Mini-Symposium: Geometric Fluid Mechanics

Abstracts

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Dual pairs in infinite dimensions with applications to fluid mechanics

The Euler equation describing ideal incompressible fluid flow is a geodesic equation on the group of volume preserving diffeomorphisms with right invariant L^2 metric. The EPDiff (or higher dimensional Camssa-Holm) equation is a geodesic equation on the group of all diffeomorphisms with right invariant H^1 metric.

The aim of this talk is to present the mathematical tools necessary for writing in a rigorous way the dual pair for Euler equation [1] and the dual pair for the EPDiff equation [2]. These are: differential forms on manifolds of functions, dual pairs of Poisson maps in infinite dimensions and central extensions of Lie algebras of vector fields.

The first dual pair requires a volume manifold S and a symplectic manifold M. It consists of two momentum maps on Emb(S,M): for the left action of the group of Hamiltonian diffeomorphisms of M and the right action of the group of (exact) volume preserving diffeomorphisms of S. When M is 2-dimensional, the left leg captures the motion of point vortices.

The second dual pair consists of two momentum maps on the cotangent bundle of the manifold of embeddings Emb(S,M): for the cotangent lifts of the left Diff(M) action and of the right Diff(S) action. Its left leg is the singular solution momentum map.

References

[1] J. E. Marsden and A. Weinstein, *Coadjoint orbits, vortices, and Clebsch variables for incompressible fluids*, Phys. D, 7, 305-323, 1983.

[2] D. D. Holm and J. E. Marsden, *Momentum maps and measure-valued solutions (peakons, filaments and sheets) for the EPDiff equation*, In The Breadth of Symplectic and Poisson Geometry, A Festshrift for Alan Weinstein, 203-235, Progr. Math., 232, J. E. Marsden and T. S. Ratiu, Editors, Birkhauser Boston, 2004.

Feride Tiglay (EPFL Lausanne)

Euler-Poincar equations on Lie groups and homogeneous spaces, their orbit invariants and applications to PDE.

We develop the necessary tools and prove abstract Noether theorems for Euler-Poncar equations on homogeneous spaces and for a generalized class of Euler-Poncar equations on Lie groups. Orbit invariants play an important role in this context and we use these invariants to prove global existence and uniqueness results for a class of PDE. This class includes integrable equations like Camassa-Holm, Degasperis-Procesi, ??CH and ??DP equations, as well as the geodesic equations with respect to Sobolev metrics on the group of diffeomorphisms of the circle.