IRTG 1529 Winter Seminar and Klausurtagung

Fluids and Snow

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Resolvent estimates for the Stokes equations in spaces of bounded functions

K. Abe, University of Tokyo

The Stokes equation is well understood in the L^p-setting for a large class of domains including bounded and exterior domains with smooth boundaries provided $1 < p < \infty$. The situation is different for the case $p = \infty$ since in this case the Helmholtz projection does not act as a bounded operator anymore. Nevertheless, it was recently shown by a contradiction argument that the Stokes operator generates an analytic semigroup on L^∞-type spaces for a large class of domains. In this talk, we present a new approach as well as new a priori estimate to the resolvent Stokes equation. They in particular implies that the Stokes operator generates an analytic semigroup of angle $\pi/2$ on L^∞-type spaces for a large class of domains. This talk is based on a joint work with Y. Giga and M. Hieber.

Analysis of the Stokes equation in a layer domain

L. von Below, TU Darmstadt

The Stokes equation in a layer domain $\mathbb{R}^n \times (0, 1)$ has been studied extensively in an L_p-setting for $1 < p < \infty$ by, among others, Abe and Shibata, Abels, Beale, Nishida and Solonnikov. However, concerning the endpoint cases L_1 and L_∞ comparably little seems to be known.

Aim of this talk is to present some new results concerning the Stokes equation and the corresponding resolvent problem in a layer domain in L_∞.

Shape Optimization with the Navier-Stokes-Equations

M. Fischer, TU Darmstadt

We consider the general setting of a shape-optimization problem with PDEs by introducing the concept of the perturbation of identity. After working out the abstract setting in general Banach spaces we apply the ideas to the instationary Navier-Stokes-Equations and present a result concerning the differentiability of the solution with respect to the transformations.

A Liouville theorem for the planer Navier-Stokes equations with the no-slip boundary condition and its application to a geometric regularity criterion

P.-Y. Hsu, University of Tokyo

In this talk, we establish a Liouville type result for a backward global solution to the Navier-Stokes equations in the half plane with the no- slip boundary condition. No assumptions on spatial decay for the vorticity nor the velocity field are imposed. We study the vorticity equations instead of the original Navier-Stokes equations. As an application, we extend the geometric regularity criterion for the Navier -Stokes equations in the three dimensional half space under the no-slip boundary

condition. This is a joint work with Yoshikazu Giga (University of Tokyo) and Yasunori Maekawa (Tohoku University).

The global Cauchy problem for nonlinear dispersive equations on modulation spaces

T. Kato

In this talk, we discuss time decay estimates for dispersive equations with non-homogeneous symbols on modulation spaces $M^s_{p,q}$ to obtain the global well-posedness of the Cauchy problems for nonlinear dispersive equations. As a result, we have a generalization of the result by B. Wang and H. Hudzik in 2007, which treated the Schrödinger equations with a nonlinearity of wider class.

Proof Mining in Nonlinear Analysis

U. Kohlenbach, TU Darmstadt

During the last 10 years a systematic use of proof-theoretic transformations to extract new information from proofs in nonlinear analysis has emerged. We will survey some of the key features involved as well as report on a few selected applications.

Proof Theory and PDEs

A. Koutsoukou-Argyraki, TU Darmstadt

Proof mining is a research program in proof theory initiated by G. Kreisel in the 50's with the original name "unwinding of proofs" that describes the process of logically analysing proofs in various branches of mathematics, so as to extract additional information that is implicit in the proofs. This information may be of quantitative or qualitative nature. These applications to mathematics are described as instances of logical phenomena by general logical metatheorems. The main focus of my research project is to apply proof mining to analysis and partial differential equations. In particular, I am currently working together with Ulrich Kohlenbach on determining rates of metastability for the strong convergence of the solutions of Cauchy problems of the form

$$u'(t) + A(u(t)) \ni f(t),\ t \in [0,\infty),\ f(\cdot) \in L^1(0,\infty,X)$$

$$u(0) = x,$$

to the zero z of A as $t \to \infty$, where X is a real Banach space and $A : X \to 2^X$ is an operator characterized by certain accretivity properties, by logically analysing the proofs in a paper by J. García-Falset.

Pressure - stabilized characteristics finite element scheme for the Peterlin viscoelastic model

H. Mizerová, University of Mainz

We consider the Peterlin viscoelastic model describing the behavior of some polymeric fluids. This system of equations models an unsteady motion of an incompressible viscoelastic fluid. The first aim of this talk is to present a pressure - stabilized characteristics finite element scheme for this model. The next part is dedicated to the error estimates. Stability and convergence results for the velocity, the pressure and the conformation tensor are shown.

Traces theorems for vector fields in special Lipschitz domains

S. Monniaux, Université Aix Marseille

Local in time unique existence theorem for the compressible fluid flow

M. Murata, Waseda University

In this talk, we consider the motion of viscous compressible barotropic fluid in a uniform $W_q^{3-1/q}$ domain with slip boundary condition. Our purpose is a local in time unique existence of solutions to nonlinear problem in the L_p in time and L_q in space framework. Our result is extension of Burnat and Zajaczkowski [1], which is proved in the L_2 framework. In order to show the purpose, we proved L_p-L_q maximal regularity for the linearized problem by $\mathcal{R}$-boundedness of solution operator to resolvent problem.

References

[1] M. Burnat and W. Zajaczkowski, *On local motion of a compressible barotropic viscous fluid with boundary slip condition*, Topological Methods in Nonlinear Analysis Journal of the Julinsz Schauder Center **10** (1997), 195–223.

P1/P1-stabilized characteristics finite element schemes for flow problems

H. Notsu, Waseda University

P1/P1-stabilized characteristics finite element schemes for flow problems are presented. Theoretical and numerical results are shown. The schemes are symmetric and employ a cheep P1/P1-element, which leads to small degrees of freedom.

L^∞-stability of traveling waves in a radiating gas model

M. Ohnawa, Waseda University

In this talk, we deal with an initial value problem to a model system of radiating gases proposed by Hamer:

$$u_t + uu_x + q_x = 0,$$
$$-q_{xx} + q + u_x = 0,$$

where $u(t,x)$ and $q(t,x)$ are real-valued functions for $t \geq 0$ and $x \in \mathbb{R}$ with

$$u(0,x) = u^0(x),$$
$$u^0(x) \to u_\pm, q(t,x) \to 0 \text{ as } x \to \pm\infty.$$

We are particularly concerned with the asymptotic stability in L^∞ of traveling wave solutions to this system for piecewise smooth initial data. It is known this system admits discontinuous traveling waves if the shock strength $\delta_S := u_- u_+$ is strictly above (supercritical) a threshold value of $\sqrt{2}$, while if it is below (subcritical) or equal to (critical) $\sqrt{2}$, traveling waves are continuous. We prove all subcritical shock waves are stable to small perturbations whereas for the critical shock wave certain types of perturbations are found to cause a blowup of the first order derivative of the solution whatever small the perturbations may be. The stability of subcritical shock waves is robust in the sense that it is not affected by possible collisions of discontinuities contained in initial data and the solution converges uniformly to a traveling wave beyond such events. We also give sufficient conditions for both occurrence and nonoccurrence of collisions of discontinuities.

Angle of bounded analyticity of the Stokes semigroup on L^∞_σ

M. Rapp, TU Darmstadt

We consider the Stokes equations

$$\begin{aligned} u_t - \Delta u &= -\nabla \pi_u && \text{in} && (0,T)\times\Omega \\ \operatorname{div} u &= 0 && \text{in} && (0,T)\times\Omega \\ u &= 0 && \text{on} && (0,T)\times\partial\Omega \\ u(t=0) &= u_0 \end{aligned}$$

in an exterior domain $\Omega \subset \mathbb{R}^d$ ($d \geq 3$) with C^m-boundary, where $m > \max\{3, \frac{d}{2}\}$. In recent years there have been a lot of results concerning the corresponding semigroup on $L^\infty_\sigma(\Omega)$, the space of bounded, weakly divergence free functions. Abe, Giga, Hieber proved that the semigroup is analytic of angle $\frac{\pi}{2}$, Maremonti that it is a bounded semigroup and Maremonti, Hieber that it is a bounded analytic semigroup. Our aim is to prove that the semigroup is bounded analytic of angle $\frac{\pi}{2}$.

Strong stability of 2D viscoelastic Poiseuille-type flows

J. Sauer, K. Schade, TU Darmstadt

We investigate L_p stability issues of small viscoelastic Poiseuille-type flows in two dimensions stemming from a model considered in Fang-Hua Lin, Chun Liu, and Ping Zhang (2005). We

show local existence of perturbed flows of locally-in-time existing Poiseuille-type flows and global existence of the peturbed flows whenever the initial perturbation is small enough. In this case the perturbed flow decays exponentially. In all cases, the perturbations immediately regularize.

Linear stability analysis of plane Couette flow for the Oldroyd-B fluid

B. She, University of Mainz

We investigate the linear stability of plane Couette flow for the Oldroyd-B fluid using a characteristic finite element method. For computing the eigenvalues, an Arnoldi-based algorithm is implemented. The perturbation decays in our calculation and no instability has shown up.

On the Computational Complexity of Poisson's Problem

F. Steinberg, TU Darmstadt

Real complexity theory expands discrete computability and complexity theory to non-countable structures as the real numbers and functions thereon. It is well known, that there is a close relationship between properties of important operators on real functions in this framework and strictness of inclusions of discrete complexity classes. We investigate the solution operator of the Dirichlet problem for Poisson's equation on a ball and show that it is essentially of the same difficulty as integration.

Global Stability Analysis of Flow around Cylinder and Spheroid

A. Tezuka, Waseda University

The Global Linear Stability Analysis clarifies the onset of the change of steady flowfield. In the latest study, the method is applied for two phase film flow. In this presentation, the numerical scheme with the combination of Arnoldi's method for approximate eigensystem analysis is explained and the example of the flowfield configuration of two-dimensional cylinder and three-dimensional spheroid are demonstrated. In the wake of the flow past two-dimensional cylinder, a periodic oscillation is observed in the wake flow when the Reynolds number is over 46. By the application of global stability analysis, it is clarified that, the temporal amplification factor changes from negative value to positive value at the critical Reynolds number. The flow around a spheroid at angle of attack becomes steady asymmetric in the certain range of Reynolds number and angle of attack. By the application of global stability analysis, it is clarified that, non-oscillatory, non-axisymmetric flow (in the case of zero angle of attack) and non-oscillatory asymmetric flow (in the case of nonzero angle of attack) are observed in a certain range of the freestream Reynolds number and angle of attack.

Global Well-posedness of an Inviscid Three-dimensional Pseudo-Hasegawa-Mima-Charney-Obukhov Model

E. S. Titi, Weizmann Institute of Science and University of California, Irvine

The three-dimensional inviscid Hasegawa-Mima model is one of the fundamental models that describe plasma turbulence. The same model is known as the Charney-Obukhov model for stratified ocean dynamics, and also appears in literature as a simplified reduced Rayleigh-Bénard convection model. The mathematical analysis of the Hasegawa-Mima and of the Charney-Obukhov equations is challenging due to the absence of any smoothing viscous terms, as well as to the presence of an analogue of the vortex stretching terms. In this talk, we introduce and study a model which is inspired by the inviscid Hasegawa-Mima and Charney-Obukhov models, which we call a pseudo-Hasegawa-Mima model. The introduced model is easier to investigate analytically than the original inviscid Hasegawa-Mima model, as it has a nicer mathematical structure. The resemblance between this model and the Euler equations of inviscid incompressible fluids inspired us to adapt the techniques and ideas introduced for the two-dimensional and the three-dimensional Euler equations to prove the global existence and uniqueness of solutions for our model. This is in addition to proving and implementing a new technical logarithmic inequality, generalizing the Brezis-Gallouet and the Berzis-Wainger inequalities. Moreover, we prove the continuous dependence on initial data of solutions for the pseudo-Hasegawa-Mima model. These are the first results on existence and uniqueness of solutions for a model that is related to the three-dimensional inviscid Hasegawa-Mima equations. (This part is a joint work with C. Cao and A. Farhat.)

If time allows I will also present some recent results concerning the two-dimensional Bousinessq equations with an-isotroic viscosity and without diffusion. (This part is joint work with A. Larios and E. Lunasin.)

Time periodic problem of the the compressible Navier-Stokes equation

K. Tsuda, Kyushu University

We investigate the time periodic problem of the compressible Navier-Stokes equation in an unbounded domain with a time periodic external force. As the first step of analysis, we consider the case of the whole space $\mathbb{R}^n$.

Ma, Ukai, and Yang (2010) [1] showed that for $n \geq 5$, if the external force is small, then there exists a time-periodic solution. They also showed that the time-periodic solution is stable under sufficiently small perturbation and they obtain the time decay rate of the perturbation.

We first consider the case such that the external force satisfies some symmetry. Then, we show that for $n \geq 3$, if the external force is small in some weighted Sobolev space, then there exists a time-periodic solution. Furthermore, we show that the time-periodic solution is stable under sufficiently small perturbation and we obtain the time decay rate of the perturbation.

On the other hand, as for the case such that the external force doesn't necessarily satisfy the symmetry, we see that for $n \geq 3$, if the external force is small enough in some weighted L^∞ space, then we obtain the existence and the energy stability of a time-periodic solution.

A part of this work was studied in a joint work with Professor Kagei, in Kyusyu University.

References

[1] H.Ma, S.Ukai, and T.Yang,(2010) Time periodic solutions of compressible Navier-Stokes equations, Journal of Differential Equations, Volume 248, Issue 9, pp. 2275–2293.

New approach to the Hadamard variational formual for the Green function of the Stokes equations

E. Ushikoshi, Tamagawa University

We consider the Hadamard variational formula for the Green function of the Stokes equations on the bounded domain Ω with the smooth boundary $\partial\Omega$. Under the perturbation with preserving its volume and keeping its topological type, we establish a more refined proof of its formula of the Green function not only for the first variation but also the second variation for both velocity and pressure. Our method gives a new systematic proof of the Hadamard variational formula, which enables us to deal with the higher derivatives with respect to the perturbation of domains.

Martingale Solutions for some Stochastic Euler Equations

S. Yokoyama, University of Tokyo

Martingale solution is known as the concept of one of the solutions of stochastic partial differential equation. In this talk, I will introduce the overview of martingale solutions. Then I will introduce the martingale solution to some stochastic Euler equation derived by the variational problem discussed by V.I. Arnold.

On the Navier-Stokes-Maxwell-Stefan system for compressible mixtures

E. Zatorska, Paris