On the ECE Lemma

Gerhard W. Bruhn, Darmstadt University of Technology, Germany

Februrary 18, 2009

(our comments in **blue**, quotations from Evans' texts in ***black italics***)

For the first time the 'ECE Lemma' appeared in 2003 in the FOPL paper [1] THE EVANS LEMMA OF DIFFERENTIAL GEOMETRY . There the essential lines are

. . . where the scalar curvature R_1 is defined by

$$-R_1\, q^a{}_\lambda := (D^\mu \omega_\mu{}^a{}_b)\, q^b{}_\lambda - (D^\mu \Gamma^\nu{}_{\mu\lambda})\, q^a{}_\lambda\,. \qquad ([1],44)$$

. . .

Define the scalar curvature R_2 by

$$-R_2\, q^a{}_\lambda := -\Gamma^\nu{}_{\nu\mu}\omega^{\mu a}{}_b\, q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu}\, \Gamma^\mu{}_\lambda{}^\nu\, q^a{}_\nu\,, \qquad ([1],48)$$

to obtain the Evans lemma

$$\Box q^a{}_\lambda = R\, q^a{}_\lambda, \qquad ([1],49)$$

where

$$R = R_1 + R_2. \qquad ([1],50)$$

The *problem* with these definitions is that there are *two indices* on the left hand side of the eqs. ([1],44) and ([1],48) running each over 0,1,2,3. Therefore each equation ([1],44) and ([1],48) de facto represents 4×4 = 16 definitions of R_1 and R_2 respectively. Since the author does not worry about the compatibility of his 2×16 definitions we must conclude that in reality the quantities R_1 and R_2 (and so R, required for the ECE Lemma ([1],49)) are *not at all well-defined* by the eqs. ([1],44) and ([1],48).

Thus there is no valid proof of of the ECE Lemma (49) in the paper [1].

Later on the author tried to give other proofs for the ECE Lemma. Especially, he now gave an explicit representation [2,3] of the quantity R, which gives opportunity to check whether Eq. ([1],49) is satisfied:

In the web note [3] we find a simplification of the consideration [1,2], now supplied with an attempt to close the gap mentioned above:

. . . i.e.

$$\Box\, q^a{}_\lambda = \partial^\mu\, (\Gamma^\nu{}_{\mu\lambda}\, q^a{}_\nu - \omega^a{}_{\mu b}\, q^b{}_\lambda) \qquad ([3],8)$$

Now define

$$R = q_a^{\lambda}\, \partial^{\mu} (\Gamma^{\nu}_{\mu\lambda}\, q^a_{\ \nu} - \omega^a_{\ \mu b}\, q^b_{\ \lambda})$$ ([3],9)

and use

$$q^a_{\ \lambda} q^{\lambda}_{\ a} = 1$$ (error! = 4 would be correct.) ([3],10)

to find by using Eq. ([3],10)

$$\begin{aligned} \Box\, q^a_{\ \lambda} &=_{(8)} 1 \cdot \partial^{\mu} (\Gamma^{\nu}_{\mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda}) \\ &=_{(10)} [q^a_{\ \lambda} q^{\lambda}_{\ a}] \cdot \partial^{\mu} (\Gamma^{\nu}_{\mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda}) \\ &=_{(?)} q^a_{\ \lambda} \cdot [q^{\lambda}_{\ a}\, \partial^{\mu} (\Gamma^{\nu}_{\mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda})] \\ &=_{(9)} R\, q^a_{\ \lambda}\,. \end{aligned}$$ ([3],11)

The choice of the indices ${}^a{}_\lambda$ and ${}^\lambda{}_a$ in $[q^a_{\ \lambda} q^{\lambda}_{\ a}]$ as dummy indices is ***inadmissible*** due to the occurrence of a and λ in the next expression (...). If other correct dummies were used instead then the next step **=(?)** would be ***impossible*** . Hence the calculation ([3],11) is *invalid:*

There is no proof for the quantity R of Eq. ([3],9) being a solution of Eq. ([1],49). However, the calculation above is quite following the author's ideas of New Math.

Summary: There is no valid proof of the ECE Lemma.

References

[1] M.W. Evans, *THE EVANS LEMMA OF DIFFERENTIAL GEOMETRY*, http://www.aias.us/documents/uft/a7thpaper.pdf

[2] M.W. Evans, *Proof Of The Evans Lemma From The Tetrad Postulate*, http://www.aias.us/documents/uft/a35thpaper.pdf

[3] M.W. Evans, *Some Key Derivations: 1. Derivations of the Lemma* , http://www.atomicprecision.com/blog/wp-filez/akeyderivations1and2.pdf

[4] G.W. Bruhn, *The ECE Lemma, Comments*, http://www.mathematik.tu-darmstadt.de/~bruhn/ECE-Lemma011007.html

[5] G.W. Bruhn, *A remark on Evans' recent web article on the ECE Lemma*, http://www.mathematik.tu-darmstadt.de/~bruhn/toMWE240507.html