Evans' "3-index, totally antisymmetric unit tensor"

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Though in the literature the n-dimensional Levi-Civita-symbol is defined only in n-dimensional space per permutations of the n coordinates, M.W. Evans defines a 3-index-$\in$-tensor in 4-dimensional spacetime in [1,(2.51)] by refering to the 4-dimensional Levi-Civita-symbol $\in_{sijk}$ for applications in context with *Local Lorentz Transforms* (LLTs) of the tetrad:

(1.1) $$\left.\begin{matrix} \in_{123} = \in_{231} = \in_{312} = 1 \\ \in_{132} = \in_{213} = \in_{321} = -1 \end{matrix}\right\} := \in_{0ijk} \qquad (i,j,k = 1,2,3 \text{ permuted})$$

(1.2) $$\left.\begin{matrix} \in_{023} = \in_{302} = \in_{230} = 1 \\ \in_{032} = \in_{320} = \in_{203} = -1 \end{matrix}\right\} := \in_{i1jk} \qquad (i,j,k = 0,2,3 \text{ permuted})$$

(1.3) $$\left.\begin{matrix} \in_{013} = \in_{301} = \in_{130} = 1 \\ \in_{031} = \in_{310} = \in_{103} = -1 \end{matrix}\right\} := \in_{ij2k} \qquad (i,j,k = 0,1,3 \text{ permuted})$$

(1.4) $$\left.\begin{matrix} \in_{012} = \in_{120} = \in_{201} = 1 \\ \in_{021} = \in_{102} = \in_{210} = -1 \end{matrix}\right\} := \in_{ijk3} \qquad (i,j,k = 0,1,2 \text{ permuted})$$

Since these 4 cases are disjoint we have

(2) $$\in_{ijk} := \in_{0ijk} + \in_{i1jk} + \in_{ij2k} + \in_{ijk3} = \in_{0ijk} - \in_{1ijk} + \in_{2ijk} - \in_{3ijk}\,.$$

Let $\in'_{i'j'k'}$ denote Evans' "3-index, totally antisymmetric unit tensor" in another coordinate system $x^{0'}$, $x^{1'}$, $x^{2'}$, $x^{3'}$:

(2') $$\in_{i'j'k'} = \in_{0'i'j'k'} - \in_{1'i'j'k'} + \in_{2'i'j'k'} - \in_{3'i'j'k'}\,,$$

and let $a^{i'}{}_{i'} = \partial x^{i'}/\partial x^{i}$ be the corresponding transformation coefficients of the coordinate transform $x^{i'} = x^{i'}(x^i)$. Since we know the transform of the Levi-Civita-symbols:

(3) $$\in_{s'i'j'k'} = a^{s}{}_{s'}\, a^{i}{}_{i'}\, a^{j}{}_{j'}\, a^{k}{}_{k'} \in_{sijk}$$

we can explicitly determine the transformation behaviour of Evans' symbols:

(4) $\in_{i'j'k'} = (a^s{}_{0'} - a^s{}_{1'} + a^s{}_{2'} - a^s{}_{3'}) \in_{sijk} a^i{}_{i'} a^j{}_{j'} a^k{}_{k'}$

which in case of correct tensor transformation behaviour of the Evans symbols should agree with

(5) $\in_{i'j'k'} = \in_{ijk} a^i{}_{i'} a^j{}_{j'} a^k{}_{k'}$.

Comparison of (5) and (6) yields the condition

(6) $(a^s{}_{0'} - a^s{}_{1'} + a^s{}_{2'} - a^s{}_{3'}) \in_{sijk} = \in_{ijk} = \in_{0ijk} - \in_{1ijk} + \in_{2ijk} - \in_{3ijk}$,

where on the right hand side at most one term can appear. This means that for each value of s we have

(7) $a^s{}_{0'} - a^s{}_{1'} + a^s{}_{2'} - a^s{}_{3'} = (-1)^s \qquad (s=0,1,2,3)$.

These conditions are fulfilled for the identity transform but even not for spatial rotations, all the more NOT for general LLTs.

Evans' "3-index, totally antisymmetric unit tensor" in 4D does not transform *covariantly*,
$\in_{ijk}$ is NO TENSOR.

In that sense the defining Eqs. (1.1-4) or (2) of Evans' "3-index $\in$-tensor in 4D" don't yield a tensor. The consequences of the *missing tensor property* for Evans' ECE theory are listed in [2].

References

[1] M.W. Evans, Geodesics and the Aharonov-Bohm effect in ECE theory, http://www.aias.us/documents/uft/a56thpaper.pdf

[2] G.W. Bruhn, Comments on Evans' Duality, http://www.mathematik.tu-darmstadt.de/~bruhn/EvansDuality.html