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Proseminar 'extra' 2018:

Symmetries in Algebra & Combinatorics

This proseminar is aimed at students in the first two semesters who are keen to explore some mathematical themes outside the regular first-year curriculum, and have an interest in the broad topic of *symmetries in algebra & combinatorics*. It offers an opportunity to explore some selected topic, to present it in front of the group, to engage in discussions and thus to gain and share mathematical knowledge and insights. Focus is on guided exploration and curiosity rather than on any set curriculum.

Instead of CP: an individual certificate for successful participants
Interested?—send email to otto@mathematik.tu-darmstadt.de

Potential core topics:

- symmetries & groups, symmetry groups, permutation groups
- Cayley graphs und -groups: algebraic constructions
- local vs. global symmetries: inverse semigroups & groupoids
- homogeneity in the random graph and 0-1-laws
- graph isomorphy and combinatorial approximations
- graph isomorphy and algebraic approximations

Available either in English or in German, dependent on participants' wishes.

Symmetrien in Algebra & Kombinatorik

Dieses Proseminar wendet sich an Studierende der ersten beiden Semester, die sich außerhalb des regulären Studienplans fuer weitergehende Fragestellungen interessieren und Lust haben, sich mit dem Thema *Symmetrien in Algebra & Kombinatorik* zu beschäftigen. Das Proseminar gibt die Gelegenheit, sich anhand verschiedener Quellen in ein ausgewähltes Thema einzulesen, einen Vortrag vorzubereiten und in der Gruppe zu präsentieren. Ziel ist das gemeinsame Erarbeiten interessanter Kenntnisse und Perspektiven, wobei nicht ein vorgebenes Lernpensum sondern das fachliche Interesse und die Neugierde der Teilnehmer(innen) im Vordergrund stehen sollen.

Statt CP: Bescheinigung über erfolgreiche Teilnahme
Interessent(inn)en melden sich bitte persönlich per Email an (s.o.)

Suggested Topics

Symmetries & groups, symmetry groups, permutation groups: examples (algebra, geometry), automorphism groups of structures; e.g. Moebius transformations; Cayley graphs; theorems of Frucht, Babai, Bouwer.

Selected passages from textbook sources together with: R. Frucht: Herstellung von Graphen mit vorgegebener abstrakter Gruppe, Compositio Mathematica, 6, S. 239–250, 1938, and Graphs of degree three with a given abstract group, Can. J. Math., 1, pp. 365-378, 1949; L. Babai: Representation of permutation groups by graphs, Combinatorial Theory and Its Applications I, pp. 55-80, 1969; Bouwer: Section graphs for finite permutation groups, J. Comb. Theo., 6, pp. 378–386, 1969.

Cayley graphs and -groups: algebraic constructions: e.g. construction of homogeneous finite groups of large girth, and possible generalisations.

Selected passages from: N. Alon: Methods from higher algebra, in Handbook of Combinatorics; Godsil: Tools from linear algebra, in Handbook of Combinatorics; M. Otto, JACM, 2012 (§ 2).
Possibly also G. Margulis: Explicit construction of graphs without short cycles and low density codes, Combinatorica, 1982; W. Imrich: Explicit construction of regular graphs without small cycles, Combinatorica, 1984.

Local vs. global symmetries: local symmetries and partial isomorphisms; generalisations of the concept of groups (motivation & examples): pseudo-groups, inverse semigroups, groupoids; elementary combinatorial proof of an extension property due to Hrushovki; back&forth systems as approximations to isomorphy (e.g. Cantor's theorem for $(\mathbb{Q}, <)$); Fraïssé limits.

Selected passages from: Lawson: Inverse Semigroups (§ 1); B. Herwig, D. Lascar: Extending partial isomorphisms and the profinite topology on free groups (§ 4.1), Transactions of the AMS, 352, pp. 1985–2021, 2000; W. Hodges: A Shorter Model Theory (§ 3.2).

Homogeneity in the random graph and 0-1–laws: concept of random structures, almost sure properties of finite graphs; concrete realisations of the Rado-graph and its combinatorial and logical properties.

Selected passages from: J. Spencer: Logic and random structures, in: Finite Model Theory and Its Applications. M. Otto: Lecture Notes on Finite Model Theory (§ 3).

Graph isomorphism and combinatorial approximations: graph isomorphism and orbits under automorphism groups of graphs; *colour refinement* (Leman–Weisfeiler), generalisations and limitations.

Selected passages from: M. Grohe: Descriptive Complexity, Canonisation, and Definable Graph Structure Theory, (§ 3, esp. pp. 76–84).
J. Cai, M. Fürer, N. Immerman: An optimal lower bound on the number of variables for graph identification, Combinatorica, 288, pp. 389–410, 1992.

Graph isomorphism and algebraic approximations: relaxations of algebraic isomorphism condition and its link with *colour refinement*; fractional isomorphism, results of Tinhofer and Ramana–Scheinerman–Ullman.

Selected passages from: G. Tinhofer: Graph isomorphism and theorems of Birkhoff type, Computing 86; E. Scheinerman, D. Ullmann: Fractional Graph Theory (chapter 6); M. Grohe, M. Otto: Pebble games and linear equations, J. Symb. Logic, 2015; M. Grohe: Descriptive Complexity, Canonisation, and Definable Graph Structure Theory, (§ 3, esp. pp. 76–84); H. Dell, M. Grohe, G. Rattan: Weisfeiler-Leman meets homomorphisms, 2018; L. Lovasz: Operations with structures, Acta Mathematica Hungarica, 18, pp. 321–328, 1967.