\mathcal{A}, \boldsymbol{a}]/{\simeq^m}$ are FO-definable at qfr-depth m

recall: the bigger picture w.r.t. games & logic

(1) semantic evaluation game (model checking game):

checking $\mathcal{A}, \boldsymbol{a} \models \varphi$ in dialogue game between verifier & refuter

(2) equivalence game (back&forth game):

checking whether $\big(\mathcal{A}, \boldsymbol{a} \models \varphi \Leftrightarrow \mathcal{B}, \boldsymbol{b} \models \varphi\big)$ for all $\varphi \in \mathrm{L}_\ell$ in back&forth game

for many logics like guarded fragment GF, k-variable fragments FO^k, ...

can typically relate levels $\equiv_{\mathrm{L}}^{\ell}$ **of L-equivalence in (2)**

to $\sim^{\ell}$ **between the game graphs of the L-evaluation game (1)**

back to bisimulation & basic modal logic ML

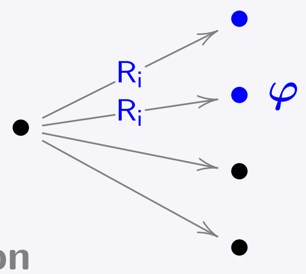
on graph-like structures

with binary accessibility relations $\boldsymbol{R} = (R_1, \dots)$ $\rightsquigarrow$ modalities $\Diamond_i / \Box_i$
and unary predicates $\boldsymbol{P} = (P_1, \dots)$ $\rightsquigarrow$ basic propositions p_i

atomic formulae: $\bot, \top$ and p_i
booleans connectives: $\wedge, \vee, \neg$
modal quantification:

$$\Diamond_i\, \varphi \;\equiv\; \exists y \big(R_i xy \wedge \varphi(y) \big)$$

$$\Box_i\, \varphi \;\equiv\; \forall y \big(R_i xy \rightarrow \varphi(y) \big)$$



relativised FO quantification

observation

- 0-bisimulation condition $\sim^0$ matches atomic equiv. $\equiv^0_{\mathrm{ML}}$
- bisimulation b&f matches modal quantification pattern

bisimulation — modal Ehrenfeucht–Fraïssé

modal Ehrenfeucht–Fraïssé/Karp thms

$\mathcal{A}, a \sim^\ell \mathcal{B}, b$	$\Leftrightarrow$	$\mathcal{A}, a \equiv^\ell_{\mathrm{ML}} \mathcal{B}, b$ *	ML-equiv. to depth ℓ
$\mathcal{A}, a \sim^\infty \mathcal{B}, b$	$\Leftrightarrow$	$\mathcal{A}, a \equiv^\infty_{\mathrm{ML}} \mathcal{B}, b$	ML_∞-equiv.

in full analogy with classical picture:

$\mathcal{A}, \mathbf{a} \simeq^\ell \mathcal{B}, \mathbf{b}$	$\Leftrightarrow$	$\mathcal{A}, \mathbf{a} \equiv^\ell_{\mathrm{FO}} \mathcal{B}, \mathbf{b}$ *	FO-equiv. to qfr-depth ℓ
$\mathcal{A}, \mathbf{a} \simeq^\infty \mathcal{B}, \mathbf{b}$	$\Leftrightarrow$	$\mathcal{A}, \mathbf{a} \equiv^\infty_{\mathrm{FO}} \mathcal{B}, \mathbf{b}$	FO_∞-equiv.

corollary

- the semantics of $\mathrm{ML} \subseteq \mathrm{ML}_\infty$ is invariant under bisimulation
- the semantics of ML-formulae of depth ℓ is invariant under $\sim^\ell$

where $\simeq^\ell$ has finite index

variations & the quintessential nature of bisimulation

- **bisimulation in game graphs for other logics**

  states: admissible assignments
  transitions: quantification patterns

  "all Ehrenfeucht–Fraïssé games are bisimulation games"

close to original (basic modal) bisimulation:

- **two-way and global bisimulation** $\approx$
  with extended challenge/response options
  (backward moves & jumps) for corresponding modalities

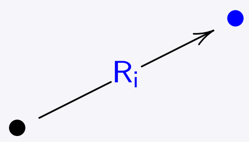
qualitatively different:

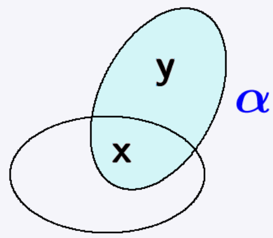
- **guarded bisimulation**
  from graphs to hypergraphs, with moves respecting overlaps

guarded bisimulation: ... hypergraph of visible patches

as an example of the systematic variability
and relationship between games (1) & (2)

access to (singleton) worlds	$\leftrightsquigarrow$	access to guarded patches
propositional information	$\leftrightsquigarrow$	local isomorphgism type
modalities in ML $\forall y(R_i xy \rightarrow \varphi(y))$	$\leftrightsquigarrow$	guarded quantification in GF $\forall \mathbf{y}\big(\alpha(\mathbf{y}) \rightarrow \varphi(\mathbf{y})\big)$
moves along accessibility edges	$\leftrightsquigarrow$	moves between patches that respect overlaps





bisimulation — modal Ehrenfeucht–Fraïssé

typical example of a bisimulation issue and its FO counterpart:

when does $\equiv_{\mathrm{ML}}$ ($\sim^\omega$) coincide with full bisimulation $\sim$?

when does $\equiv_{\mathrm{FO}}$ ($\simeq^\omega$) coincide with partial isomorphy $\simeq^\infty$?

Hennessy–Milner thm (the modal answer)

over suitably saturated models, $\sim^\omega$ ($\equiv_{\mathrm{ML}}$) coincides with $\sim$ ($\equiv^\infty_{\mathrm{ML}}$)

- **finitely branching**
- **modally or ω-saturated** (ω-saturation is good also for $\simeq^\omega/\simeq^\infty$)
- **recursively saturated pairs** (also good for $\simeq^\omega/\simeq^\infty$)

crucial in classical model-theoretic arguments for modal logics

model theory of modal logics

thesis: information-theoretically, Kripke structures are meant to represent bisimulation types

just as transition systems stand for possible system behaviours

modal model theory = bisimulation invariant model theory

here briefly look at:

- tree unfoldings
- tree model property & finite model property
- **expressive completeness** **(classical and fmt)**

tree unfoldings (cf. game trees)

tree unfolding: unfolding $\mathcal{A}$ into $\mathcal{A}^*_a$

based on the set of labelled directed paths σ rooted at a in $\mathcal{A}$ with natural projection to endpoints as a homomorphism

$$\begin{array}{rcl} \pi\colon \mathcal{A}^*_a & \longrightarrow & \mathcal{A} \\ \sigma & \longmapsto & \pi(\sigma) \end{array}$$

that induces a bisimulation $\mathcal{A}^*_a, a \sim \mathcal{A}, a$

$\pi\colon \mathcal{A}^*_a \longrightarrow \mathcal{A}$ is an example of a bisimilar covering:

- **π is a homomorphism:** **the forth-property**
- **π has lifting property:** **the back-property**

  for its graph $\{(\sigma, \pi(\sigma)) \colon \sigma \in \mathcal{A}^*_a\}$: a bisimulation relation

tree unfoldings and tree model property

$\left.\begin{array}{r}\text{bisimilar unfoldings into tree structures}\\ \text{preservation under bisimulation}\end{array}\right\} \Rightarrow$ tree model property

tree model property:

for all $\sim$-invariant logics $\mathsf{ML}, \ldots, \mathsf{L}_{\mu}, \ldots \mathsf{ML}_{\infty}$:
every satisfiable formula has a tree model

important: can employ good model-theoretic and algorithmic properties of trees, MSO on trees, tree automata, ...
for robust decidability and complexity results for modal logics

finite (tree) model property

for basic modal logic ML (and some close relatives)
even get finite tree models, hence the

finite model property:

every satisfiable formula of ML has a finite (tree) model

ad-hoc method: for $\varphi \in \mathsf{ML}$ of depth ℓ,
truncate tree model at depth ℓ (preserving $\sim^{\ell}$)
and prune $\sim^{\ell}$-equivalent siblings (finite index)

more generic method: passage to $\sim^{\ell}$-quotient of any model yields a finite model (usually not a tree model)

generalises to some extensions
but not, in this simple form, e.g. to GF ($\rightarrow$ Grädel, 1999)

expressive completeness of modal logics

... relative to FO, consider

FO/$\sim$ $:=$ $\left\{\begin{array}{l}\text{the classes of } \sim\text{-invariant FO-properties of} \\ \text{(just finite, or all) ptd Kripke structures}\end{array}\right.$

remark:
semantic classes corresponding to undecidable conditions like $\sim$-invariance are at the heart of classical 'preservation theorems', which really concern the quest for **syntactic representation**

in this case, the positive answer underpins the role of ML, twice:

FO/$\sim$ $\equiv$ ML **classically, van Benthem** (1983)
FO/$\sim$ $\equiv$ ML **in fmt, Rosen** (1997)

expressive completeness: FO/$\sim$ $\equiv$ ML

it suffices to show that for $\varphi(x) \in$ FO
$\sim$-invariance implies $\sim^\ell$-invariance for some finite level $\ell \in \mathbb{N}$

a non-classical compactness property (!)

then $\varphi \equiv \varphi' \in$ ML by Ehrenfeucht–Fraïssé:
ML-definability of $\sim^\ell$-classes & finite index

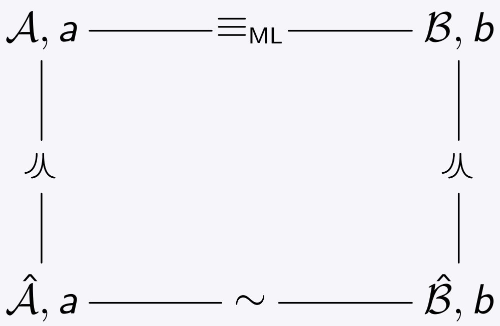
NB: two, a priori independent, readings: classical & fmt

expressive completeness: generic classical approach

$\sim$-invariance $\Rightarrow$ $\sim^{\ell}$-invariance for some ℓ

$(*)$

classical compactness argument with upgrading along $\equiv_{\mathsf{FO}}$-axis
through Hennessy–Milner property for ω-saturated structures



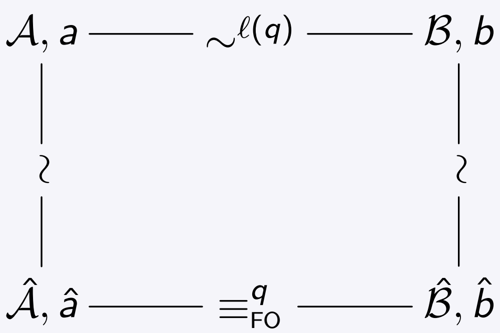
ω-saturated extns

elegant and smooth, but no information regarding target ℓ
and not an option for fmt version

expressive completeness: a constructive approach

$\sim$-invariance $\Rightarrow$ $\sim^{\ell}$-invariance for some ℓ

upgrading along $\sim$-axis of $\sim^{\ell(q)}$ $(\equiv^{\ell}_{\mathsf{ML}})$ to $\simeq^{q}$ $(\equiv^{q}_{\mathsf{FO}})$
through $\sim$-preserving model transformations



bisimilar companions

more constructive, potentially suitable for fmt,
also yielding information regarding $\ell(q)$

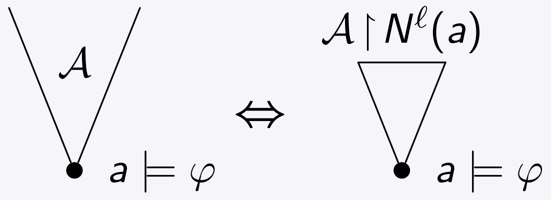
expressive completeness: FO/$\sim$ $\equiv$ ML

a simple argument (good classically & fmt)
using the locality of FO/$\sim$ & Ehrenfeucht–Fraïssé

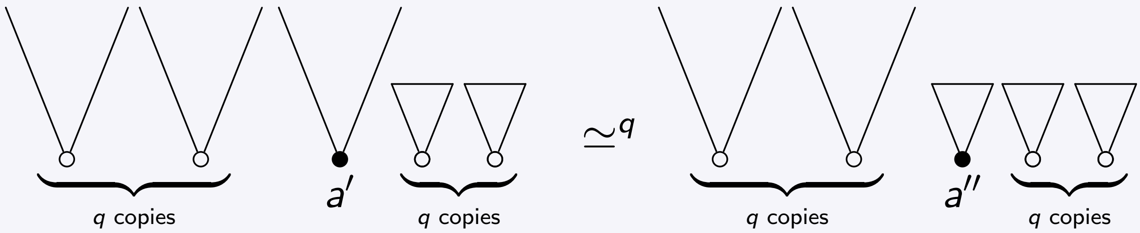
$\sim$-invariance $\Rightarrow$ $\sim^{\ell}$-invariance for $\ell = 2^q - 1$

(optimal)

show that



in q-round FO game on:



a more generic constructive approach

upgrading in

$$\begin{array}{ccc} \mathcal{A}, \boldsymbol{a} & \text{———} \sim^{\ell(q)} \text{———} & \mathcal{B}, \boldsymbol{b} \\ \wr & & \wr \\ \hat{\mathcal{A}}, \hat{\boldsymbol{a}} & \text{———} \equiv^q_{\mathrm{FO}} \text{———} & \hat{\mathcal{B}}, \hat{\boldsymbol{b}} \end{array}$$

requires (finite) model transformations $\mathcal{A}/\mathcal{B} \longmapsto \hat{\mathcal{A}}/\hat{\mathcal{B}}$ that are

- compatible with bisimulation:
  ideally want $\approx$ coverings (for symmetry & homogeneity)
- suitable to eliminate all obstacles for $\simeq^q$ ($\equiv^q_{\mathrm{FO}}$)
  that are *not controlled* by any level of $\sim^{\ell}$:

  need to avoid short cycles **& small multiplicities**

part II: variations, generalisations & challenges

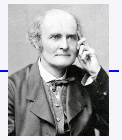
in this part (survey style):

- **technical variations: finite bisimilar coverings**
  avoiding short cycles in graph & hypergraph coverings
  in products with finite Cayley graphs
  … for dealing with global and guarded bisimulation

- **classically beyond FO to MSO:** Janin–Walukiewicz
  … and a big **?** in finite model theory

- **essentially modal variations, within & beyond FO:**
  … team & inqusitive semantics, common knowledge

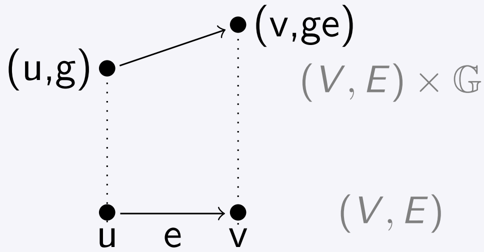
combinatorics of finite coverings



for local acyclicity in bisimilar coverings

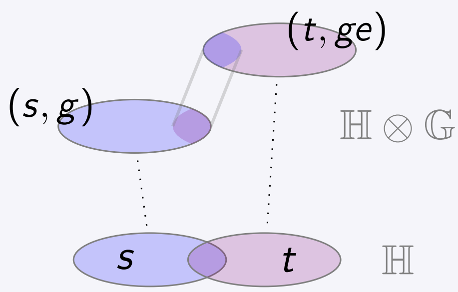
of **Kripke frames** (=graphs):

in products with Cayley graphs of groups w/o short generator cycles



of **guarded frames** (=hypergraphs):

in products with Cayley graphs of groups w/o short coset cycles
much trickier – why?



… and the construction of finite groups (better still: groupoids) that avoid certain patterns (equalities, relations) is a non-trivial algebraic-combinatorial challenge (with further applications)

from FO to MSO

theorem (Janin–Walukiewicz, 1996)

MSO/$\sim$ $\equiv$ L_μ

modal μ-calculus L_μ is expressively complete for the class of all $\sim$-invariant MSO-definable properties of pointed Kripke structures

proof based on

(1) tree model property (for any $\sim$-invariant phenomenon!)

(2) analysis of MSO model-checking by tree automata

OPEN: status in finite model theory
where neither (1) nor (2) applies, so that
known finite coverings do not seem to help

from MSO to GSO

joint work with Achim Blumensath & Erich Grädel

via analysis of game trees for guarded bisimulation, guarded tree unfoldings, and reduction to Janin–Walukiewicz get

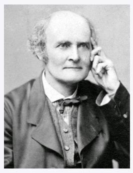
$\rightsquigarrow$ $\boxed{\mathrm{GSO}/\sim_g \;\equiv\; \mu\mathrm{GF}}$
over the class of all guarded structures

again: classical setting only!

essentially modal variations within FO (1)

joint work with Anuj Dawar

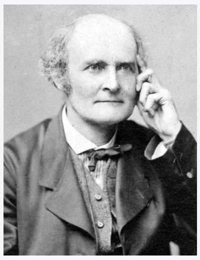
- global finite coverings allow for local acyclicity (and finitely boosted branching) throughout



$\rightsquigarrow$ $\boxed{\mathrm{FO}/\!\approx \;\equiv\; \mathrm{ML}[\forall, -]}$

for classical & fmt analogue of van Benthem–Rosen

- restrictions to several relevant classes of (finite) frames: reflexive, irreflexive, symmetric ... as you would expect



- quite different: bisimilar hypergraph coverings based on coset-acyclicity in Cayley graphs

$\rightsquigarrow$ $\boxed{\mathrm{FO}/\!\sim_g \;\equiv\; \mathrm{GF}}$ (O_2003)

for classical & fmt analogue of van Benthem–Rosen

essentially modal variations within FO (2)

joint work with Anuj Dawar

- over rooted transitive frames (which defeat locality):

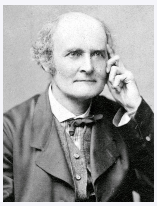
$\rightsquigarrow$ $\boxed{\mathrm{FO}/\!\sim \;\equiv\; \mathrm{ML}[^*] \;\equiv\; \mathrm{MSO}/\!\sim}$

over finite or wellfounded rooted transitive frames

(finite) Löb and Grzegorczyk frames also motivated by information & proof theory

- through global finite coverings for multi-agent S5-frames:

equivalence classes (information states) $\leftrightsquigarrow$ hyperedges with pre-processed simple overlaps



$\rightsquigarrow$ $\boxed{\mathrm{FO}/\!\sim \;\equiv\; \mathrm{ML}}$

over (finite) multi-agent epistemic S5 models

motivated by knowledge representation

non-classically beyond FO (1): team semantic ML

treat sets X of worlds in Kripke structures as information states
arbitrary rather than relationally encoded subsets X

- bisimulation $\rightsquigarrow$ team bisimulation (element-wise match of sets)

- basic team ML (with team disjunction & just nnf negation)
  is "flat" with standard translations $\forall x(x \in X \rightarrow \varphi(x))$,

  hence too weak to cover all $\sim$-invariant team properties
  that are FO-definable in the form $\psi(X)$ (FO^{T}-definable)

- augmented by strict negation, get ML[non] with

  $\rightsquigarrow$ $\boxed{\mathrm{FO}^{\mathsf{T}}/\!\sim \;\equiv\; \mathrm{ML[non]}}$

  full team-semantic analogue of van Benthem–Rosen
  with 'constructive' proof lifted to (scattered) teams

non-classically beyond FO (2): inquisitive ML

joint work with Ivano

inquisitive Kripke frames give worlds access to sets of
information states rather than sets of worlds

$\rightsquigarrow$ one level up & akin with team semantic concepts

- inquisitive modal logic $\textsc{InqML}$ extends basic (team) ML
  and defines persistent state properties that are (obviously!)
  invariant under the inquisitive variant of bisimulation

- natural 2-sorted relational encodings of models give FO
  access to some MSO-features, and in this context

  $\rightsquigarrow$ $\boxed{\mathrm{FO}^{\downarrow}/\!\sim \;\equiv\; \textsc{InqML}}$
  over (finite) relational inquisitive models

  full inquisitive analogue of van Benthem–Rosen
  over non-elementary classes of relational structures,
  on FO/MSO borderline esp. in the epistemic S5 version (!)

non-classically beyond FO (3): common knowledge

joint work with Felix Canavoi

common knowledge logic ML[CK]:

multi-modal S5 with 'common knowledge' modalities
$\Box_\alpha$ for sets α of agents

intuition: "among α, everybody knows that everybody knows that everybody knows that ..." (ad infinitum)

- the new $\Box_\alpha$ is the box modality for $R_\alpha = \mathbf{TC}(\bigcup_{i\in\alpha} R_i)$

  beyond FO due to non-elementary nature of TC (!)

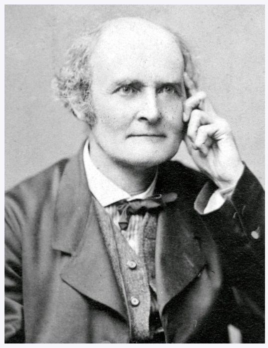
  but with the usual standard translation into FO over the richer non-elementary class of CK-frames with the new R_α

non-classically beyond FO (3): ML over CK-frames

ML[CK] is just ML over CK-frames:
S5-frames with induced equivalences R_α

which really seem to defeat locality!

where once more
Cayley helps a lot



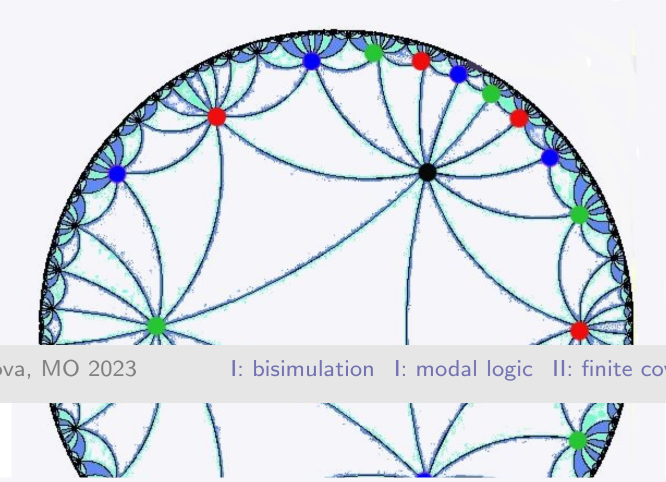
- need tractable forms of local acyclicity, simultaneously at all levels α (at nested levels of granularity)
- using finite bisimilar coverings in products with Cayley graphs of finite groups w/o short coset cycles, can show:

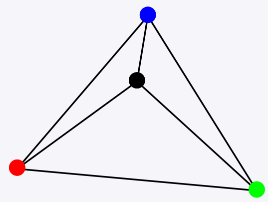
  $\rightsquigarrow$ $\boxed{\mathrm{FO}/\!\sim\ \equiv\ \mathrm{ML}\ \equiv\ \mathrm{ML[CK]}}$
  over the class of all (finite) CK-models

  full analogue of van Benthem–Rosen
  in a very non-classical setting

kind of a summary

- forms of bisimulation reflect what matters **(up to what?)**
- bisimulation (generic E–F) as the back&forth **(how similar?)**
- variations on modal accessibility **(access to what?)**
- semantic characterisations **(what up to what?)**
- bisimilar coverings & model transformations **(combinatorics!)**





some pointers my hobby horses

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